A Higher Arithmetic

STONE-MALLORY
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BENJ. H. SANBORN & CO.

A HIGHER ARITHMETIC

BY

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PREFACE

This book has been prepared for use in high schools and in teacher-training schools. It has been the aim of the authors to meet three major objectives of a final course in arithmetic: (1) to show how to get skill, and to develop skill, in computation; (2) to develop the power to analyze a new arithmetical situation, to see the quantitative relations of the numbers given, and thus to discover what process or processes to apply; and (3) to give that information about personal, civic, business, and industrial problems that we meet in the everyday affairs of life, or meet in general reading.

Since the problems that arise in any specialized vocation are based upon the fundamental principles of arithmetic, but vary somewhat in the different vocations, the authors have sought to lay a solid foundation for the arithmetic work that will be met in the various vocations rather than to stress the work of any particular vocation. They hold that by obtaining a thorough grounding in the fundamental principles of arithmetic, by a mastery of rapid and accurate computation, and by power to interpret quantitative relations, one is much better fitted for any kind of arithmetical work than if he had merely developed skill in some particular type of work.

To develop speed and accuracy in computation, the book takes up every type of skill that enters into the four fundamental processes with whole numbers, fractions, and decimals. The practice exercises of each type should be used until the desired skill is obtained.

To develop power to see quantitative relations, a chapter is devoted to, "How to analyze and solve a problem." Most problems that we have to solve in life are met so often that we know at once what processes to apply. There are others, however, that are met so rarely that it takes very careful thinking to know what to do. This chapter contains problems that you will meet in the everyday affairs of life, and also problems rarely, if ever, met. The purpose of the second kind is to give you training in reasoning out what to do when you meet a new problem.

Most of the book, however, is devoted to that informational side of

the subject needed by all intelligent citizens in order to interpret those personal, social, industrial, and business problems that are frequently discussed, or which are met in general reading.

The authors assume that the users of this text have a knowledge of the arithmetic of the first eight grades, and hence they use many of the simpler concepts of per cent and measurement in problems before the formal treatment of these subjects is taken up in the text.

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A HIGHER ARITHMETIC

CHAPTER I

TESTING SKILLS IN FUNDAMENTAL OPERATIONS FOR SPEED AND ACCURACY

In all mathematics, there is constant need of the ability to perform the four fundamental processes of arithmetic accurately and with a reasonable degree of speed. You need to add, subtract, multiply, and divide whole numbers, fractions, decimals and sometimes compound numbers.

Compound numbers are such numbers as 2 ft. 8 in.; 3 gal. 1 qt.; 5 bu. 3 pk.; etc.

In this chapter, work in the four processes is grouped for review and drill. It is very necessary for you to understand what a process means and to be able to use it accurately before attempting to solve the problems.

Skill in computation comes through practice. While all problems that you solve require computation, to get practice in every skill it will be advisable to return to this chapter frequently.

The meaning of the letters is: E = excellent; G = good; F = fair. When a student finds that he is deficient in his work, he should practice on these exercises until a satisfactory standard has been attained.

THE ADDITION OF WHOLE NUMBERS

The numbers added are called addends and the result is called the sum. Addition is finding the sum of two or more numbers without counting.

Write the sums. Do not copy the numbers.

E = 2 min. **G = $2\frac{1}{2}$ min.** **F = 3 min.**

1.	2.	3.	4.	~~5.~~	~~6.~~
485	516	963	768	~~642~~	~~896~~
693	398	847	297	579	430
587	467	796	564	865	574
926	843	284	829	476	268
180	954	325	547	391	345

CHECKING ADDITION

Every computation to be of any use must be accurate. This means that all work must be checked.

To check addition, add a second time in the opposite direction. If the results do not agree, find your error.

Add the following and check the results. If you have more than one answer wrong, you need more practice.

E = 8 min. **G = 10 min.** **F = 12 min.**

NOTE TO TEACHER AND STUDENT. — The time given for these tests has been computed from the known range for such combinations. In written work, where several consecutive combinations must be made, the time ranges from about 30 to 50 combinations per minute. After the answer has been found, the work should be checked very carefully.

1.	2.	3.	4.	5.	6.	7.
296	831	209	625	375	908	416
398	935	491	287	152	470	863
764	247	647	536	609	735	248
587	472	362	169	926	153	748
386	619	728	847	748	617	382
297	350	953	491	483	849	625
564	705	835	918	831	526	591
381	928	974	723	517	284	930

8.	9.	10.	11.	12.	13.	14.
924	175	836	826	519	473	693
408	617	935	604	753	298	305
635	942	718	460	137	982	482
297	427	361	712	490	536	161
753	869	196	385	972	657	918
317	293	502	941	246	849	913
146	731	850	927	628	735	483
819	358	624	573	385	179	759

Checking Addition by Casting Out Nines

A second method of checking addition is by casting out nines.

The remainder found by dividing a number by 9 is the same remainder that will result from dividing the sum of the digits of that number by 9.

Thus, $524 \div 9$ will give the same remainder as that obtained from $(5 + 2 + 4) \div 9$.

This remainder is called the excess of nines.

The check depends upon the fact that

The excess of nines in the sum equals the excess in the sum of the excesses of the addends.

246	3	$2 + 4 + 6 = 12.$	*The sum of these two digits is 3, or the excess.*
735	6	$7 + 3 + 5 = 15.$	*The sum of these two digits is 6, or the excess.*
287	8	$2 + 8 + 7 = 17.$	*The sum of these two digits is 8, or the excess.*
630	0	$6 + 3 + 0 = 9.$	*When the sum is exactly divisible by 9, there is no excess.*
1898	8		

The sum of the excesses in the addends is $3 + 6 + 8 = 17$. *The sum of these two digits is* 8, *or the excess.*

The sum of the digits in the sum is $1 + 8 + 9 + 8 = 26$. *The sum of the digits of* 26 *is* 8, *or the excess.*

Add and check by casting out nines.

E = 4 min. **G = 5 min.** **F = 6 min.**

1.	**2.**	**3.**	**4.**	**5.**
6534	4362	5763	3465	3568
8765	8461	4611	2846	7281
9098	1789	7652	9762	4563
1467	6754	9875	8346	8712
5764	1768	6381	9581	9630
4381	7235	5763	4268	8145
2659	1763	4567	3405	6453
7544	7845	7692	9630	9036

THE ADDITION OF FRACTIONS

You know that only like things can be added. Hence you know that

Before fractions can be added they must all have the same denominator.

1. Find the sum of $\frac{3}{8}$, $\frac{7}{8}$, and $\frac{5}{8}$.

$\frac{3}{8}$ $\frac{7}{8}$ $\frac{5}{8}$ $\frac{15}{8} = 1\frac{7}{8}$	*Here 3, 7, and 5 were added just as if they had been like numbers of any kind.* *The sum $\frac{15}{8}$ was changed to $1\frac{7}{8}$ by dividing 15 by 8.*

2. Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$.

$\frac{1}{2} = \frac{4}{8}$ $\frac{3}{4} = \frac{6}{8}$ $\frac{7}{8} = \frac{7}{8}$ $\frac{17}{8} = 2\frac{1}{8}$	*Here you changed $\frac{1}{2}$ to $\frac{4}{8}$ by multiplying each term by 4. How was $\frac{3}{4}$ changed to $\frac{6}{8}$? If you can make the changes mentally and add without writing down the changed fractions, much time is saved.*

3. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.

$\frac{2}{3} = \frac{8}{12}$ $\frac{3}{4} = \frac{9}{12}$ $\frac{5}{6} = \frac{10}{12}$ $\frac{27}{12} = 2\frac{3}{12}$ $= 2\frac{1}{4}$	*In Exercise 2 the common denominator (8) was seen. Here you have to think of a number that will contain each of the given denominators. It is 12. Tell how each fraction was changed to 12ths.* *The $\frac{3}{12}$ in the sum was changed to $\frac{1}{4}$ by dividing each term by 3.*

Tests in Addition of Fractions

A

E = 5 min. G = 6 min. F = 7 min.

1.	2.	3.	4.	5.	6.	7.	8.
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$
$\frac{1}{4}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$

9.	10.	11.	12.	13.	14.	15.	16.
$\frac{2}{3}$	$\frac{1}{4}$	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{2}{3}$
$\frac{5}{6}$	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{1}{9}$	$\frac{5}{12}$	$\frac{7}{12}$

17.	18.	19.	20.	21.	22.	23.	24.
$\frac{1}{2}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{5}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{3}{4}$
$\frac{5}{16}$	$\frac{9}{16}$	$\frac{11}{12}$	$\frac{9}{16}$	$\frac{2}{9}$	$\frac{2}{3}$	$\frac{8}{9}$	$\frac{5}{6}$

25.	26.	27.	28.	29.	30.	31.	32.
$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$

33.	34.	35.	36.	37.	38.	39.	40.
$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{1}{4}$	$\frac{3}{8}$
$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{1}{16}$	$\frac{7}{16}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{9}{16}$

41.	42.	43.	44.	45.	46.	47.	48.
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{5}{12}$	$\frac{2}{3}$	$\frac{7}{8}$
$\frac{3}{4}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{7}{8}$	$\frac{5}{6}$
$\frac{5}{8}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{16}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{3}{4}$

Tests in Adding Mixed Numbers

B

E = 3 min. **G = $3\frac{1}{2}$ min.** **F = 4 min.**

1.	**2.**	**3.**	**4.**	**5.**	**6.**
$3\frac{1}{2}$	$1\frac{2}{3}$	$2\frac{3}{4}$	$3\frac{1}{2}$	$3\frac{3}{4}$	$1\frac{1}{2}$
$4\frac{2}{3}$	$2\frac{5}{6}$	$5\frac{1}{8}$	$4\frac{3}{4}$	$2\frac{5}{6}$	$3\frac{5}{8}$
$1\frac{5}{6}$	$3\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{1}{2}$	$2\frac{3}{4}$

7.	**8.**	**9.**	**10.**	**11.**	**12.**
$2\frac{2}{3}$	$2\frac{5}{8}$	$1\frac{1}{3}$	$3\frac{1}{8}$	$2\frac{5}{9}$	$1\frac{2}{3}$
$5\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{1}{6}$	$4\frac{1}{4}$	$3\frac{2}{3}$	$3\frac{3}{4}$
$3\frac{1}{2}$	$4\frac{1}{4}$	$5\frac{3}{4}$	$2\frac{1}{2}$	$4\frac{1}{6}$	$2\frac{1}{6}$

13.	**14.**	**15.**	**16.**	**17.**	**18.**
$4\frac{3}{8}$	$3\frac{2}{3}$	$2\frac{3}{4}$	$5\frac{1}{3}$	$1\frac{1}{3}$	$5\frac{1}{2}$
$6\frac{1}{4}$	$4\frac{5}{6}$	$3\frac{1}{2}$	$2\frac{7}{12}$	$2\frac{5}{12}$	$6\frac{2}{3}$
$3\frac{1}{2}$	$1\frac{1}{4}$	$4\frac{1}{3}$	$3\frac{1}{2}$	$6\frac{3}{4}$	$4\frac{5}{6}$

ADDING DECIMALS

You know that when adding decimals, the decimal points must be under each other so as to bring like units under each other.

Exercises

1.	**2.**	**3.**	**4.**	**5.**
9.36	17.3	46.08	.076	.09
18.7	9.86	19.8	9.8	.165
7.65	48.095	7.645	4.36	.39
8.015	8.365	9.46	18.486	1.8

THE ADDITION OF COMPOUND NUMBERS

To add compound numbers, add each group separately, and, if necessary, change the sums to the next higher order. For example, in Exercise 1, the sum of the inches is 19 in. or 1 ft. 7 in. Write 7 in. and carry 1 ft. to the next group.

1.	2.	3.	4.
3 ft. 4 in.	6 bu. 3 pk.	5 yd. 2 ft.	12 lb. 6 oz.
6 ft. 8 in.	7 bu. 2 pk.	6 yd. 2 ft.	8 lb. 10 oz.
9 ft. 7 in.	9 bu. 3 pk.	4 yd. 1 ft.	6 lb. 8 oz.

Test in Copying and Adding

See if you can copy these 10 exercises and add them without making a mistake. After adding all 10, check to see if all are correct.

1. Add $12\frac{1}{2}$, $26\frac{1}{4}$, $38\frac{1}{2}$, $46\frac{3}{4}$, $25\frac{1}{2}$, and $81\frac{1}{4}$.
2. Add $14\frac{1}{3}$, $21\frac{5}{6}$, $18\frac{1}{3}$, $40\frac{1}{6}$, $17\frac{2}{3}$, and $70\frac{5}{6}$.
3. Add $21\frac{5}{8}$, $16\frac{1}{2}$, $24\frac{3}{4}$, $30\frac{1}{8}$, $16\frac{1}{2}$, and $32\frac{7}{8}$.
4. Add $30\frac{1}{3}$, $14\frac{3}{4}$, $8\frac{7}{12}$, $9\frac{5}{6}$, $17\frac{1}{2}$, and 15.
5. Add $13\frac{3}{4}$, $16\frac{7}{8}$, $21\frac{1}{2}$, $18\frac{2}{3}$, 16, and $16\frac{5}{6}$.
6. Add 3.4, 16.5, 7.21, 14.8, 19, and .84.
7. Add 36.7, .975, 8.16, 19.8, 36, and .74.
8. Add .98, .067, 9.6, .935, 4.2, and 16.
9. Add 3 ft. 8 in.; 6 ft. 10 in.; 4 ft. 4 in.; and 9 ft. 8 in.
10. Add 3 lb. 6 oz.; 7 lb. 8 oz.; 9 lb. 8 oz.; and 7 lb. 10 oz.

THE SUBTRACTION OF WHOLE NUMBERS

Subtraction is used to find the difference between two numbers; to find the remainder when one number is taken from another; and to find how much must be added to a number to make it equal another.

These uses are illustrated by the following problems:

1. John wants to buy a sled costing \$9.75. He has only \$7.95. How much more money does he need?

\$9.75 7.95 \$1.80	*The result shows the* ***amount to be added.***

2. John earned \$12.80 one week and \$9.75 the next. Find how much more he earned the first week.

\$12.80 9.75 \$ 3.05	*The result shows the* ***difference*** *between the earnings.*

3. Frank earned \$12.50 and spent \$3.85 of it. How much had he left?

\$12.50 3.85 \$ 8.65	*The result shows the* ***remainder*** *left after spending part of it.*

The terms used in subtraction are:

76 is called the **minuend**
35 is called the **subtrahend**
41 is called the **result**

Result + subtrahend = minuend

There are three distinct methods used in subtraction. They are illustrated in the following exercises.

FIRST METHOD:

6027 3574 2453	*Think,* "4 *from* 7 *is* 3. 7 *from* 12 *is* 5. 6 *from* 10 *is* 4. 4 *from* 6 *is* 2."

SECOND METHOD:

6027 3574 2453	*Think,* "4 *from* 7 *is* 3. 7 *from* 12 *is* 5. 5 *from* 9 *is* 4. 3 *from* 5 *is* 2."

THIRD METHOD:

6027 3574 2453	*Think,* "4 *and* 3 *make* 7, *write* 3. 7 *and* 5 *make* 12, *write* 5 *and carry* 1. 6 *and* 4 *make* 10, *write* 4 *and carry* 1. 4 *and* 2 *make* 6, *write* 2."

In the following exercises use the method with which you are familiar. In this book the third method, known as the additive method, will be used.* If you wish to change from one of the other methods to the additive method, or from any of the three methods to either of the other two, be sure to practice using the new method for several days in order to insure accuracy and speed.

*See Stone's pamphlet *How We Subtract*, published by Benj. H. Sanborn & Co.

To check subtraction, go over the work a second time, or see if the sum of the two smaller numbers is equal to the larger number.

A Test in Subtraction

Subtract and check:

1.	2.	3.	4.	5.	6.
498	936	726	650	700	806
146	185	158	176	137	120

7.	8.	9.	10.	11.
3008	4013	5106	7100	8100
1624	1643	1730	3786	1594

12.	13.	14.	15.	16.
53,275	19,806	82,456	37,813	64,100
14,968	10,927	39,149	22,035	25,461

17. What must be added to each of the following to make 100?

60	72	80	35	27	81	92	84	57	37
82	75	61	39	45	56	28	77	63	94
19	44	42	91	38	66	15	25	14	76
32	16	53	24	88	59	47	36	21	89

A Time Test in Subtraction

This test contains all the difficult combinations in subtraction and should be used until 100% is made in the time limit.

E = 4 min. **G = 5 min.** **F = 6 min.**

1.	2.	3.	4.
74,906,387	63,911,600	50,840,035	81,396,328
16,758,059	15,876,398	29,386,429	79,638,965

5.	6.	7.	8.
85,239,710	73,874,129	43,865,564	61,521,449
29,784,725	36,897,346	18,966,397	26,749,397

9.	10.	11.	12.
72,225,938	41,604,885	51,097,229	61,701,327
49,668,242	25,015,167	11,322,871	42,604,836

THE SUBTRACTION OF FRACTIONS

Before fractions can be subtracted they must have the same denominator.

I. $\frac{4}{5}$ $\frac{1}{5}$ $\frac{3}{5}$	*Since subtraction is the reverse of addition, think, "$\frac{1}{5}$ and $\frac{3}{5}$ make $\frac{4}{5}$." Hence write $\frac{3}{5}$. These fractions have the same denominator.*
II. $\frac{5}{8}$ $\frac{3}{8}$ $\frac{1}{4}$	*Here you thought, "$\frac{3}{8}$ and $\frac{2}{8}$ make $\frac{5}{8}$," but you reduced $\frac{2}{8}$ to $\frac{1}{4}$ and wrote $\frac{1}{4}$.*
III. $\frac{3}{4}$ $\frac{1}{8}$ $\frac{5}{8}$	*Here the denominators are unlike but the denominator to which you changed $\frac{3}{4}$ is seen.*
IV. $\frac{2}{3}$ $\frac{1}{4}$ $\frac{5}{12}$	*Here the denominators are unlike and the denominator to which you changed the fractions is not seen. You changed $\frac{2}{3}$ to $\frac{8}{12}$ and $\frac{1}{4}$ to $\frac{3}{12}$ and subtracted as in I.*

Practice Exercises

Subtract:

1.	2.	3.	4.	5.	6.	7.	8.
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{2}{3}$
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$

9.	10.	11.	12.	13.	14.	15.	16.
$\frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{9}$	$\frac{7}{9}$	$\frac{8}{9}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$

17.	18.	19.	20.	21.	22.	23.	24.
$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$
$\frac{1}{6}$	$\frac{3}{8}$	$\frac{2}{9}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{5}{9}$	$\frac{5}{16}$

SUBTRACTION OF MIXED NUMBERS

The additive method is illustrated below. It is based upon the fact that the result added to the smaller of the two given numbers must make the larger.

I.	$6\frac{3}{4}$ $4\frac{1}{4}$ $2\frac{1}{2}$	*Think, "$\frac{1}{4}$ and $\frac{2}{4}$ make $\frac{3}{4}$." Write $\frac{1}{2}$. Think, "4 and 2 make 6." Write* 2.
II.	7 $4\frac{2}{3}$ $2\frac{1}{3}$	*Think, "$\frac{2}{3}$ and $\frac{1}{3}$ make* 1." *Write $\frac{1}{3}$ and carry* 1. *Think, "5 and 2 make 7." Write* 2.
III.	$6\frac{1}{8}$ $2\frac{3}{8}$ $3\frac{3}{4}$	*Since $\frac{1}{8}$ is less than $\frac{3}{8}$, think, "$\frac{3}{8}$ and $\frac{5}{8}$ make 1" as in* II. *Now add the $\frac{1}{8}$ above making $\frac{6}{8}$ or $\frac{3}{4}$, and write $\frac{3}{4}$. Carry 1 and think, "3 and 3 make 6." Write* 3.
IV.	$7\frac{1}{4}$ 3 $4\frac{1}{4}$	*Since there is no fraction below, write the fraction given above, or $\frac{1}{4}$. Then 3 and 4 make 7, write* 4.

Exercises

Subtract:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**
1	1	1	1	1	1	1	1	1
$\frac{1}{4}$	$\frac{2}{3}$	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{3}{5}$	$\frac{2}{7}$	$\frac{5}{9}$	$\frac{7}{12}$	$\frac{5}{16}$

10.	**11.**	**12.**	**13.**	**14.**	**15.**
$5\frac{7}{8}$	$6\frac{1}{4}$	$7\frac{1}{2}$	$4\frac{5}{6}$	$5\frac{7}{8}$	$8\frac{5}{6}$
$3\frac{1}{4}$	$3\frac{1}{8}$	$4\frac{1}{6}$	$2\frac{1}{4}$	$1\frac{1}{2}$	$4\frac{3}{4}$

16.	**17.**	**18.**	**19.**	**20.**	**21.**
$8\frac{3}{4}$	$9\frac{5}{6}$	$7\frac{1}{2}$	$4\frac{7}{8}$	$9\frac{3}{4}$	$8\frac{1}{2}$
$6\frac{1}{4}$	$7\frac{1}{6}$	$4\frac{1}{4}$	$2\frac{1}{2}$	$4\frac{2}{3}$	$4\frac{1}{3}$

22.	**23.**	**24.**	**25.**	**26.**	**27.**
6	8	5	7	8	9
$1\frac{1}{2}$	$6\frac{3}{4}$	$2\frac{5}{8}$	$3\frac{2}{3}$	$4\frac{7}{8}$	$3\frac{1}{4}$

28.	**29.**	**30.**	**31.**	**32.**	**33.**
$7\frac{1}{4}$	$6\frac{1}{2}$	$7\frac{1}{3}$	$9\frac{1}{8}$	$4\frac{1}{6}$	$8\frac{3}{8}$
$5\frac{3}{4}$	$2\frac{3}{4}$	$4\frac{2}{3}$	$5\frac{3}{4}$	$2\frac{2}{3}$	$4\frac{3}{4}$

34.	**35.**	**36.**	**37.**	**38.**	**39.**
$8\frac{3}{8}$	$5\frac{1}{2}$	$7\frac{5}{8}$	$9\frac{5}{6}$	$5\frac{3}{4}$	$6\frac{2}{3}$
6	3	5	3	4	1

Time Tests in Subtraction of Fractions and Mixed Numbers

A

E = 5 min. G = 6 min. F = 7 min.

1.	2.	3.	4.	5.	6.
$3\frac{2}{3}$	$4\frac{5}{8}$	$5\frac{5}{6}$	$6\frac{2}{3}$	$5\frac{2}{3}$	$6\frac{1}{3}$
$1\frac{1}{2}$	$1\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{1}{6}$	$2\frac{1}{4}$	$3\frac{1}{4}$

7.	8.	9.	10.	11.	12.
$7\frac{1}{2}$	$6\frac{1}{2}$	$5\frac{1}{2}$	$6\frac{3}{4}$	$7\frac{3}{4}$	$6\frac{3}{4}$
$5\frac{1}{4}$	$3\frac{3}{8}$	$3\frac{1}{3}$	$4\frac{1}{3}$	$2\frac{1}{2}$	$3\frac{1}{8}$

B

E = $2\frac{1}{2}$ min. G = 3 min. F = $3\frac{1}{2}$ min.

1.	2.	3.	4.	5.	6.
5	4	3	6	4	7
$3\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{3}{4}$	$2\frac{1}{3}$	$2\frac{2}{3}$	$2\frac{3}{8}$

7.	8.	9.	10.	11.	12.
5	3	4	7	4	8
$2\frac{1}{6}$	$1\frac{5}{6}$	$2\frac{7}{8}$	$3\frac{5}{8}$	$1\frac{1}{8}$	$3\frac{1}{9}$

13.	14.	15.	16.	17.	18.
6	9	5	8	7	6
$4\frac{5}{9}$	$6\frac{5}{12}$	$1\frac{7}{12}$	$2\frac{9}{16}$	$5\frac{3}{16}$	$2\frac{7}{9}$

C

E = 5 min. G = 6 min. F = 7 min.

1.	2.	3.	4.	5.	6.
$5\frac{1}{2}$	$6\frac{1}{4}$	$4\frac{3}{8}$	$5\frac{3}{8}$	$6\frac{1}{2}$	$5\frac{1}{4}$
$2\frac{3}{4}$	$3\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{3}{4}$	$3\frac{5}{8}$	$3\frac{3}{8}$

7.	8.	9.	10.	11.	12.
$6\frac{1}{4}$	$5\frac{1}{2}$	$6\frac{1}{3}$	$7\frac{1}{3}$	$6\frac{1}{3}$	$8\frac{1}{3}$
$3\frac{7}{8}$	$3\frac{7}{8}$	$4\frac{2}{3}$	$5\frac{1}{2}$	$3\frac{5}{6}$	$6\frac{3}{4}$

13.	14.	15.	16.	17.	18.
$5\frac{1}{3}$	$6\frac{1}{4}$	$8\frac{1}{3}$	$6\frac{1}{6}$	$5\frac{1}{6}$	$8\frac{1}{4}$
$3\frac{5}{9}$	$3\frac{5}{6}$	$2\frac{5}{12}$	$3\frac{1}{3}$	$3\frac{3}{4}$	$3\frac{2}{3}$

D

E = 1 min. G = $1\frac{1}{4}$ min. F = $1\frac{1}{2}$ min.

1.	2.	3.	4.	5.	6.
$4\frac{3}{4}$	$6\frac{7}{8}$	$9\frac{1}{3}$	$7\frac{5}{6}$	$9\frac{1}{4}$	$5\frac{1}{2}$
2	4	5	3	7	2

7.	8.	9.	10.	11.	12.
$3\frac{1}{3}$	$8\frac{1}{9}$	$7\frac{2}{5}$	$6\frac{3}{4}$	$8\frac{5}{6}$	$7\frac{5}{16}$
1	4	3	2	5	5

E

E = 8 min. G = 10 min. F = 12 min.

1.	2.	3.	4.	5.	6.	7.	8.
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	6	$\frac{7}{8}$	7	$6\frac{4}{5}$
$\frac{1}{2}$	$\frac{1}{8}$	$\frac{4}{9}$	$\frac{1}{4}$	$4\frac{3}{4}$	$\frac{5}{16}$	$2\frac{5}{12}$	$2\frac{3}{4}$

9.	10.	11.	12.	13.	14.	15.	16.
$8\frac{1}{2}$	5	$8\frac{3}{4}$	7	$7\frac{1}{2}$	8	$8\frac{2}{3}$	3
$2\frac{5}{6}$	$2\frac{2}{3}$	$5\frac{2}{3}$	$3\frac{7}{12}$	$4\frac{7}{8}$	$5\frac{5}{6}$	$4\frac{5}{9}$	$1\frac{3}{8}$

17.	18.	19.	20.	21.	22.	23.	24.
$8\frac{5}{6}$	7	$9\frac{2}{5}$	$6\frac{1}{3}$	$6\frac{5}{8}$	$9\frac{1}{3}$	7	$6\frac{1}{4}$
$5\frac{3}{4}$	$4\frac{1}{6}$	4	$3\frac{2}{3}$	$3\frac{1}{2}$	$7\frac{5}{6}$	$2\frac{7}{8}$	$5\frac{1}{2}$

25.	26.	27.	28.	29.	30.	31.	32.
$7\frac{1}{16}$	$8\frac{1}{12}$	$9\frac{1}{9}$	$2\frac{1}{2}$	8	$6\frac{1}{6}$	$\frac{7}{8}$	$\frac{8}{9}$
$4\frac{5}{8}$	$5\frac{3}{4}$	$3\frac{1}{3}$	$1\frac{5}{6}$	$5\frac{5}{8}$	$2\frac{3}{4}$	$\frac{3}{4}$	$\frac{5}{6}$

33.	34.	35.	36.	37.	38.	39.	40.
$7\frac{5}{6}$	8	7	$\frac{1}{2}$	$7\frac{1}{6}$	$9\frac{1}{4}$	$9\frac{1}{3}$	$8\frac{1}{8}$
3	$6\frac{7}{8}$	$4\frac{1}{4}$	$\frac{2}{5}$	$4\frac{3}{8}$	$4\frac{1}{8}$	$2\frac{3}{5}$	$5\frac{3}{16}$

THE SUBTRACTION OF DECIMALS

You know that in adding decimals, the decimal points must be under each other. In subtracting decimals, the same thing is true.

Exercises

Subtract:

1.	2.	3.	4.	5.	6.
56.21	34.28	47.09	36.8	54.3	17.85
32.46	14.08	16.7	15.48	19.46	9.9

7.	8.	9.	10.	11.	12.
13.5	19.5	112.0	0.01	5.63	185.2
0.02	4.078	4.28	0.001	0.63	96.57

THE SUBTRACTION OF COMPOUND NUMBERS

In subtracting compound numbers, begin at the right and subtract each group separately. Use the method you used in mixed numbers.

There are four types, as follows:

I.	6 ft. 9 in. 2 ft. 7 in. 4 ft. 2 in.	*Think, "7 in. and 2 in. make 9 in." Write 2 in.* *Think, "2 ft. and 4 ft. make 6 ft." Write 4 ft.*
II.	8 ft. 5 ft. 8 in. 2 ft. 4 in.	*Think, "8 in. and 4 in. make 1 ft." Write 4 in.; carry 1 ft.* *Think, "6 ft. and 2 ft. make 8 ft." Write 2 ft.*
III.	9 ft. 3 in. 5 ft. 7 in. 3 ft. 8 in.	*Think, "7 in. and 5 in. make 1 ft.," as in* II. *Then add the 5 in. and 3 in. and write 8 in. Carry 1 ft.* *Think, "6 ft. and 3 ft. make 9 ft." Write 3 ft.*
IV.	7 ft. 9 in. 5 ft. 2 ft. 9 in.	*Think, "There are no inches to subtract." So bring down 9 in.* *Think, "5 ft. and 2 ft. make 7 ft." Write 2 ft.*

Exercises

Subtract:

1.	2.	3.	4.
8 lb. 10 oz.	6 ft. 8 in.	12 gal.	4 yd.
4 lb. 3 oz.	3 ft. 5 in.	8 gal. 3 qt.	2 yd. 2 ft.

5.	6.	7.	8.
12 lb. 4 oz.	6 bu. 2 pk.	6 lb. 2 oz.	4 yd. 1 ft.
8 lb. 10 oz.	4 bu. 3 pk.	4 lb.	3 yd.

REVIEW OF MULTIPLICATION

Multiplying by a whole number saves adding when the addends are all alike.

Multiplying by a fraction is both division and multiplication.

\$24.60	is called the **multiplicand**
7	is called the **multiplier**
\$172.20	is called the **product**

Test in the 100 Multiplication Facts

E = $1\frac{1}{2}$ min. **G = 2 min.** **F = $2\frac{1}{2}$ min.**

Multiply:

2	3	2	1	3	5	2	0	5	2
1	0	2	4	1	2	3	0	1	8
8	2	2	1	1	3	6	7	9	4
1	6	7	3	5	3	1	1	2	1
3	1	0	9	4	1	4	1	6	0
2	7	3	1	2	6	4	8	2	1
5	2	0	7	5	7	4	8	2	5
5	4	2	3	3	2	5	2	9	6
3	0	3	6	0	4	5	1	5	3
4	4	5	6	6	3	7	0	4	7

0	4	0	3	9	2	9	7	9	8
5	6	8	6	5	5	0	5	3	5
6	7	5	7	4	7	0	8	6	8
4	0	9	7	8	6	9	8	0	4
2	1	4	6	9	6	0	8	5	6
0	1	0	7	4	8	7	9	0	5
8	9	5	7	9	7	4	9	4	3
3	9	8	4	8	8	7	6	9	9
8	6	9	8	8	7	3	2	1	6
7	9	7	0	6	9	8	8	9	3

To check your work in multiplication, go over the work a second time.

Practice Exercises

Multiply and check:

1.	**2.**	**3.**	**4.**	**5.**	**6.**
16	29	37	46	58	73
7	8	6	9	7	9
7.	**8.**	**9.**	**10.**	**11.**	**12.**
60	348	406	580	6095	7806
7	5	9	7	9	8
13.	**14.**	**15.**	**16.**	**17.**	**18.**
258	972	329	4028	1009	9274
4	3	6	5	7	8

Tests for Speed and Accuracy

A

E = 2 min. **G = $2\frac{1}{2}$ min.** **F = 3 min.**

Multiply, then go over the work a second time to see if all are correct.

1.	2.	3.	4.	5.
2893	5486	9138	3795	6438
9	6	7	8	5

6.	7.	8.	9.	10.
8793	6845	2938	6457	9638
6	7	8	9	4

B

E = 8 min. **G = 9 min.** **F = 10 min.**

1.	2.	3.	4.	5.	6.
7593	8624	9357	2684	3957	6248
84	48	75	57	39	93

7.	8.	9.	10.	11.	12.
5379	8246	9137	2614	6821	3197
26	62	97	86	79	98

Checking Multiplication by Casting Out Nines

The following principles control the checking of multiplication by casting out nines.

1. **The remainder arising from dividing a number by 9 (called the excess of nines) is the same as the remainder found by dividing the sum of the digits by 9.**

2. The excess of nines in the product equals the excess in the product of the excesses of the multiplier and multiplicand.

1. Find 248 × 3764 and check by casting out nines.

3764	2	*The excess in 3764 is 2.*
248	5	*The excess in 248 is 5.*
30112		
15056		
7528		
933472	10	*The excess in the product of excesses is 1.*
1	1	*The excess in the product is 1.*

Very probably the work is correct.

The fact that the excess in each product is the same is not an ***absolute*** *check, but if they are* ***not*** *alike, there has been an error.*

Multiply and check by casting out nines:

2. 789 × 643
3. 862 × 947
4. 375 × 896
5. 762 × 934
6. 538 × 476
7. 928 × 647
8. 329 × 728
9. 453 × 784
10. 762 × 847
11. 768 × 307
12. 708 × 943
13. 276 × 709
14. 846 × 709
15. 247 × 806

MULTIPLYING FRACTIONS AND MIXED NUMBERS

The following are the most common examples of the multiplication of fractions and mixed numbers.

I. $3 \times \frac{3}{4} = 2\frac{1}{4}$	*This is* 3×3 *fourths, or* $\frac{9}{4}$, *or* $2\frac{1}{4}$.
II. $3 \times \frac{2}{9} = \frac{2}{3}$	*If multiplied as in* I, *you get* $\frac{6}{9}$, *or* $\frac{2}{3}$ *when reduced. The same answer is obtained by dividing* 9 *by* 3, *leaving the numerator unchanged.*
III. $3 \times 4\frac{1}{2} = 13\frac{1}{2}$	*Here* $3 \times \frac{1}{2}$ *is* $1\frac{1}{2}$. *Write the* $\frac{1}{2}$ *and carry* 1 *to* 3×4.
IV. $\frac{2}{3} \times 12 = 8$	*Here we are to find* 2 *of the* 3 *equal parts into which* 12 *has been divided, or* 8.
V. $\frac{3}{4} \times 5 = 3\frac{3}{4}$	*Here we are to find* 3 *of the* 4 *equal parts into which* 5 *has been divided, as in* IV; *but since* 5 *is not exactly divisible by* 4, *it is easier to multiply by* 3 *and then divide by* 4.
VI. $\begin{array}{r} 27 \\ 5\frac{3}{4} \\ \hline 4\,)\,81 \\ 20\frac{1}{4} \\ 135 \\ \hline 155\frac{1}{4} \end{array}$	*Here we multiply by the fraction as in* V, *and then by the whole number and add the partial products.*
VII. $\frac{3}{4} \times \frac{5}{8} = \frac{15}{32}$	*Here the products of the numerators and the denominators are found.*
VIII. $\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{2}{3}$	*Proceed as in* VII, *except cancelling out common factors whenever possible.*
IX. $3\frac{1}{2} \times 1\frac{7}{8} =$ $\frac{7}{2} \times \frac{15}{8} = \frac{105}{16}$ $= 6\frac{9}{16}$	*Here the mixed numbers were changed to fractions and then multiplied as in* VII.

Exercises

Multiply:

1. $4 \times \frac{3}{8}$
2. $5 \times \frac{5}{6}$
3. $7 \times \frac{3}{4}$
4. $4 \times 7\frac{1}{2}$
5. $5 \times 3\frac{1}{4}$
6. $8 \times 12\frac{1}{3}$
7. $12 \times 4\frac{2}{3}$
8. $5 \times 40\frac{2}{9}$
9. $\frac{3}{4} \times 8$
10. $\frac{5}{6} \times 4$
11. $\frac{2}{3} \times 18$
12. $\frac{7}{8} \times 20$
13. $\frac{5}{9} \times 15$
14. $\frac{3}{4} \times 9$
15. $\frac{3}{8} \times 12$
16. $1\frac{1}{2} \times 13$
17. $4\frac{3}{4} \times 15$
18. $8\frac{3}{8} \times 24$
19. $7\frac{2}{3} \times 16$
20. $\frac{2}{3} \times \frac{5}{9}$
21. $\frac{4}{5} \times \frac{3}{4}$
22. $\frac{3}{8} \times \frac{1}{2}$
23. $\frac{7}{8} \times \frac{4}{9}$
24. $\frac{5}{6} \times \frac{4}{5}$
25. $\frac{2}{3} \times \frac{7}{8}$

Time Tests in Multiplication of Fractions and Mixed Numbers

A

E = 5 min. **G = 6 min.** **F = 7 min.**

1. $\frac{1}{8} \times 6\frac{3}{4}$
2. $\frac{3}{4} \times 7\frac{1}{2}$
3. $\frac{5}{6} \times 3\frac{2}{3}$
4. $\frac{2}{3} \times 5\frac{3}{4}$
5. $\frac{7}{8} \times 2\frac{1}{2}$
6. $\frac{5}{6} \times 1\frac{2}{3}$
7. $\frac{5}{8} \times 7\frac{1}{2}$
8. $2\frac{1}{2} \times \frac{5}{6}$
9. $3\frac{1}{2} \times \frac{5}{7}$
10. $4\frac{3}{4} \times \frac{1}{2}$
11. $\frac{7}{8} \times 3\frac{2}{3}$
12. $\frac{2}{9} \times 4\frac{1}{2}$
13. $\frac{5}{6} \times 5\frac{1}{3}$
14. $\frac{7}{8} \times 3\frac{1}{3}$
15. $\frac{3}{5} \times 4\frac{2}{7}$

B

E = 5 min. **G = 6 min.** **F = 7 min.**

1. $2\frac{1}{2} \times 4\frac{1}{5}$
2. $3\frac{1}{3} \times 2\frac{1}{2}$
3. $4\frac{2}{3} \times 3\frac{1}{7}$
4. $2\frac{3}{5} \times 4\frac{1}{3}$
5. $3\frac{1}{2} \times 3\frac{2}{3}$
6. $2\frac{1}{4} \times 3\frac{1}{3}$
7. $1\frac{1}{3} \times 2\frac{1}{4}$
8. $1\frac{1}{6} \times 2\frac{1}{7}$
9. $3\frac{3}{4} \times 2\frac{2}{5}$
10. $7\frac{1}{2} \times 2\frac{1}{3}$

MULTIPLICATION OF DECIMALS

When multiplying decimals, proceed as in the multiplication of whole numbers. Then point off as many decimal places in the product as there are in both the multiplicand and the multiplier.

The reason for this rule is easily seen. Thus to find 5×4.35, the result is the same as if 4.35 were written five times and added, giving 21.75.

To find $.4 \times 3.46$ means find $\frac{4}{10}$ of 3.46, which you can find by multiplying by 4 and dividing by 10. This would give 1.384.

Or, the truth of the rule may be seen by taking several examples and reasoning as follows:

$$\begin{array}{r} 7.16 \\ 4.2 \\ \hline 1432 \\ 2864 \\ \hline 30.072 \end{array}$$

The result must be a little larger than 4×7, *or* 28, *and not so large as* 5×8, *or* 40. *Hence the only reasonable answer is* 30.072, *not* 3.0072 *or* 300.72.

Multiply:

1.	2.	3.	4.	5.	6.
36.04	3.26	406	7.25	4.628	782
7	.06	.9	.8	5	3.6

7.	8.	9.	10.	11.	12.
367	.28	5.27	.021	47.5	23
.42	5.3	49	57	1.8	.06

13.	14.	15.	16.	17.	18.
7.12	6.04	18	\$4.20	\$12.65	\$5.95
23	58	.46	5.3	32.6	3.4

MULTIPLICATION OF COMPOUND NUMBERS

When multiplying compound numbers, multiply each group separately, beginning at the right. If necessary, change each group to its next higher order.

Multiply:

1. 3 bu. 3 pk. × 5	2. 6 ft. 5 in. × 6	3. 9 lb. 6 oz. × 8
4. 12 bu. 1 pk. × $2\frac{1}{2}$	5. 6 yd. 2 ft. × 4	6. 9 lb. 8 oz. × $3\frac{3}{4}$
7. 5 ft. 4 in. × 6	8. 4 yd. 1 ft. × $5\frac{1}{2}$	9. 7 yd. 2 ft. × $6\frac{1}{2}$

Saving Work in Computation

Ability to see number relations that will save writing so many figures is one of the ways to save computation. In this section some examples of how to save work are given.

MULTIPLYING BY SOME POWER OF 10

You know that a zero annexed to a whole number multiplies it by 10; that two zeros annexed multiplies it by 100; and so on.

You know also that moving the decimal point one place to the right in decimals multiplies the number by 10; moving it two places to the right multiplies it by 100; and so on.

Give without a pencil:

1. 10×35	6. 100×17	11. 10×0.06
2. 10×3.5	7. 100×20	12. 10×0.6
3. 10×40	8. 100×3.65	13. 100×0.245
4. 10×1.75	9. 100×4.2	14. 10×3.12
5. 10×0.56	10. 100×0.35	15. 100×3.6

MULTIPLYING BY ALIQUOT PARTS OF 10 AND 100

The aliquot part of a number is a number that is contained in it a whole number of times.

Thus,

$5 = \frac{1}{2}$ of 10	$50 = \frac{1}{2}$ of 100	$33\frac{1}{3} = \frac{1}{3}$ of 100
$2\frac{1}{2} = \frac{1}{4}$ of 10	$25 = \frac{1}{4}$ of 100	$16\frac{2}{3} = \frac{1}{6}$ of 100
$3\frac{1}{3} = \frac{1}{3}$ of 10	$12\frac{1}{2} = \frac{1}{8}$ of 100	$8\frac{1}{3} = \frac{1}{12}$ of 100

These facts may be used to save work.

Thus,

$2\frac{1}{2} \times 28 = \frac{1}{4} \times 280 = 70$; $16\frac{2}{3} \times 48 = \frac{1}{6} \times 4800 = 800$;
$33\frac{1}{3} \times 36 = \frac{1}{3} \times 3600 = 1200$; $12\frac{1}{2} \times 56 = \frac{1}{8} \times 5600 = 700$.

Give without a pencil:

1. 5×84	**7.** 50×64	**13.** $33\frac{1}{3} \times 96$
2. $2\frac{1}{2} \times 64$	**8.** 50×75	**14.** $33\frac{1}{3} \times 45$
3. $2\frac{1}{2} \times 45$	**9.** 25×86	**15.** $33\frac{1}{3} \times 36$
4. $3\frac{1}{3} \times 48$	**10.** 25×96	**16.** $16\frac{2}{3} \times 96$
5. $3\frac{1}{3} \times 25$	**11.** $12\frac{1}{2} \times 24$	**17.** $16\frac{2}{3} \times 24$
6. 25×32	**12.** $12\frac{1}{2} \times 18$	**18.** $8\frac{1}{3} \times 36$

DIVIDING BY POWERS OF 10

Reversing the rule for multiplying by powers of 10, we see that all division by such numbers may be done by removing zeros or by moving the decimal point. Thus,

$340 \div 10 = 34$; $35 \div 10 = 3.5$; $34.2 \div 100 = .342$.

Give without a pencil:

1. $380 \div 10$	**6.** $12.5 \div 10$	**11.** $340 \div 100$
2. $298 \div 10$	**7.** $38.2 \div 10$	**12.** $265 \div 100$
3. $745 \div 10$	**8.** $.45 \div 10$	**13.** $42.6 \div 100$
4. $32.5 \div 10$	**9.** $1.65 \div 10$	**14.** $32.5 \div 100$
5. $2.46 \div 10$	**10.** $175 \div 10$	**15.** $4.5 \div 100$

SPECIAL CASES IN MULTIPLICATION

Squaring a Two-figured Number Ending in 5

To square any two-figured number ending in 5, multiply the tens' digit by the next consecutive number and annex 25.

$$\begin{array}{r} 75 \\ 75 \\ \hline 5625 \end{array}$$

Think, "7 × 8 = 56. *Then annex* 25. *The product is* 5625."

Applications of the special method are given below.

Give the products:

1. 45 × 45
2. .35 × .35
3. 6.5 × 6.5
4. 25 × 25
5. .15 × .15
6. 5.5 × 5.5
7. 95 × 95
8. .85 × .85
9. .75 × .75
10. 3.5 × 3.5

Find the products:

1. 35 × 3.5
2. 75 × .75
3. 1.5 × .15
4. .65 × 6.5
5. 55 × .55
6. .85 × 8.5
7. 25 × .25
8. .45 × 45
9. .95 × 9.5
10. 65 × 6.5

This same principle is true for any two-figured numbers in the same decade, the sum of whose unit digits is 10.

$$\begin{array}{r} 46 \\ 44 \\ \hline 2024 \end{array}$$

Think, "4 × 5 = 20." *Annex the product of* 4 × 6 *and then write the product,* 2024.

Find the product:

1. 38×32	6. 28×22
2. 43×47	7. 14×16
3. 91×99	8. 48×42
4. 58×52	9. 74×76
5. 67×63	10. 93×97

Multiply the following decimals using the above method:

1. $18 \times .12$	6. $.48 \times 4.2$
2. 36×3.4	7. 3.3×37
3. $5.3 \times .57$	8. $56 \times .54$
4. $.94 \times 9.6$	9. $13 \times .17$
5. $61 \times .69$	10. 2.6×2.4

REVIEW OF DIVISION

Divide and express the remainders as common fractions in lowest terms:

1.	2.	3.	4.	5.	6.
3)10¢	4¢)12¢	5)21 ft.	6 in.)40 in.	2)56 in.	\$8)\$60

Give the missing numbers:

7. 56 lb. = ——— times 8 lb.
8. 56 lb. = 7 parts, each ———.
9. 42 ft. = ——— times 7 ft.
10. 42 ft. = 6 parts, each ———.
11. 45¢ will buy ——— 5-cent oranges.
12. 45¢ will buy 9 oranges at ——— each.
13. ? × 9¢ = 72¢.
14. 8 × ——— = 72¢.

By division we find

1. **How many times as large one number is as another; or,**
2. **The size of one of the equal parts into which a number has been divided.**

15. Tell what was found in each of the division exercises from 1 to 14 inclusive.

16. Give five problems in which the first use of division is used.

17. Give five problems in which the second kind of division is used.

27 is the **quotient**
34)936 is the **dividend**
68
256
238 34 is the **divisor**
18 is the **remainder**

To check division, see if the product of the divisor and quotient plus the remainder equals the dividend. Or, go over the work a second time.

Division may also be checked by casting out nines but this method is seldom used. It depends upon the fact that

The excess in the product of the excesses in the divisor and quotient, plus the excess in the remainder, is equal to the excess in the dividend.

146
32)4697
32
149
128
217
192
25

The excess in 32 *is* 5. *The excess in* 146 *is* 2. *The excess in* 5 × 2 *is* 1. *The excess in* 25 *is* 7. 1 + 7 = 8. *The excess in 4697 is* 8. *The work very probably is correct.*

Divide and check:

1. 96,321 ÷ 97
2. 39,267 ÷ 72
3. 84,603 ÷ 53
4. 36,475 ÷ 67
5. 64,789 ÷ 48
6. 92,462 ÷ 69
7. 37,306 ÷ 76
8. 89,225 ÷ 65
9. 92,462 ÷ 39
10. 90,401 ÷ 71

A Test in Division

You ought to divide 8 of these in 5 minutes. After working for 5 minutes check the work by one of the two methods given and see how many are correct.

1. $32\overline{)3654}$
2. $46\overline{)3364}$
3. $55\overline{)8164}$
4. $96\overline{)9067}$
5. $78\overline{)2363}$
6. $54\overline{)1328}$
7. $67\overline{)4870}$
8. $92\overline{)3000}$
9. $43\overline{)3927}$
10. $61\overline{)6930}$

A Time Test in Division

E = 4 min.	**G = 5 min.**	**F = 6 min.**
1.	2.	3.
$82\overline{)48626}$	$56\overline{)40656}$	$97\overline{)80898}$
4.	5.	6.
$63\overline{)32697}$	$94\overline{)37224}$	$86\overline{)23882}$

DIVISION OF A FRACTION BY A WHOLE NUMBER

There are only two types of fractions to be divided by a whole number: that in which the numerator will contain the divisor without a remainder, and that in which it will not.

I. $2\,\overline{)\,\frac{4}{5}}$
$\quad\frac{2}{5}$

II. $2\,\overline{)\,\frac{5}{6}}$
$\quad\frac{5}{12}$

Here the 4 is divided by 2, giving 2. Then the answer is $\frac{2}{5}$, *just as* $4 ÷ 2 = $2.

Here 5 will not contain 2, but $\frac{5}{6} = \frac{10}{12}$, *in which 10 will contain 2, giving* $\frac{5}{12}$ *as in I. We see that the result is the same as if we had left the 5 of* $\frac{5}{6}$ *unchanged and multiplied 6 by 2.*

To divide a fraction by a whole number, divide the numerator by the divisor, if possible; otherwise, multiply the denominator by it.

Exercises

Give the quotients without a pencil:

1. $2\,\overline{)\,\frac{3}{4}}$	**2.** $2\,\overline{)\,\frac{5}{8}}$	**3.** $4\,\overline{)\,\frac{5}{6}}$	**4.** $4\,\overline{)\,\frac{4}{5}}$
5. $5\,\overline{)\,\frac{5}{16}}$	**6.** $6\,\overline{)\,\frac{5}{6}}$	**7.** $8\,\overline{)\,\frac{1}{2}}$	**8.** $2\,\overline{)\,\frac{7}{16}}$
9. $3\,\overline{)\,\frac{9}{16}}$	**10.** $4\,\overline{)\,\frac{7}{8}}$	**11.** $4\,\overline{)\,\frac{2}{5}}$	**12.** $6\,\overline{)\,\frac{4}{7}}$
13. $6\,\overline{)\,\frac{3}{5}}$	**14.** $8\,\overline{)\,\frac{4}{9}}$	**15.** $2\,\overline{)\,\frac{5}{8}}$	**16.** $6\,\overline{)\,\frac{8}{9}}$
17. $2\,\overline{)\,\frac{1}{6}}$	**18.** $9\,\overline{)\,\frac{3}{4}}$	**19.** $7\,\overline{)\,\frac{7}{12}}$	**20.** $2\,\overline{)\,\frac{1}{3}}$

DIVISION OF A MIXED NUMBER BY A WHOLE NUMBER

There are three types of mixed numbers that may be divided by a whole number.

I.	$4\,\overline{)\,3\frac{1}{3}}$ $\frac{5}{6}$	*Think of* $3\frac{1}{3}$ *as* $\frac{10}{3}$, *then* $4\,\overline{)\,\frac{10}{3}}$ *is* $\frac{5}{6}$.
II.	$4\,\overline{)\,12\frac{1}{2}}$ $3\frac{1}{8}$	*Think,* "12 ÷ 4 *is* 3, *then* $4\,\overline{)\,\frac{1}{2}}$ *is* $\frac{1}{8}$."
III.	$4\,\overline{)\,13\frac{1}{4}}$ $3\frac{5}{16}$	*Think,* "13 ÷ 4 *is* 3 *and* 1 *remaining; then* $4\,\overline{)\,1\frac{1}{4}}$ *is* $\frac{5}{16}$."

To divide a mixed number by a whole number, if the whole number of the dividend is less than that of the divisor, change the mixed number to the fraction form and then divide. If the whole number of the dividend is greater than the divisor, proceed as in the division of whole numbers until you get a remainder less than the divisor and then divide the fraction, or mixed number, by the divisor.

Exercises

Find the quotients:

1. $4\,\overline{)\,3\frac{1}{2}}$	**9.** $3\,\overline{)\,1\frac{1}{2}}$	**17.** $6\,\overline{)\,5\frac{1}{2}}$
2. $3\,\overline{)\,2\frac{1}{2}}$	**10.** $4\,\overline{)\,10\frac{1}{2}}$	**18.** $8\,\overline{)\,16\frac{1}{2}}$
3. $8\,\overline{)\,6\frac{1}{4}}$	**11.** $3\,\overline{)\,8\frac{1}{4}}$	**19.** $3\,\overline{)\,24\frac{1}{3}}$
4. $2\,\overline{)\,12\frac{1}{3}}$	**12.** $3\,\overline{)\,7\frac{1}{2}}$	**20.** $3\,\overline{)\,20\frac{1}{4}}$
5. $4\,\overline{)\,8\frac{1}{4}}$	**13.** $3\,\overline{)\,16\frac{3}{4}}$	**21.** $6\,\overline{)\,8\frac{1}{2}}$
6. $5\,\overline{)\,10\frac{5}{6}}$	**14.** $5\,\overline{)\,12\frac{5}{6}}$	**22.** $2\,\overline{)\,1\frac{5}{6}}$
7. $3\,\overline{)\,15\frac{2}{3}}$	**15.** $3\,\overline{)\,7\frac{1}{8}}$	**23.** $3\,\overline{)\,33\frac{1}{3}}$
8. $9\,\overline{)\,18\frac{3}{4}}$	**16.** $8\,\overline{)\,4\frac{1}{4}}$	**24.** $8\,\overline{)\,30\frac{1}{4}}$

HOW TO MULTIPLY A FRACTION BY A FRACTION

Now that we can multiply and divide fractions by whole numbers, we can see how to multiply a fraction by a fraction. For, just as we may find $\frac{3}{4} \times 15$ by multiplying by 3 and dividing by 4, we may find $\frac{3}{4} \times \frac{3}{5}$ by multiplying by 3 and dividing by 4.

$$\text{That is, } \frac{3}{4} \times \frac{3}{5} = (3 \times \frac{3}{5}) \div 4 = 4\overline{)\tfrac{9}{5}} \quad \frac{9}{20}$$

That is, we may get the result by multiplying the numerators and the denominators.

DIVISION OF A WHOLE NUMBER BY A FRACTION

There is only one distinct type to be found in exercises involving the division of a whole number by a fraction.

$\frac{3}{4}\overline{)\,12}$ $\frac{4}{3} \times 12 = 16$	*By multiplying both dividend and divisor by* $\frac{4}{3}$, *the quotient will not be changed.* $\frac{4}{3} \times \frac{3}{4}$ *is* 1. *Then we have* $1\overline{)\,\frac{4}{3} \times 12}$ *giving as its quotient* $\frac{4}{3} \times 12$. *This is the same as multiplying the dividend by the reciprocal of the divisor. If the product of any two numbers is* 1, *either one may be considered the reciprocal of the other.*

To divide a whole number by a fraction, multiply the number by the reciprocal of the fraction.

Find the quotient:

1. $\frac{3}{4}\overline{)\,6}$ **2.** $\frac{2}{3}\overline{)\,8}$ **3.** $\frac{3}{8}\overline{)\,12}$ **4.** $\frac{5}{6}\overline{)\,4}$

5. $\frac{7}{8}\overline{)\,2}$ 6. $\frac{2}{9}\overline{)\,4}$ 7. $\frac{3}{5}\overline{)\,15}$ 8. $\frac{3}{4}\overline{)\,5}$

9. $\frac{4}{5}\overline{)\,16}$ 10. $\frac{5}{12}\overline{)\,15}$ 11. $\frac{7}{8}\overline{)\,6}$ 12. $\frac{5}{9}\overline{)\,3}$

DIVISION OF A FRACTION BY A FRACTION

There are but two types of problems involving division of a fraction by a fraction.

I. $\frac{3}{4}\overline{)\,\frac{2}{5}}$ *The reciprocal of the divisor is* $\frac{4}{3}$.

$\frac{4}{3} \times \frac{2}{5} = \frac{8}{15}$.

II. $\frac{5}{6}\overline{)\,\frac{5}{9}}$ *The reciprocal of the divisor is* $\frac{6}{5}$.

$$\frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{9}}} = \frac{2}{3}$$

To divide a fraction by a fraction, multiply the dividend by the reciprocal of the divisor. Cancel out common factors where it is possible to save reduction of the quotient.

Find the quotient:

1. $\frac{1}{3}\overline{)\,\frac{1}{5}}$
2. $\frac{1}{2}\overline{)\,\frac{2}{3}}$
3. $\frac{1}{2}\overline{)\,\frac{3}{8}}$
4. $\frac{3}{5}\overline{)\,\frac{3}{8}}$
5. $\frac{5}{9}\overline{)\,\frac{5}{6}}$
6. $\frac{2}{5}\overline{)\,\frac{3}{4}}$
7. $\frac{5}{6}\overline{)\,\frac{7}{8}}$
8. $\frac{3}{4}\overline{)\,\frac{7}{8}}$
9. $\frac{1}{2}\overline{)\,\frac{2}{5}}$
10. $\frac{5}{9}\overline{)\,\frac{5}{18}}$
11. $\frac{3}{4}\overline{)\,\frac{7}{12}}$
12. $\frac{2}{3}\overline{)\,\frac{1}{6}}$
13. $\frac{2}{3}\overline{)\,\frac{4}{5}}$
14. $\frac{1}{8}\overline{)\,\frac{3}{16}}$
15. $\frac{2}{7}\overline{)\,\frac{2}{21}}$
16. $\frac{6}{7}\overline{)\,\frac{9}{14}}$
17. $\frac{3}{4}\overline{)\,\frac{9}{10}}$
18. $\frac{3}{4}\overline{)\,\frac{5}{16}}$
19. $\frac{3}{4}\overline{)\,\frac{3}{16}}$
20. $\frac{1}{3}\overline{)\,\frac{2}{5}}$
21. $\frac{5}{6}\overline{)\,\frac{4}{9}}$
22. $\frac{1}{3}\overline{)\,\frac{1}{6}}$
23. $\frac{2}{3}\overline{)\,\frac{2}{9}}$
24. $\frac{5}{8}\overline{)\,\frac{15}{16}}$

Tests in Division of Fractions

A

E = 5 min.	G = $6\frac{1}{2}$ min.	F = 8 min.
1. $2\overline{)\frac{3}{4}}$	11. $6\overline{)26\frac{2}{3}}$	21. $6\overline{)33\frac{1}{3}}$
2. $7\overline{)\frac{7}{8}}$	12. $3\overline{)13\frac{1}{3}}$	22. $9\overline{)8\frac{1}{2}}$
3. $\frac{3}{4}\overline{)\frac{5}{6}}$	13. $\frac{5}{6}\overline{)8}$	23. $5\overline{)2\frac{2}{3}}$
4. $4\overline{)8\frac{1}{3}}$	14. $4\overline{)3\frac{1}{8}}$	24. $\frac{3}{4}\overline{)\frac{7}{12}}$
5. $5\overline{)11\frac{1}{2}}$	15. $\frac{1}{2}\overline{)\frac{3}{8}}$	25. $3\overline{)13\frac{1}{2}}$
6. $\frac{2}{3}\overline{)\frac{4}{9}}$	16. $\frac{3}{4}\overline{)\frac{9}{16}}$	26. $\frac{2}{5}\overline{)6}$
7. $4\overline{)\frac{8}{9}}$	17. $\frac{3}{4}\overline{)\frac{7}{32}}$	27. $\frac{7}{2}\overline{)\frac{7}{5}}$
8. $\frac{2}{3}\overline{)8}$	18. $6\overline{)3\frac{1}{2}}$	28. $\frac{5}{2}\overline{)\frac{5}{4}}$
9. $\frac{2}{3}\overline{)\frac{1}{2}}$	19. $\frac{3}{4}\overline{)9}$	29. $\frac{21}{4}\overline{)\frac{7}{5}}$
10. $9\overline{)18\frac{3}{5}}$	20. $4\overline{)121\frac{1}{4}}$	30. $8\overline{)576\frac{2}{3}}$

B

E = 3 min.	G = $4\frac{1}{2}$ min.	F = 6 min.
1. $5\overline{)34\frac{1}{2}}$	5. $8\overline{)34\frac{3}{4}}$	9. $9\overline{)30\frac{3}{5}}$
2. $4\overline{)36\frac{3}{4}}$	6. $2\overline{)14\frac{3}{4}}$	10. $7\overline{)15\frac{2}{5}}$
3. $3\overline{)14\frac{2}{3}}$	7. $4\overline{)26\frac{5}{6}}$	11. $5\overline{)9\frac{1}{6}}$
4. $6\overline{)27\frac{3}{4}}$	8. $5\overline{)16\frac{2}{3}}$	12. $6\overline{)7\frac{1}{5}}$

C

E = 4 min.	F = 5 min.	G = 6 min.
1. $\frac{1}{2}\overline{)\frac{3}{4}}$	6. $\frac{1}{3}\overline{)\frac{3}{4}}$	11. $\frac{7}{8}\overline{)1\frac{3}{4}}$
2. $\frac{5}{6}\overline{)\frac{2}{3}}$	7. $\frac{2}{3}\overline{)3\frac{1}{2}}$	12. $3\overline{)5\frac{1}{2}}$
3. $\frac{2}{3}\overline{)\frac{3}{4}}$	8. $4\overline{)1\frac{1}{2}}$	13. $4\overline{)2\frac{1}{2}}$
4. $\frac{1}{3}\overline{)\frac{2}{5}}$	9. $\frac{4}{5}\overline{)2\frac{1}{2}}$	14. $\frac{5}{6}\overline{)\frac{15}{16}}$
5. $\frac{3}{4}\overline{)\frac{2}{5}}$	10. $7\overline{)4\frac{1}{2}}$	15. $\frac{2}{3}\overline{)\frac{4}{9}}$

Drill Exercises in Division

Divide and check:

1. $5\overline{)\frac{3}{4}}$	13. $3\overline{)2\frac{1}{2}}$	25. $\frac{2}{3}\overline{)\frac{3}{4}}$
2. $4\overline{)\frac{2}{3}}$	14. $2\overline{)3\frac{1}{3}}$	26. $\frac{3}{4}\overline{)\frac{5}{6}}$
3. $5\overline{)\frac{15}{16}}$	15. $3\overline{)4\frac{1}{5}}$	27. $\frac{2}{3}\overline{)\frac{1}{2}}$
4. $3\overline{)\frac{1}{2}}$	16. $5\overline{)2\frac{2}{3}}$	28. $\frac{3}{4}\overline{)\frac{5}{8}}$
5. $2\overline{)\frac{5}{6}}$	17. $6\overline{)4\frac{1}{3}}$	29. $\frac{5}{6}\overline{)\frac{2}{9}}$
6. $5\overline{)\frac{3}{7}}$	18. $8\overline{)5\frac{1}{2}}$	30. $\frac{4}{5}\overline{)\frac{3}{8}}$
7. $8\overline{)\frac{4}{5}}$	19. $4\overline{)6\frac{3}{4}}$	31. $\frac{3}{4}\overline{)\frac{1}{2}}$
8. $2\overline{)\frac{7}{8}}$	20. $6\overline{)7\frac{1}{2}}$	32. $\frac{1}{2}\overline{)\frac{3}{4}}$
9. $10\overline{)\frac{5}{6}}$	21. $5\overline{)8\frac{2}{3}}$	33. $\frac{2}{3}\overline{)\frac{7}{8}}$
10. $12\overline{)\frac{6}{7}}$	22. $4\overline{)2\frac{3}{4}}$	34. $\frac{7}{9}\overline{)\frac{5}{6}}$
11. $3\overline{)\frac{3}{8}}$	23. $3\overline{)2\frac{2}{5}}$	35. $\frac{3}{4}\overline{)\frac{15}{16}}$
12. $8\overline{)\frac{7}{8}}$	24. $5\overline{)12\frac{1}{2}}$	36. $\frac{5}{6}\overline{)\frac{5}{9}}$

37. $1\frac{1}{2}\overline{)2\frac{2}{3}}$	47. $8\overline{)34\frac{1}{2}}$
38. $1\frac{3}{4}\overline{)3\frac{1}{2}}$	48. $9\overline{)76\frac{2}{3}}$
39. $2\frac{1}{2}\overline{)5\frac{2}{3}}$	49. $7\overline{)79\frac{3}{4}}$
40. $5\frac{1}{3}\overline{)6\frac{1}{4}}$	50. $5\overline{)86\frac{2}{3}}$
41. $3\frac{5}{6}\overline{)8\frac{2}{3}}$	51. $8\overline{)102\frac{5}{6}}$
42. $4\frac{5}{8}\overline{)9\frac{1}{4}}$	52. $9\overline{)134\frac{3}{8}}$
43. $5\frac{1}{3}\overline{)7\frac{1}{2}}$	53. $8\overline{)276\frac{4}{5}}$
44. $2\frac{1}{3}\overline{)6\frac{3}{4}}$	54. $5\overline{)120\frac{5}{6}}$
45. $5\frac{1}{2}\overline{)3\frac{1}{4}}$	55. $4\overline{)139\frac{1}{3}}$
46. $1\frac{2}{3}\overline{)1\frac{7}{8}}$	56. $3\overline{)281\frac{3}{4}}$

DIVISION OF DECIMALS

The actual computation in the division of decimals is the same as that for whole numbers. The chief difficulty met in division is the proper placing of the decimal point in the quotient.

Placing the Decimal Point in Quotients

1. From 25 peach trees John's father picked 61.5 bu. of peaches. Find the average number of bushels per tree.

$$
\begin{array}{r}
2.46\\
25\overline{)61.50}\\
\underline{50}\\
11\,5\\
\underline{10\,0}\\
1\,50\\
\underline{1\,50}
\end{array}
$$

You know that the answer is more than 2 *and less than* 3. *Why?*

So it must be 2.46 *and not* .246 *or* 24.6.

If the divisor is a whole number, the decimal point in the quotient is directly above the decimal point in the dividend when each quotient figure is placed directly above the right-hand figure of the number being divided.

2. From 2.5 acres Frank's father raised 61.5 bu. of wheat. Find the average number of bushels per acre.

$$\begin{array}{r} 24.6 \\ 2.5\,\overline{)\,61.50} \end{array}$$

Since 20 × 2.5 = 50 *and* 30 × 2.5 = 75, *you know that the quotient is between* 20 *and* 30; *so it is* 24.6 *and not* 2.46 *or* 246.

3. From .25 of an acre a farmer in Maine raised 61.5 bu. of potatoes. Find the yield per acre.

$$\begin{array}{r} 246 \\ .25\,\overline{)\,61.50} \end{array}$$

Since 200 × .25 = 50 *and* 300 × .25 = 75, *you know the quotient is between* 200 *and* 300; *so it is* 246.

The decimal point in the quotient is as many places to the right of the decimal point in the dividend as there are decimal places in the divisor, if each quotient figure is placed directly over the right-hand figure of the number being divided (the partial dividend used).

Tell where the decimal point should be placed in the quotient (annex or prefix zeros if necessary):

1. $\begin{array}{r} 101 \\ 1.25\,\overline{)\,12625} \end{array}$

2. $\begin{array}{r} 101 \\ 1.25\,\overline{)\,126.25} \end{array}$

3. $\begin{array}{r} 101 \\ 12.5\,\overline{)\,12625} \end{array}$

4. $\begin{array}{r} 101 \\ 125\,\overline{)\,1.2625} \end{array}$

5. $\begin{array}{r} 101 \\ .125\,\overline{)\,12.625} \end{array}$

6. $\begin{array}{r} 101 \\ 125\,\overline{)\,126.25} \end{array}$

7. $\begin{array}{r} 101 \\ 12.5\,\overline{)\,.12625} \end{array}$

8. $\begin{array}{r} 101 \\ .125\,\overline{)\,126.25} \end{array}$

9. $\begin{array}{r} 101 \\ 12.5\,\overline{)\,1262.5} \end{array}$

10. $\begin{array}{r} 101 \\ .125\,\overline{)\,12625} \end{array}$

11. $\begin{array}{r} 101 \\ 12.5\,\overline{)\,12.625} \end{array}$

12. $\begin{array}{r} 101 \\ 125\,\overline{)\,.12625} \end{array}$

13. $\begin{array}{r} 101 \\ .125\,\overline{)\,1.2625} \end{array}$

14. $\begin{array}{r} 101 \\ 1.25\,\overline{)\,12.625} \end{array}$

15. $\begin{array}{r} 101 \\ .125\,\overline{)\,1262.5} \end{array}$

A Test in Pointing off Quotients

In the following, prefix or annex zeros when necessary and place the decimal point where it should be to make the equation true:

1. $4.41 \div 0.42 = 105$
2. $97.28 \div 3.2 = 304$
3. $752 \div 0.16 = 47$
4. $95.7 \div 2.9 = 33$
5. $25.11 \div 0.27 = 93$
6. $7.011 \div 5.7 = 123$
7. $70.918 \div 0.059 = 1202$
8. $8.7 \div 0.087 = 1$
9. $20.976 \div 0.76 = 276$
10. $44.472 \div 1.02 = 436$
11. $0.7866 \div 85.5 = 92$
12. $879.79 \div 0.907 = 97$
13. $472.131 \div 6.27 = 753$
14. $6.2222 \div 58.7 = 106$
15. $1747.2 \div 0.312 = 56$
16. $57.629 \div 0.0143 = 403$
17. $2898.75 \div 0.0125 = 2319$
18. $3.9346 \div 1.03 = 382$
19. $125.6640 \div 3.1416 = 40$
20. $18.225 \div 13.5 = 135$

Exercises for Practice

1. $4.56 \div 2.7$
2. $9.07 \div 4.3$
3. $8.96 \div .56$
4. $19.75 \div 9.3$
5. $17.03 \div .76$
6. $2.965 \div .89$
7. $35.62 \div .175$
8. $568.2 \div 7.96$
9. $2.845 \div 91.7$
10. $184.6 \div 9.13$
11. $1.732 \div 75.2$
12. $1.09 \div .687$

Test in Division

See if you can get the correct result for each. If there are any that you cannot get, find out how to divide them and practice until you can do them quickly.

1.	**2.**	**3.**	**4.**	**5.**
$8\,)\,\underline{560}$	$5\,)\,\underline{2515}$	$.6\,)\,\underline{19.62}$	$.7\,)\,\underline{6.3}$	$.0004\,)\,\underline{.8}$
6.	**7.**	**8.**	**9.**	**10.**
$\frac{2}{7}\,)\,\underline{\frac{6}{7}}$	$\frac{2}{3}\,)\,\underline{\frac{3}{4}}$	$3\,)\,\underline{\frac{6}{7}}$	$3\,)\,\underline{178\frac{3}{4}}$	$\frac{3}{4}\,)\,\underline{2\frac{1}{3}}$

11.	12.	13.
$1\frac{1}{2}\overline{)\frac{5}{8}}$	$2\frac{1}{3}\overline{)8\frac{1}{2}}$	$\frac{1}{2}\overline{)\frac{5}{8}}$
14.	**15.**	**16.**
$\frac{3}{4}\overline{)6.5}$	$1\frac{1}{2}\overline{)8.4}$	$2\frac{1}{4}\overline{)7.2}$

DIVISION OF COMPOUND NUMBERS

When dividing compound numbers by an abstract divisor, we may change the dividend to units of the smaller group and divide as in whole numbers. It may be necessary to change the quotient to a higher group. Or, such numbers may be divided as shown below.

1. If a piece of ribbon 9 yd. 12 in. long is cut into 6 equal pieces, what is the length of each?

6) 9 yd. 12 in. 1 yd. 20 in.	*Such numbers need not be changed to a single unit before dividing.* *Think, "9 yd.* ÷ 6 = 1 *yd. and* 3 *yd. remaining."* *Write* 1 *yd. Think, "*3 *yd.* 12 *in.* = 120 *in.;* 120 *in.* ÷ 6 = 20 *in." Write* 20 *in.*

2. John has 5 yd. 6 in. of copper wire which he wishes to divide into 3 equal pieces. How long will each piece be?

3. Nell has 4 yd. 8 in. of ribbon which she wishes to cut into badges 8 inches long. How many will it make?

4 yd. 8 in. = 152 in. $8\,\overline{)\,152}$ $\quad 19$ Or, 4 yd. 8 in. = $4\frac{2}{9}$ yd. 8 in. = $\frac{2}{9}$ yd. $4\frac{2}{9} \div \frac{2}{9} = \frac{9}{2} \times \frac{38}{9} = 19.$ $\frac{2}{9}\,\overline{)\,4\frac{2}{9}}$ $\quad 19$	*In such problems, both dividend and divisor must be expressed in like units.* *Which of the two ways given here do you prefer?*

4. Henry has 3 bu. 2 qt. of feed for his rabbits. If he uses 2 qt. per day, how long will it last?

Divide:

5. 8 yd. 2 ft. by 4.
6. 8 yd. 2 ft. by 4 ft.
7. 10 gal. 2 qt. by 6.
8. 10 gal. 2 qt. by 6 qt.
9. 8 hr. 20 min. by 4.
10. 8 hr. 20 min. by 4 min.
11. 8 bu. 3 pk. by 5.
12. 8 bu. 3 pk. by 5 pk.

HOW TO USE THE REMAINDERS

1. From 17 yd. of material, how many dresses each requiring 4 yd. can be made?

$4\,\overline{)\,17}$ 4, 1 rem.	*Here the quotient is 4 and 1 yd. remaining. Hence the answer is, "4 dresses and 1 yd. remaining."* *To have answered, "$4\frac{1}{4}$ dresses," would have been foolish.*

2. If 17 yd. of material is cut into 4 equal pieces, what is the length of each?

$4\overline{)17}$ $4\frac{1}{4}$	*Here the quotient should be expressed as a fraction. Hence the answer is $4\frac{1}{4}$ yd.*

3. From $9\frac{1}{2}$ yd., how many aprons can be made each requiring $1\frac{7}{8}$ yd.?

$9\frac{1}{2} \div 1\frac{7}{8} = \frac{8}{15} \times \frac{19}{2} = 5\frac{1}{15}$ $\frac{1}{15} \times 1\frac{7}{8}$ yd. $= \frac{1}{8}$ yd. So $\frac{1}{8}$ yd. remains.	*Here the 1 remaining when dividing 76 by 15 was not yards. But $5\frac{1}{15}$ shows that the $9\frac{1}{2}$ yd. piece is $5\frac{1}{15}$ times $1\frac{7}{8}$ yd.* *So there could be 5 aprons and $\frac{1}{15}$ of $1\frac{7}{8}$ yd. left.*

4. Mrs. Brown bought a bolt of curtain material containing $22\frac{3}{4}$ yd. How many curtains each containing $2\frac{1}{4}$ yd. can she make? How should you express the remainder?

5. Thirty children went on a picnic. The total cost was \$12.35. If they wish to share equally, how much should each pay?

Since they cannot share equally, show how much each should pay.

6. Mrs. Brown bought 12 oranges for 50¢. What did each orange cost her?

$12\overline{)50¢}$ $4\frac{1}{6}¢$	*Here you wish the exact cost, which is $4\frac{1}{6}$¢.*

7. Mrs. Smith paid \$1.75 for a remnant containing 4 yd. Find the cost of each yard. How should the answer be expressed?

8. Mrs. Brown bought $\frac{1}{2}$ yd. of ribbon marked 95¢ per yd. How much will she have to pay for it?

$2\,\overline{)\,95¢}$ $47\frac{1}{2}¢$ So, 48¢.	*Here she could not pay $47\frac{1}{2}$¢, for $\frac{1}{2}$¢ pieces are not coined. So she would have to pay 48¢, the next higher whole number.*

9. Ralph sold 20 hens and received \$21.75 for them. What did that average for each?

10. To raise \$21.75 to buy a picture, a school gave an entertainment. How many tickets must be sold at 20¢ each to raise the amount?

11. At 20¢ each, how many toys can a dealer buy for \$21.75?

The answer to a problem may require:

1. **The exact quotient in which the last remainder is expressed as a fraction;**
2. **The next whole number above or below the quotient when there is a remainder; or,**
3. **The quotient and a remainder.**

MAKING PROBLEMS

1. Make a division problem whose answer shows the size into which something has been divided.

2. Make a division problem in which the answer will show the relation of the dividend to the divisor.

3. Make a division problem that will require a fraction in the quotient to give the real answer to the question.

4. Make a division problem in which the quotient must be a whole number, leaving a remainder not divided.

5. Make a division problem that requires the nearest, or the next whole number to the exact quotient, to answer the question.

DIVIDING BY MULTIPLES OF 10 OR 100

Before dividing by such numbers as 200, 300, etc., cut off the zeros from the divisor and move the decimal point in the dividend as many places to the left. What fundamental principle allows you to do this?

Thus,

$$346 \div 20 = 34.6 \div 2 = 17.3;$$
$$48.5 \div 20 = 4.85 \div 2 = 2.425.$$

Give without a pencil:

1. $738 \div 20$	6. $618 \div 200$	11. $354 \div 300$
2. $534 \div 30$	7. $375 \div 300$	12. $810 \div 900$
3. $456 \div 40$	8. $564 \div 400$	13. $72.9 \div 900$
4. $365 \div 50$	9. $86.4 \div 400$	14. $65.6 \div 800$
5. $396 \div 60$	10. $76.8 \div 200$	15. $720 \div 800$

DIVIDING BY ALIQUOT PARTS OF 10 OR 100

1. Show that

$$465 \div 25 = 4 \times 4.65 = 18.6.$$
$$536 \div 33\tfrac{1}{3} = 3 \times 5.36 = 16.08.$$
$$394 \div 12\tfrac{1}{2} = 8 \times 3.94 = 31.52.$$

2. Tell how to divide by each of the aliquot parts of 10 and 100.

Find the quotients:

3. $436 \div 25$	8. $342 \div 5$	13. $36.4 \div 25$
4. $176 \div 12\frac{1}{2}$	9. $426 \div 33\frac{1}{3}$	14. $73.4 \div 33\frac{1}{3}$
5. $348 \div 25$	10. $175 \div 16\frac{2}{3}$	15. $82.3 \div 12\frac{1}{2}$
6. $196 \div 8\frac{1}{3}$	11. $240 \div 12\frac{1}{2}$	16. $54.6 \div 16\frac{2}{3}$
7. $135 \div 3\frac{1}{3}$	12. $580 \div 25$	17. $75.2 \div 8\frac{1}{3}$

A REVIEW OF TOPICS LEARNED IN THIS CHAPTER

1. What does addition mean? Give two ways of checking it.

2. Illustrate the three uses of subtraction. Give the ways of checking subtraction.

3. Give three ways of checking multiplication.

4. Illustrate the two uses of division.

5. Make three problems to illustrate each of the three ways of expressing a remainder.

6. What is meant by an aliquot part? Give the most commonly used aliquot parts.

7. Show how to multiply and divide two numbers by using the method of aliquot parts.

8. Show how to square a two-figured number ending in 5.

9. Give a rule for pointing off products in decimals.

10. Show how to place the decimal point in the quotient.

11. What is the reciprocal of a number?

A Test in Checking the Work

Check the following to see if the work is correct. Check the multiplication and division by casting out nines. Then check the work by some other method and see which method is the more reliable.

1.	2.	3.	4.
356	487	83,564	$437.62
672	635	24,638	129.15
491	803	58,826	$308.47
538	971		
747	748		
285	557		
3089	4201		

5.

$$\begin{array}{r} 349 \\ 9 \\ \hline 3041 \end{array}$$

6.

$$\begin{array}{r} 458 \\ 307 \\ \hline 16{,}946 \end{array}$$

7.

$$\begin{array}{r} 436 \\ 48 \\ \hline 20{,}928 \end{array}$$

8.

$$\begin{array}{r} 393 \\ 37\overline{)14{,}539} \end{array}$$

9.

$$\begin{array}{r} 8.55 \\ 9.2\overline{)786.60} \end{array}$$

10.

$$\begin{array}{r} 204 \\ 34\overline{)6276} \end{array}$$

11.

$$\begin{array}{r} 2\frac{1}{4} \\ 3\frac{1}{2} \\ 6\frac{7}{8} \\ 7\frac{3}{4} \\ \hline 21\frac{3}{8} \end{array}$$

12.

$$\begin{array}{r} 4\frac{3}{4} \\ 5\frac{7}{16} \\ 8\frac{1}{2} \\ 3\frac{1}{4} \\ \hline 23\frac{7}{16} \end{array}$$

A Test in Mixed Fundamentals

1. $8.5 + 0.735 + 16.04 + 23 =$
2. $4\frac{1}{8} + 6\frac{1}{2} + 7\frac{3}{4} + 2\frac{3}{8} =$
3. $58{,}603 - 39{,}856 =$
4. $9\frac{1}{4} - 6\frac{7}{8} =$
5. $34 \times 6.56 =$
6. $.087 \times 93.5 =$
7. $4\frac{1}{8} \times 2\frac{2}{3} =$
8. $3 \times 4\frac{4}{5} =$
9. $53{,}729 \div 48 =$
10. $764.03 \div 5.8 =$
11. $6\frac{3}{4} \div 4 =$
12. $7\frac{1}{8} \div 3\frac{1}{2} =$

A Test in Arranging in Order of Value

Arrange in order of value, beginning with the largest:

1. $\frac{5}{13}$, $\frac{8}{13}$, $\frac{2}{13}$, $\frac{9}{13}$, $\frac{4}{13}$, $\frac{7}{13}$.
2. $\frac{5}{8}$, $\frac{5}{13}$, $\frac{5}{9}$, $\frac{5}{7}$, $\frac{5}{6}$.
3. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{5}{6}$, $\frac{7}{12}$.

4. .4, .45, .413, .4058, .48, .482.

5. $\frac{1}{2}$, .48, .534, $\frac{3}{5}$, $\frac{11}{20}$.

6. .735, $\frac{3}{4}$, .8, $\frac{2}{3}$, .704.

7. .635, $\frac{2}{3}$, .65, $\frac{13}{18}$, $\frac{3}{4}$.

8. 1.35, $1\frac{1}{2}$, $\frac{149}{100}$, $1\frac{3}{5}$, $1\frac{1}{3}$.

9. $\frac{37}{100}$, .365, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$.

10. $\frac{3}{4}$, $\frac{2}{3}$, .675, .7, $\frac{65}{100}$.

A True and False Test

DIRECTIONS: In this test some of the statements are *true* and some are *false*. Read them carefully and decide which are *true* and which are *false*.

For example, in the following, statement A is *true* and statement B is *false*.

A. The sum of two numbers is always greater than either one of the numbers.

B. The difference between two numbers is always less than the smaller of the two.

E = 16. **G = 15.** **F = 14.**

1. If your work is correct in the addition of columns, the sum will be the same when beginning at the top and adding down as when beginning at the bottom and adding up.

2. Casting out nines is an absolute check for accuracy in multiplication.

3. The reciprocal of a fraction is a fraction with its terms interchanged.

4. If the decimal point in the dividend is moved to the right as many places as there are decimal places in the divisor, the point in the quotient is placed directly above the point in the dividend.

5. If you know the product of two numbers and one of them, you can find the other by subtracting the given number from the product.

6. If you multiply or divide both the dividend and the divisor by the same number, the quotient will remain the same.

7. If you add or subtract the same number from the dividend and the divisor, the quotient will remain the same.

8. If you diminish the divisor and leave the dividend unchanged, the quotient will be diminished by the same amount.

9. If you know the sum of three numbers and two of them, you can find the third number by finding the difference between the sum of the given numbers and the given sum.

10. Multiplying both terms of a fraction by the same number does not change its value.

11. Adding or subtracting the same number from each term of a fraction does not change the value of the fraction.

12. To multiply two fractions with like denominators, multiply the numerators only and place the product over the denominator of either.

13. To divide a fraction by a whole number, multiply the numerator by the number.

14. You may add two fractions by adding the numerators and the denominators.

15. You may divide any number by multiplying it by the reciprocal of the divisor.

16. In multiplication or division of fractions, if all common factors are cancelled out, the product or the quotient cannot be reduced.

A Completion Test

E = 18. **G = 16.** **F = 14.**

Fill the blank spaces with the correct word.

1. If you know the larger of two numbers and their difference, you can find the smaller by ———— the ———— from the ———— number.

2. If you know the difference between two numbers and the smaller, you can find the larger by ———— the difference ———— the ———— number.

3. If you know the product of three numbers and two of them, you can find the third number by ———— the given product by the ———— of the other two.

4. If you know the quotient and the divisor, you can find the dividend (number that was divided) by ———— the ———— by the ————.

5. When the quotient and the dividend are known, the divisor may be found by ———— the dividend by the ————.

6. If the product and the multiplicand (number that was multiplied) are known, the multiplier may be found by ———— the ———— by the ————.

7. If you divide the divisor by any number, the quotient will be ———— by the ———— number.

8. If you multiply the divisor by any number, the quotient will be ———— by the ———— number.

9. If you divide the dividend by any number, the quotient will be ———— by that number.

10. If you multiply the dividend by any number, the quotient will be ———— by that number.

11. If you multiply the multiplier by any number, the product will be ———— by that number.

12. If you divide the multiplier by any number, the product will be ———— by that number.

13. If you divide the multiplicand by any number, the product will be ———— by that number.

14. If you multiply the multiplicand by any number, the product will be ———— by that number.

15. If you multiply both multiplier and multiplicand by the same number, the product will be ———— by the number ————.

16. If you multiply both dividend and divisor by the same number, the quotient will be ————.

17. If you divide a number by a fraction less than one the quotient will be ———— than the ————.

18. If a fraction less than one is multiplied by a whole number, the product will be ———— than the ————.

CHAPTER II

GRAPHIC WAYS OF SHOWING RELATIONS

The relation between a series of numbers is often shown by the use of **graphs.** In that way important facts are stressed which might be overlooked in a table of numerical values.

In this chapter we shall study several of the ways in which number relations may be pictured. Notice how clearly the graphs tell the story represented by the numbers.

In constructing any kind of graph you must be careful of two things:

1. The graph must truthfully represent the data given by the numbers.

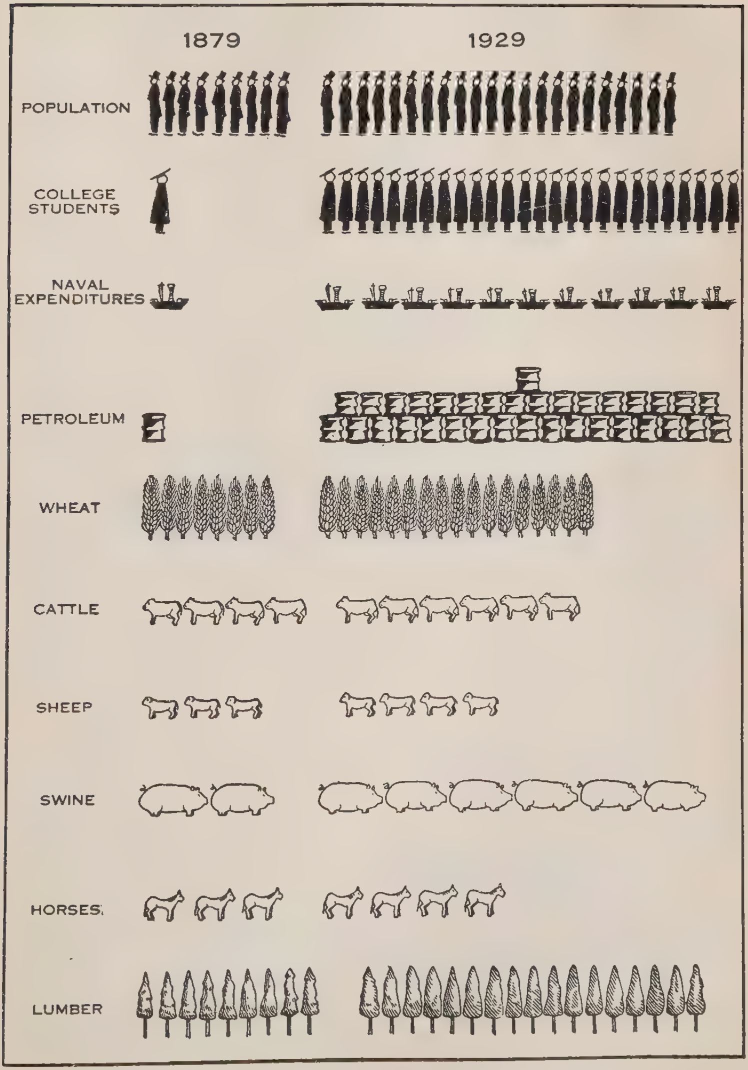
2. The graph must be so simple that the relations it represents are clearly seen.

THE PICTURE GRAPH

The graph on page 53 was made from the following table of values:

	1879	1929
Population	50,155,783	121,000,000
College students.........	37,000	973,000
Naval expenditures......	$38,116,916	$416,901,546

FIFTY YEARS OF PROGRESS IN THE UNITED STATES



Production of:	1879	1929
Petroleum	26,286,000 bbl.	902,000,000 bbl.
Wheat	425,054,000 bu.	903,000,000 bu.
Cattle	35,925,000	55,751,000
Sheep	35,192,000	47,171,000
Swine	17,682,000	54,956,000
Horses	10,357,000	14,029,000
Lumber	18,091,000,000 bd. ft.	34,142,000,000 bd. ft.

By referring to the graph answer the following questions:

1. Which items have increased more rapidly than the population?
2. Which items have increased less rapidly than the population?
3. How great has been the increase in the production of oil? How do you account for it?
4. How great has been the increase in enrollment of college students? Can you give some reasons for this increase?
5. Compare the rate of increase of the population with the rate of increase in the number of horses. How can this difference be accounted for?
6. Make other comparisons, giving reasons where possible, for the relatively slow or rapid growth.

How to Make a Picture Graph

In making a picture graph the following suggestions should be followed:

1. **Round off the numbers in the given data.**
2. **Determine the ratio in whole numbers which shows the increase or decrease.**
3. **Represent the numbers found in (2) by pictures.**

Thus, in showing the production of petroleum in the preceding graph the production given in the table is 26,286,000

bbl. for 1879 and 902,000,000 bbl. for 1929. Rounding these numbers off, we have 26 million and 900 million, and the ratio of these two numbers is about 1 to 34. Hence one oil barrel represents the production in 1879 and thirty-four the production in 1929.

NOTE. — If the production in 1879 were shown by a small barrel, that in 1929 could be shown by a barrel the same shape and 3.2 times as high. A barrel the same shape and 34 times as high would contain nearly 40,000 times as much oil!

EXERCISES

Make picture graphs illustrating the following facts:

1. In the United States 13 billion cu. ft. of soft woods are cut each year, while only 3 billion cu. ft. is added to our supply by new growth.

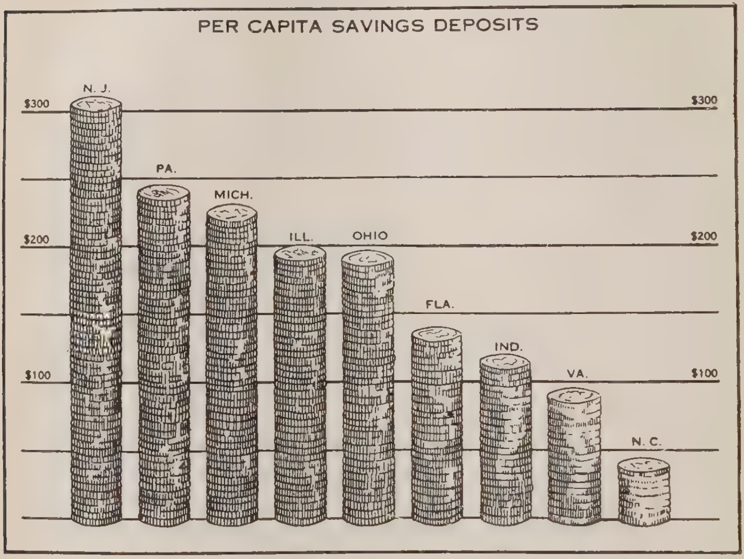
2. The supply of soft woods in the United States is 385 billion cu. ft. At the rate shown in Ex. 1 how long will this last?

3. The production of wheat throughout the world in 1928 is shown in the following table:

Europe (exc. Russia)	381,000,000 bu.
Russia	860,000,000 bu.
United States	902,000,000 bu.
Canada	310,000,000 bu.
Argentine	310,000,000 bu.
India	290,000,000 bu.
Australia	159,000,000 bu.
Other countries	236,000,000 bu.

4. In 1928 there was, in the United States, one automobile for each 5 inhabitants; in England, one for each 32; in Germany, one for each 118; in France, one for each 37; and in Canada, one for each 9.

5. The following graph shows the per capita savings bank deposits in some of the states.



(a) Assuming the population of New Jersey to be 4,000,000, what is the total amount in savings banks?

(c) If the population of Illinois is 7,500,000, what is the total amount deposited there?

6. The number of airplane passengers carried for hire in the United States is shown in the following table:

YEAR	NUMBER OF PASSENGERS
1926	5,782
1927	8,679
1928	33,414

NOTE. — Round off the numbers to the nearest thousand.

7. The following table shows how the public debt of the United States changed during a ten-year period:

1919	$25,482,000,000
1921	23,976,000,000
1923	22,350,000,000
1925	20,516,000,000
1927	18,510,000,000
1929	16,931,000,000

HINT.—Round off the numbers to the nearest billion. Then let a stack of 25 coins represent the debt for 1919.

8. In 1913 a dollar would purchase 17.9 lb. of bread. Picture graphically the amounts which could be purchased in other years, as shown in the table.

YEAR	LB.	YEAR	LB.
1913	17.9	1923	10
1915	12.9	1925	8.9
1917	7.8	1927	9.9
1919	7.5	1929	10.1
1921	9.3		

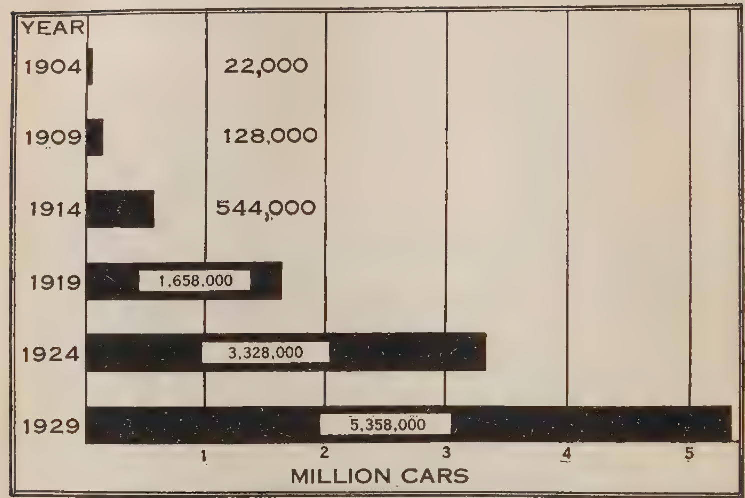
9. Protein in food is necessary for health. Represent in a picture graph the fact that there is the same amount of protein in each of the following foods: 2 lb. of potatoes; 5 oz. of eggs; 2.6 oz. of beans; 2.2 oz. of cheese.

THE BAR GRAPH

A simple way of representing clearly the relations expressed by numbers is by means of the bar graph.

In making the bar graph shown on page 58, the original numbers were first rounded off. It was then determined how long a space should be used to represent the largest number, the 5,400,000 cars produced in 1929. The scale used

PRODUCTION OF AUTOMOBILES IN THE UNITED STATES



above is $\frac{1}{8}''$ for each 200,000 cars. By referring to the graph, answer the following questions:

1. The number of cars produced in 1909 was how many times as great as the number produced in 1904? Make the same comparison for each five-year period and show your results in a bar graph.

2. If for each 8 cars produced in the United States a man is employed throughout the year, compare the number of men employed for each year shown in the graph and represent your results graphically.

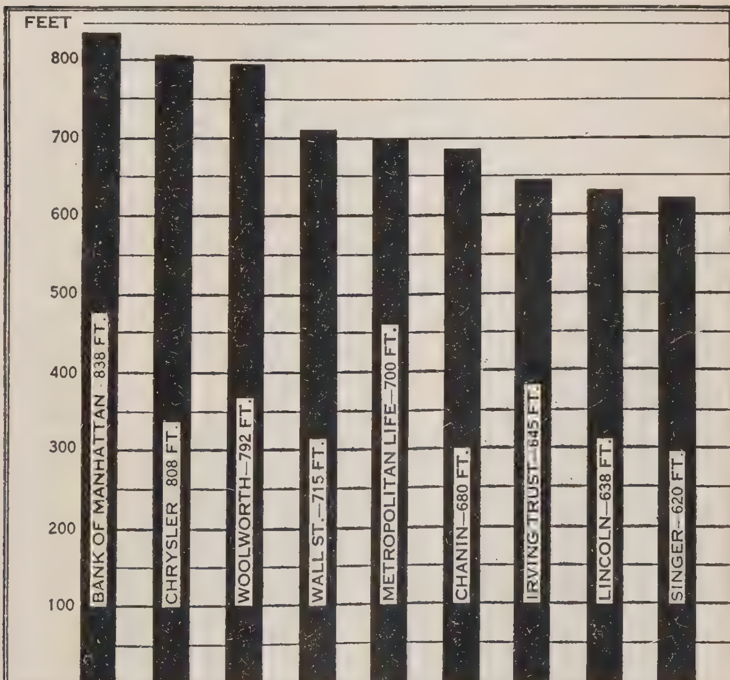
3. In 1924 there were 15,528 fatalities due to automobile accidents. In 1929 there were 31,060. Compare the rate of increase in the fatalities for the two years to the rate of increase in the number of cars.

The following principles should be observed in constructing bar graphs:

1. **The bars should all begin along the same line.**
2. **The bars should all be the same width.**
3. **The spaces between the bars should all be the same.**
4. **The scale used should begin at zero (0).**

In many cases a vertical bar graph will show the relations you wish to picture better than a horizontal graph.

HEIGHTS OF SOME TALL BUILDINGS



In the exercises, represent by bar graphs the statistics given:

1. The rapid growth in the use of automobiles in recent years has been accompanied by an increase in the number of fatal accidents caused by automobiles.

YEAR	FATALITIES	YEAR	FATALITIES
1924	15,528	1927	21,160
1925	17,751	1928	26,720
1926	18,871	1929	31,060

2. Example 1 shows the increase in the number of fatal accidents due to automobiles. The following table shows the number of such accidents per 100,000 population.

1924	157	1927	195
1925	170	1928	226
1926	179	1929	259

3. The heights of some famous structures are given in the following table. Represent them by a vertical bar graph.

STRUCTURE	LOCATION	HEIGHT
Eiffel Tower	Paris	984 ft.
Bank of Manhattan	New York	838 ft.
Manufacturers Club	Philadelphia	640 ft.
The Penobscot	Detroit	557.2 ft.
The Pittsfield	Chicago	557 ft.
Washington Monument	Washington, D. C.	555 ft.
U. S. Customs Tower	Boston	496 ft.
Pyramid of Cheops	Egypt	461 ft.
City Hall	Los Angeles	452 ft.
St. Peter's Cathedral	Rome	435 ft.
The highest building in your city		(?)

4. The United States Chamber of Commerce has made a study of reasons why customers enter a store and leave without making a purchase.

REASON FOR LEAVING	PER CENT
Indifference and discourtesy	16
Prices too high	15
Tactless handling of customers	14
Delays in store service	13

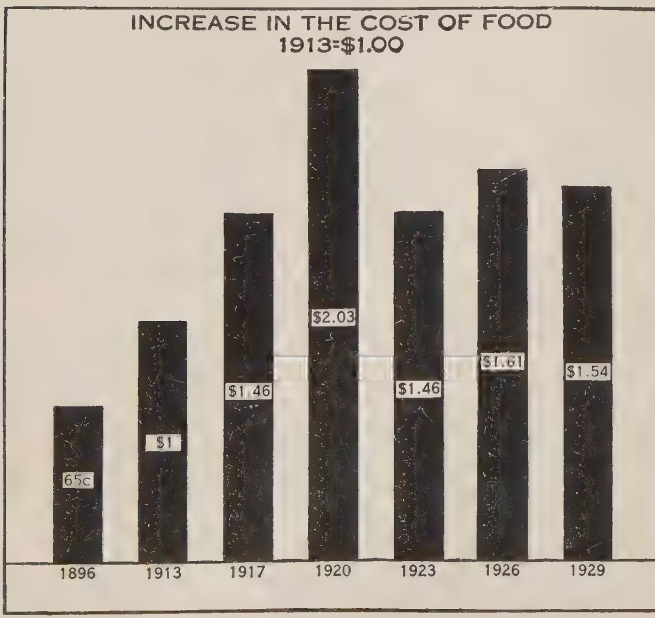
Objections to store policies 12
Tricky methods 12
Quality poor 10
Errors in service 5
Ignorance concerning goods 3

5. Make a bar graph to show the relative value of the following fractions:

$\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{16}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{7}{16}$, and $\frac{5}{8}$.

6. Make a similar graph for the following:

$\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{3}{4}$, and 1.



1. From the graph, in what year did food cost the most?

2. How much money would it take in 1929 to purchase food which would have cost one dollar in 1896?

3. In 1896, \$5 would be equivalent to how much in 1920?

4. If a family spent \$1500 for food in 1896, how much would a family of the same size require in 1920? In 1929?

5. If an income of \$2000 a year from money invested was just adequate to meet the needs of a widow in 1896, by how much would it fall short of meeting her needs in 1929?

6. A man's salary in 1913 was \$1800. In 1929 it was \$2500. Did this sum represent an increase in the amount of food he could purchase?

7. A man in 1913 retired with savings of \$50,000 invested at 6%. The income he received was just sufficient for his living expenses. By how much did his income fall short in 1929 of enough to live comfortably?

8. The degree of comfort in which a man can live is determined, not alone by the amount of money he receives for his labor, but by the amount of the necessities of life which his money will purchase. If the average amount of food which could be purchased by a week's work in 1913 is called 100, then by statistics of the U. S. Department of Labor it is shown that a week's work would purchase the following amounts of food:

YEAR	PURCHASING POWER
1913	100
1916	93.4
1919	79.5
1922	129.2
1925	141.2
1928	155.9

Represent the data graphically.

9. We know by the graph on page 61 that the cost of food has increased since 1913. We also know that wages have increased. What does the table in Example 8 show about the relative increase in both?

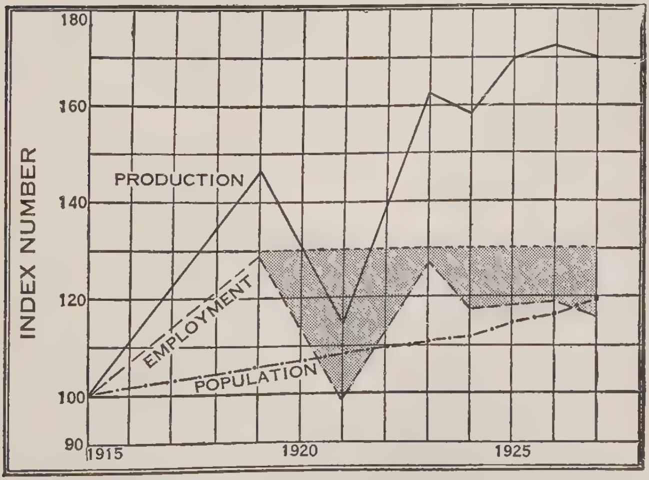
10. Make a vertical bar graph to show the number of business failures in the United States from 1915 to 1924.

1915....5,250	1919....1,650	1923....4,600
1916....4,200	1920....2,750	1924....5,350
1917....3,350	1921....4,900	
1918....2,500	1922....5,750	

11. Make a bar graph to show *Why School Costs Have Increased,* using the data given for the enrollment in the years 1912 and 1922.

	ENROLLMENT 1912	1922
Elementary School	17,078,000	20,360,000
High School	1,105,000	2,873,000
Elementary and High School	18,183,000	23,233,000

THE BROKEN-LINE GRAPH



Another means of graphically representing statistics is shown above. The solid line shows the output of American

manufacturing plants, while the broken line shows the number of workers employed in them. The growth in the population is shown by the dot and dash line.

Notice that the value of manufactured products has increased since 1914 even though the number of workers has decreased. The shaded portion of the graph shows the number of persons employed in manufacturing in 1919 for whom there has since been no need.

Exercises

1. From the graph on page 63, in which year were production and employment at the lowest level?

2. In which year was production at the peak?

3. What may have been the reason for this increased production with fewer laborers? How has this affected the problem of unemployment?

Make broken line graphs to represent the following:

4. The variation in the temperature during one cold day in the winter season is shown below:

12 (noon)	8°	12 (midnight)	−2°
2 P.M.	7°	2 A.M.	−1°
4 P.M.	6°	4 A.M.	2°
6 P.M.	4°	6 A.M.	5°
8 P.M.	1°	8 A.M.	9°
10 P.M.	0°	10 A.M.	13°
		12 (noon)	16°

5. Using 100 to represent the price of steak in 1913, the price has varied since 1913 as follows: 1913, 100; 1914, 102; 1915, 103; 1916, 108; 1917, 124; 1918, 153; 1919, 164; 1920, 172; 1921, 153; 1922, 147; 1923, 161; 1924, 168; 1925, 156.

6. A health poster stated: "If you would enjoy good health, get the proper amount of sleep." The following daily amounts were given for children of different ages:

4 yr.	12 hr.
5-7 yr.	11 hr.
8-11 yr.	10 hr.
12-14 yr.	9 hr.
adult	8 hr.

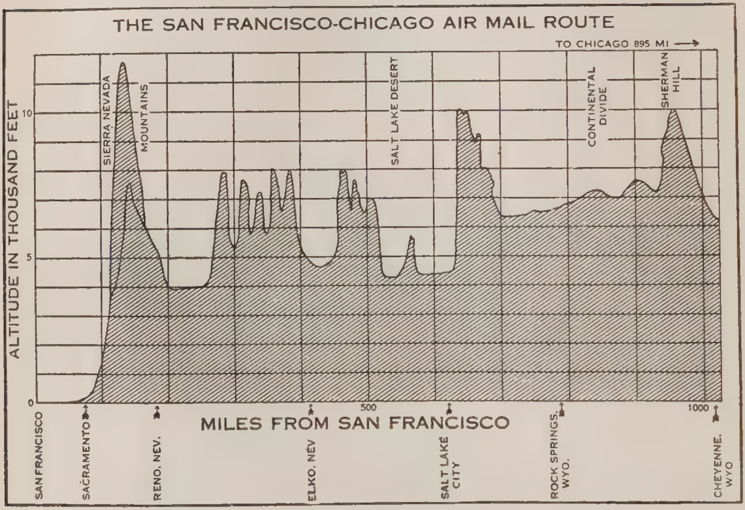
7. The average daily attendance in Miss Brown's room for one week was as follows: Mon. 96%; Tues. 98%; Wed. 91%; Thurs. 100%; Fri. 94%. During the same week, the attendance for the entire school was as follows: Mon. 95%; Tues. 90%; Wed. 93%; Thurs. 96%; Fri. 88%. On the same paper, make a broken-line graph to compare the attendance in Miss Brown's room with the attendance of the whole school.

8. The following table represents the amount by which the railroads speeded up the movement of freight between 1921 and 1929.

YEAR	AVERAGE DAILY MOVEMENT OF A FREIGHT CAR
1921	22.3 mi.
1922	24.1
1923	26.1
1924	27.8
1925	28.8
1926	29.6
1927	30.3
1928	31.2
1929	32.4

The following chart shows the altitudes of the mountains over which pilots driving mail planes must fly in going from Chicago to San Francisco. On account of clouds it is necessary

sometimes to ascend to a height of 15,000 or more feet. This graph is modeled after one which appeared in the *New York Times*, January 1, 1928.



9. From the graph, how far is it from San Francisco to Cheyenne, Wyo.?

10. How far is it from Reno, Nev., to Salt Lake City?

11. What is the approximate height of the Continental Divide?

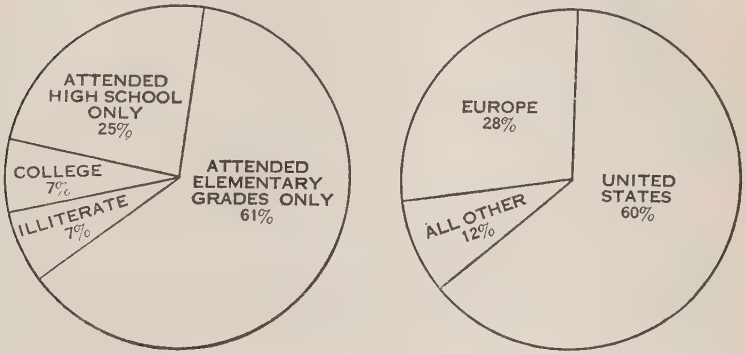
12. How high above sea level is the Salt Lake Desert?

13. About what must be the height of the plane in order to clear the highest peak along the route?

CIRCLE OR PIE GRAPHS

The "pie" graphs shown on page 67 illustrate a method of graphical representation used when it is desired to show the parts into which a whole quantity is divided. Thus the

graph on the left shows the amount of education of 69,000,000 persons in the United States. The graph at the right shows



the distribution of the world's telephones. Any graph which shows how a whole is divided into parts is called a **component parts** graph.

In making the Education graph the following steps were taken:

EDUCATION	NUMBER	%	ANGLE
Obtained in elementary grades only	42,198,000	61.13	220°
Obtained in high school only (1-4 yr.)	17,340,000	25.08	90°
Obtained in college (1-4 yr.)	4,637,000	6.69	25°
Illiterate	4,825,000	7.10	25°
Total	69,000,000	100%	360°

How to Make a Circle Graph

1. **Find the sum of the quantities to be used in making the graph.**

Thus, the total number of people over 21 years is 69,000,000.

2. **Find what per cent each item is of the whole.**

Thus, 42,198,000 ÷ 69,000,000 **equals** 61.13%; etc.

3. **Multiply each percentage by 360°.**

Thus, 61.13% of 360° = 220°; etc.

4. **Construct at the center of a circle angles equal to those found in (3).**

5. **Correctly label each sector, placing the percentage in each.**

Exercises

1. Of the 69,000,000 persons included in this study, only 27% graduated from elementary school. This is what per cent of those whose entire education was obtained in elementary grades?

2. Of those who attended high school, one-fourth graduated. What per cent of the entire 69,000,000 persons graduated from high school only?

3. Of those who attended college, 32% graduated. What per cent of the entire 69,000,000 persons are college graduates?

4. How many times as many telephones are there in the United States as in Europe?

5. The population of Europe is 480,000,000 and that of the United States about 120,000,000. If there is one telephone in the United States for each five persons, what is the ratio in Europe?

Construct circle graphs to represent the following data:

6. Over a 5-year period we consumed a yearly average of 180 lb. of meat per person, distributed as follows: pork, 92 lb.; beef, 82 lb.; mutton, 6 lb.

7. Make a circular graph to show how a certain city spent each dollar of its revenue in a recent year. The expenditures

were as follows: education, 30.8¢; interest, 20.8¢; fire department, 12.0¢; police, 9.0¢; streets, 9.0¢; sanitation, 6.9¢; general expenses, 7.9¢; parks, 2.6¢; library, 1.0¢.

8. In a certain boys' boarding school, the following program represents the daily time schedule in use. Make a circle and divide it into 24 equal parts to represent each hour during the day and then graph the schedule. The time allotment was as follows: sleep, 8 hr.; meals, 4 hr.; work, $1\frac{1}{2}$ hr.; school, $5\frac{1}{2}$ hr.; study, $2\frac{1}{2}$ hr.; recreation, $2\frac{1}{2}$ hr.

9. The year after John graduated from high school he tried to find out what his classmates were doing. He found that 40% were in college, 6% were in normal school, 21% were working, 12% were not accounted for, 8% were at home, 10% were attending business school, and 3% were in art and music schools.

10. In the continuation schools in New York State in 1929, it was found that many pupils were contributing weekly to the family income:

AMOUNT CONTRIBUTED	PER CENT
Over $16	2.8
$11–$15	25.
$6–$10	37.1
$1–$5	15.
Less than 50¢	9.3
Nothing	10.8

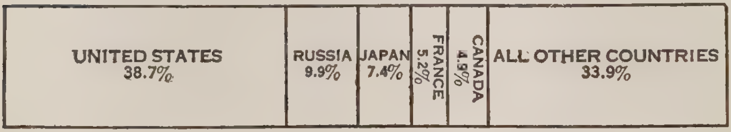
11. In 1929 the tonnage of ships constructed in the leading shipbuilding countries was:

Great Britain	1,560,254
Germany	253,256
Holland	231,934
Japan	183,570
United States	179,062
France	167,177

COMPONENT PARTS SHOWN BY A BAR GRAPH

The bar graph represents the parts of a whole even more clearly than the circle graph does, because it is easier to compare the relative lengths of the bars. If all the data are placed on your graph it is very easy for any one to make any comparisons he pleases.

THE IMPROVED ROADS OF THE WORLD



This graph shows the relative mileage of the improved roads of the different countries. It is made from the following statistics:

COUNTRY	MILEAGE	% OF TOTAL
United States	1,681,900	38.7
Russia	430,300	9.9
Japan	321,600	7.4
France	226,000	5.2
Canada	213,000	4.9
Other countries	1,473,200	33.9
Total	4,346,000	100.0

The entire bar shown in the graph above is 3.75 in. long. The part representing the mileage of the United States is $.387 \times 3.75 = 1.45$ in.; the part representing the mileage of Russia is $.099 \times 3.75 = .37$ in.; the part representing the mileage of Japan is $.074 \times 3.75 = .28$ in.; etc.

Thus the steps in making a component parts bar graph are:

1. **Find what percent each number in the given data is of the sum of all the numbers.**

2. **Select a convenient length for the whole bar graph. Multiply this length by the percentages found in (1). This will give the length of each part into which the bar is divided.**

3. **Properly label the graph and place the given data on it.**

Exercises

Show by means of bar graphs the component parts of the following:

1. In 1929 there was the following production of motor vehicles in the United States:

Passenger cars	4,586,020
Trucks	754,762
Taxicabs	17,389

2. Out of each 8¢ fare a street car company showed that the following amounts are expended:

Wages	4.27¢	Taxes	.67¢
Power, etc.	1.53¢	Stockholders	.64¢
Interest on bonds	.89¢		

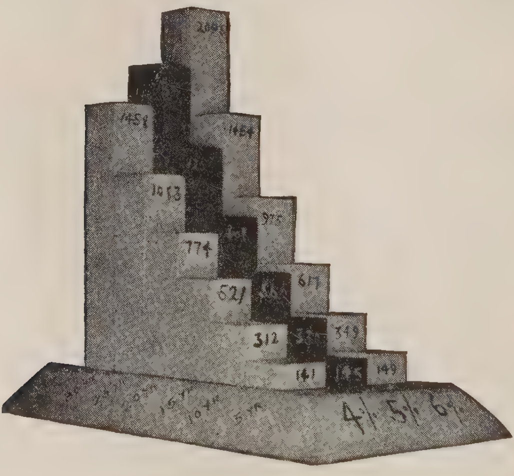
3. The student fee of $10 at a certain college is expended as follows:

		Societies:	
Athletics	$4.37	Literary	$1.07
Publications	2.54	Modern Language	1.09
		Scientific	.93

4. The number of acres of federal and state forest lands in several of the states follows:

Arizona	13,458,090	Minnesota	1,489,380
California	20,230,958	Montana	18,894,489
Colorado	14,445,882	New York	2,453,828

THE RESULTS OF SYSTEMATIC SAVINGS



The figure above shows a component parts graph of a more complicated form. The height of the prisms shows the amounts which can be accumulated in the different periods of time at different rates of interest over a period of years.

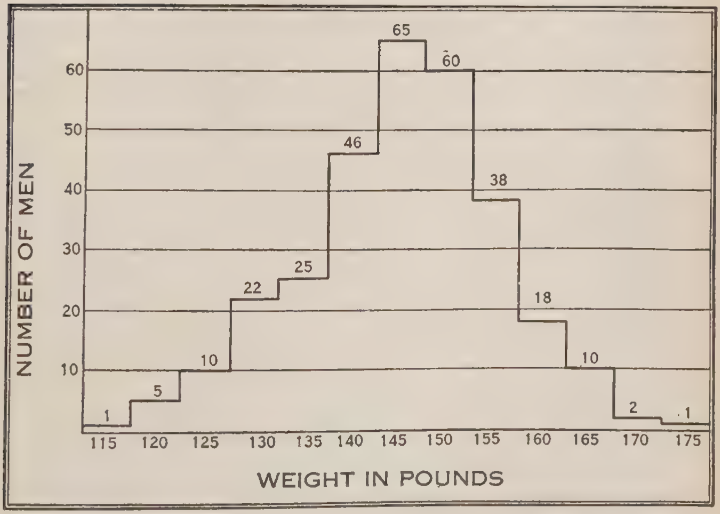
Thus, if \$25 be saved each year at 4% compound interest, in 5 years it will amount to \$141; in 10 years, to \$312; in 15 years, to \$521; in 30 years, to \$1458; while at 6% it will amount to \$149 in 5 years; to \$349 in 10 years; to \$975 in 20 years, etc.

Such a graph can be made of paper or wood or soap. You may wish to make one from the following data:

1. Amount accumulated by saving $5 a month at interest compounded every month:

	4%	5%	6%
10 yr.	$ 736	$ 776	$ 819
20 yr.	1834	2055	2310
30 yr.	3420	4161	5023
40 yr.	5910	7630	9958

DISTRIBUTION GRAPHS



The graph above shows the **distribution** of the weights of 303 men of a certain height and age. It shows that 1 of the men weighs about 115 lb., 5 weigh about 120 lb., . . . 65 weigh 145 lb., etc. From the graph it is evident that we do not find many men with a very small weight nor many men whose weight is very large. Most of them weigh from 140 to 150 lb.

In the same way you might make a distribution graph showing how the heights of the pupils in your class vary. You would find that only a few are very tall or very short. Or, if the members of the class took a test in one of the skills in arithmetic a few would be very poor while the large majority would have marks between the highest and the lowest. You would find the same result in studying many other things. If you counted the kernels on a large number of ears of corn you would again find that they would be distributed much as in the graph above.

Of course, it is impossible to represent by a bar each of the individuals in a large group. Thus, in the graph above, the 65 men whose weight is given as 145 lb., weighed from 142.5 lbs. to 147.5 lb., and they were classified into a smaller number of groups before the graph was drawn.

1. What percentage of the entire group of men are in the 145 lb. group?

2. What part of the entire group weighs from 140 lb. to 155 lb.?

3. If the men weighing more than 155 lb. are overweight, what percent is overweight?

4. If those weighing less than 140 lb. are underweight, what percent are underweight?

5. In a certain school the following grades are used: P = poor; F = fair; G = good; VG = very good; E = excellent. In a class of 45 pupils in arithmetic there were 5 P's, 11 F's, 22 G's, 4 VG's, and 3 E's. Show this by a distribution graph.

6. The grades made by 2000 pupils in arithmetic for one month were: 161 P; 553 F; 714 G; 492 VG; 80 E. Show the result by a distribution graph.

7. Make an age table for your class, grouping the class into five sections according to age. Make a distribution graph showing the result.

8. Make a distribution graph of the marks on the report cards of your class.

9. The height of 450 boys 12 years of age was taken. This is what was found:

NO.	HEIGHT IN INCHES	NO.	HEIGHT IN INCHES
3	47	85	54
5	48	58	55
12	49	38	56
30	50	22	57
38	51	12	58
60	52	4	59
80	53	3	60

Represent this by a distribution graph.

10. The following table shows the weight of boys of a certain age grouped in a *class interval* of 10 lb.

WT. IN LB.	NO. OF BOYS	WT. IN LB.	NO. OF BOYS
55-65	5	95-105	350
65-75	35	105-115	180
75-85	102	115-125	80
85-95	200	125-135	12

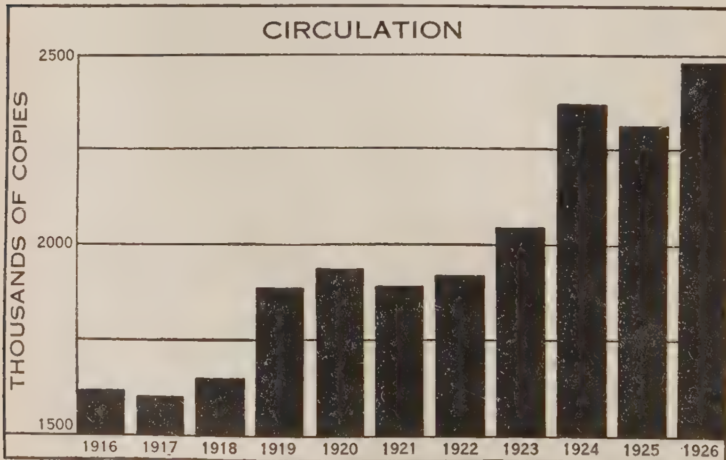
11. Make a distribution graph showing the results of an arithmetic test in which there were 12 problems to be solved in 40 minutes. The results were as follows:

NO. OF PUPILS	NO. OF PROBLEMS CORRECT	NO. OF PUPILS	NO. OF PROBLEMS CORRECT
2	0	27	7
5	1	24	8
9	2	15	9
14	3	7	10
17	4	6	11
21	5	3	12
25	6		

TESTING THE ACCURACY OF GRAPHS

You have now studied all of the general types of graphs and should be able to interpret any graph that you see in books, magazines, or newspapers. There is no general agreement as to the type of graph for a special type of data.

But the real purpose of a graph is to give a clear picture of the number relations existing. Hence it is necessary that a graph when constructed shall give a true picture of the facts. The following graph, taken from the *New York Herald-Tribune*, March 2, 1927, shows the increase in the circulation of one of the leading magazines in this country.



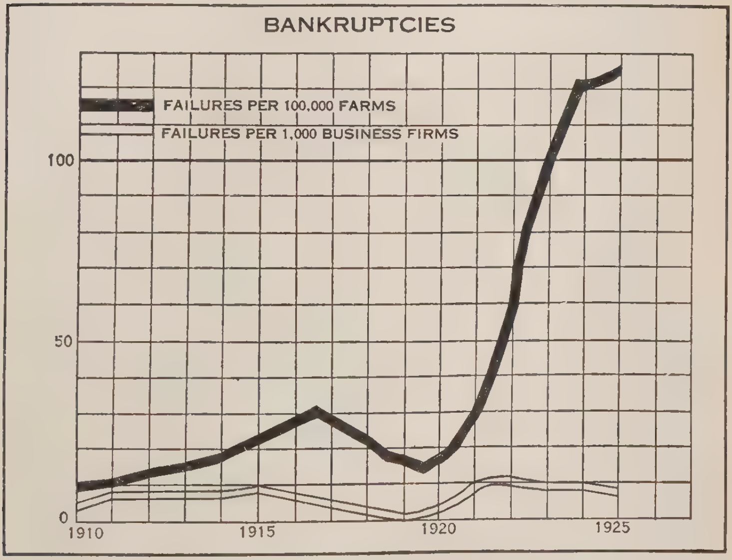
Exercises

1. Measure the lengths of the columns used to represent the circulation in 1916 and in 1926 and compare the results. According to your measurements, the circulation in 1926 is how many times as great as that in 1916?

2. The actual circulation in 1926 is approximately how many times as great as that in 1916?

3. Do you see what caused the facts to be misrepresented in this manner? (See page 59 [4].)

4. Construct this graph as it should have been made.



The above graph, which appeared in a daily newspaper, attempts to compare bankruptcies on farms with bankruptcies in business.

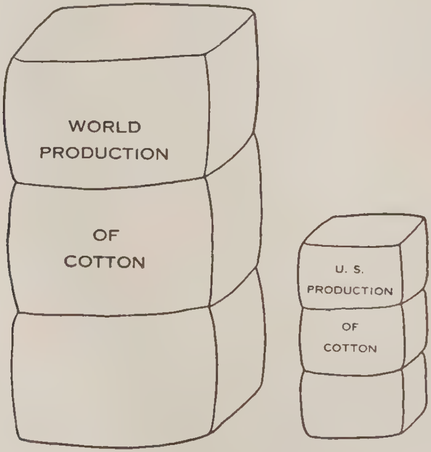
1. Does the graph give a true picture of the facts? Why not?

2. If, in 1924, there were 10 failures for every 1000 business enterprises, how many would that be for each 100,000 business enterprises?

3. How many farm bankruptcies per 100,000 farmers were there in 1924? How does this compare with the business failures?

4. Make a table showing the number of failures per 100,000 for farms and for business enterprises.

5. From the table you have made, draw a graph to correctly represent the facts.



Sometimes a picture graph like the one above is used to show relations. This one pretends to show that the United States produces half of the cotton crop of the world.

There are several objections to such a graph. While the large bale is twice as high as the small one its area is four times as great, and its weight would be eight times the weight of the small one. Therefore it is more difficult to get a true picture of the facts than it would be if the world production of cotton were shown by two bales and that of the United States by one bale, all of the bales being the same size.

CHAPTER III

HOW TO ANALYZE AND SOLVE A PROBLEM

Being able to compute is of no value unless one knows what processes to apply in solving a problem.

You study problems in school for three purposes: (1) to develop the power to see relationships which are expressed by numbers as given in a problem; (2) to learn some of the uses of arithmetic in everyday life, business, and industry; and (3) to learn to appreciate and solve problems that are necessary for an understanding of many phases of modern life.

No definite form of procedure can be given that will enable you to solve all problems. Some types of problems are met so often that you recognize at once what to do. There are other types of problems, however, that consist of several steps, and that have not been met before, which require very careful analysis before you know what to do. The following illustrations are given to show you some of the ways used in analyzing a problem. Not all of the types of problems met by the student are given, but it is hoped that a careful study of these will help you in analyzing and solving problems that you have not met before. Study each solution carefully, and then try to get from it a method of solving any problem. In general, the solution given in the text is not the only way to solve the problem. Try to find other solutions.

NOTE. — The problems of this chapter assume a general knowledge of the simplest phases of percentage and mensuration learned in the grades. If you meet any problem that you cannot solve for lack of such knowledge, either turn to the treatment of these in later chapters and look up the needed knowledge, or mark them and solve later in the course.

1. John can read 30 pages while James can read 20 pages. How many pages can John read while James reads 60 pages?

Think, "John reads 30 pages while James reads 20. Hence John reads $1\frac{1}{2}$ times as fast as James." Then think, "If John reads $1\frac{1}{2}$ times as fast as James, he can read $1\frac{1}{2}$ times as many pages."

$1\frac{1}{2} \times 60$ *pages* $= 90$ *pages.*

Or think, "If James can read 20 pages in a given time, to read 60 pages it will require just 3 times as long." Then think, "If James reads 3 times as long, John will read 3 times as long, so he will read 3 times his original amount, or 3×30 pages $= 90$ pages."

2. When John was 12 years old his father was 3 times as old. How old will John be when his father is 45 years of age? Will the father still be 3 times as old as the son?

This problem cannot be solved by any fixed form or type that you may have studied. Think, "The father must be 36 years of age." Then think, "It will be 9 years before the father will be 45 years of age. Then John will be 9 years older, hence he will be 21 years of age."

Now you can see that for the father to be 3 times as old as John he would need to be 63 years of age. Can you find a reason why the father's age would not always remain 3 times the age of the son?

3. A boy bought papers at 90¢ per hundred and sold them at 2¢ each. How many papers does he need to sell to make $5.50?

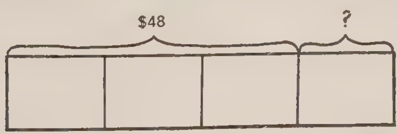
Think, "The selling price for 100 *papers is* \$2, *hence there is a profit of* \$1.10 *on every hundred papers that he sells." Then think, "If the profit is* \$1.10 *per hundred, to make* \$5.50 *it will be necessary to find the ratio of* \$5.50 *to* \$1.10. \$5.50 ÷ \$1.10 = 5."

Then think, "It will be necessary to sell 5 *times as many papers to make* \$5.50 *as to make* \$1.10, *hence* 500 *papers."*

Or think, "If 100 *papers cost* 90¢, *one paper costs* $\frac{9}{10}$¢. *Hence the profit on one paper will be* $1\frac{1}{10}$¢." *Then think, "If* $1\frac{1}{10}$¢ *is the profit from one paper, to make* \$5.50 *he would need to sell as many as* 550¢ ÷ $1\frac{1}{10}$¢ *or* 500, *the number of papers.*

4. At a "special $\frac{1}{4}$-off sale" a chair cost \$48. Find the regular price.

In all problems of this type, a diagram will help you to see the relations. Draw a figure to represent the former

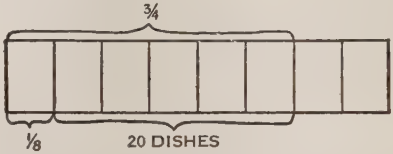


price and divide it into four equal parts. Then "$\frac{1}{4}$ *off" will leave* 3 *of these parts to represent the* \$48. *Hence the former price is* $\frac{4}{3}$ *as much as the reduced price. Thus the solution is* $\frac{4}{3}$ × \$48 *or* \$64, *the former price.*

Or, from the diagram, you see that the "$\frac{1}{4}$ *off" is just* $\frac{1}{3}$ *of* \$48, *or* \$16. *Add this to the* \$48 *and you find the former price, or* \$64.

5. After 20 dishes of ice cream had been taken from a can that was $\frac{3}{4}$ full, it was still $\frac{1}{8}$ full. How many dishes remain?

Make a diagram to show the relation of the part taken out to the part remaining.



From the figure it is readily seen that the part remaining is $\frac{1}{5}$ of the amount taken out. Thus the solution is $\frac{1}{5}$ of 20 dishes, or 4 dishes.

Can you solve this by any other method?

6. A merchant's margin was $\frac{1}{3}$ of the selling price of an article. What part of the cost was it?

Problems of this type are generally simplified if a diagram is made to represent the different relations. Draw a diagram to represent the selling price and divide it into

3 equal parts. Since the selling price is made up of cost and margin, and since one of these parts represents the margin, the other two parts represent the cost. Then it is seen that the margin is equal to one-half of the cost.

In order to see if your solution is correct, select some selling price, as \$15. There was a margin of \$5, hence the cost was \$10. Then you see that the margin was equal to $\frac{1}{2}$ of the cost.

7. The receipts from a high school football game amounted to \$625. There were 520 grand-stand seats sold at 75¢ each. The bleachers sold at 50¢ each. How many bleacher seats were sold?

> *There are three steps, each requiring a different process. Find out what they are before you begin the computation.*

8. A formula for mixed feed called for 8 qt. of cracked corn to 5 qt. of oats. How much of each is needed to make 26 qt. of mixed feed?

> *Draw a diagram to represent the cracked corn and divide it into 8 equal parts. Then add 5 of these equal parts to represent the oats. Now it is seen that there are 13 parts in the mixture. From this you see that the cracked corn is $\frac{8}{13}$ of the mixture and the oats $\frac{5}{13}$ of it. Hence the solution will be $\frac{8}{13} \times 26$ qt. and $\frac{5}{13} \times 26$ qt.*
>
> *See if you can think of two or three other ways of solving this problem.*

9. A tank will hold 150 gallons. What is the capacity in cubic feet?

> *Here a needed fact is not given. You need to know the relation between a gallon and a cubic foot.*
>
> *Knowing that "1 cu. ft. = 7.48 gal.," what process would you use?*
>
> *From the above formula, find what the relation is between one gallon and one cubic foot.*
>
> *If you could not find that "1 cu. ft. = 7.48 gal.", but you could find that "231 cu. in. = 1 gal.", could you solve the problem?*

10. A bin that is 4 ft. wide and 6 ft. long will be filled to what depth with 60 bu. of grain, allowing .8 bu. to the cu. ft.?

> *Think, "The area of the base times the height will equal the number of cu. ft. in the bin." Then,* 24 *times some number is equal to the number of cu. ft. in the bin. So you must know the number of cubic feet. To find this, think,* ".8 × ? = 60, *the number of bushels."*
>
> *From this relation, you see that the solution will be* 60 ÷ .8 *or* 75, *the number of cu. ft.*
>
> *Since you know the capacity of the bin, think,* "24 × ? = 75." *Then the solution will be* 75 ÷ 24 *or* $3\frac{1}{8}$, *the number of ft. in the depth.*
>
> *Students often find it difficult to determine whether they should multiply or divide* 60 *by* .8. *Think, "Since only* .8 *bu. can be put in one cu. ft., there will need be more cu. ft. than bu., hence it must be divided."*

11. A man made a trip in his car in $3\frac{1}{4}$ hours at an average speed of 24 miles per hour. If he had increased his speed $\frac{1}{4}$, how long would it have taken him to make the trip?

> *There are three steps involved:* (1) *to find the total distance traveled;* (2) *to find the increased speed;* (3) *to find the time it would require to travel the distance at the new speed.*

12. A man planned to make an automobile trip in 4 hours, but he increased his speed $\frac{1}{4}$. How long did it actually take him?

Assume some rate of travel, as 20 mi. per hour. Then in 4 hr. he can travel 80 mi. If he increased his rate $\frac{1}{4}$, his new speed will be 25 mi. per hr.

Then the time required will be 80 mi. ÷ 25 mi. = $3\frac{1}{5}$, the number of hours required.

Assume some other rate and see if the same result is obtained.

There are usually several methods of solving a problem. A different method of attack for Problem 12 may be used.

You know that if you go twice as fast, it will take but $\frac{1}{2}$ as long; if you go 3 times as fast, it will take $\frac{1}{3}$ as long; if you go $\frac{1}{4}$ as fast, it will take you 4 times as long; if you go $\frac{1}{5}$ as fast, it will take you 5 times as long; and so on until you see that there is a ***reciprocal relation*** *between* ***speed*** *and* ***time.***

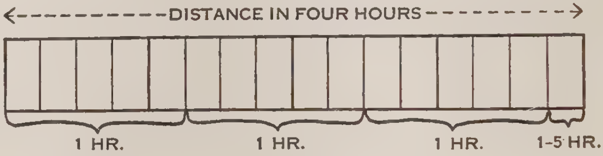
Now think, "An increase of $\frac{1}{4}$ of the original speed will make the new speed $\frac{5}{4}$ of the original speed."

Since you have seen that there is a reciprocal relation between speed and time, if the speed is $\frac{5}{4}$ as great, the time will be $\frac{4}{5}$ as much.

Then the time will be $\frac{4}{5} \times 4$ hr. = $3\frac{1}{5}$ hr., the same answer that you obtained by the other method.

A diagram to show the distances traveled at each rate may be of help.

Draw a diagram and divide it into 4 equal parts. Let each part represent the distance traveled at the end of each hour at the original rate.



Since the rate is $\frac{1}{4}$ more, he would have been $\frac{1}{4}$ farther at any given time. From the diagram you can see that he would have finished the trip by the increased speed at the same time that he would have made $\frac{4}{5}$ of it at the original speed, or in $3\frac{1}{5}$ hours.

13. A man made an automobile trip between two cities. When going he averaged but 20 miles per hour, and when returning he averaged 30 miles per hour. What was the average number of miles per hour for the entire trip?

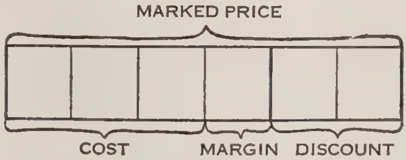
Not knowing the distance, assume some number that is divisible by both 20 and 30, as 60. The time going would be 3 hours. The time returning would be 2 hours.

Then he traveled 120 miles in 5 hours, or an average of 24 miles per hour.

14. A merchant marked goods "$\frac{1}{3}$ off" and sold them at a margin of $\frac{1}{4}$ of the selling price. He marked them to make a margin of what part of the cost?

It is readily seen that the selling price is $\frac{2}{3}$ of the marked price.

Think, "If the margin is $\frac{1}{4}$ of the S. P., it will be $\frac{1}{4} \times \frac{2}{3}$ of M. P. or $\frac{1}{6}$ of M. P."



Draw a diagram to represent the marked price and divide it into 6 equal parts. One of these parts represents the actual margin. Since he discounted the goods $\frac{1}{3}$ or $\frac{2}{6}$ of the M. P., 2 of these parts represent the discount. Then the other 3 parts represent the cost. Hence the cost is equal to $\frac{1}{2}$ of the M. P. and the margin plus the discount the other $\frac{1}{2}$.

Now you can see that he marked the goods to give a margin equal to the cost.

STEPS IN SOLVING A NEW PROBLEM

1. Read the problem carefully. Find what is given and what is wanted.

2. Try to determine the relation between what is given and what is wanted.

3. Suggest ways that might lead to the part that is wanted.

4. If there are several steps involved, decide upon the order in which the different processes are to be used.

5. Try some of the suggested ways and see if they bring a result that is reasonable.

6. Be careful that the computation has been accurate. Check it to see if there has been an error.

7. In some problems you are able to check the result of your reasoning and computation by substituting another set of numbers for the ones given. Then see if the result will be the same as the one you obtained.

Problems for Analysis

Many of the problems given in the following exercises may be new to you. Most of them require careful analysis and cannot be solved by remembering some fixed type. You may not be able to solve all of them without help. Much of the value you derive from this chapter can be measured by the amount of success you achieve from your own effort. Mark those that you were unable to solve and see if at the end of the year, you have developed sufficient reasoning ability to solve those you marked.

A

1. Henry can walk 3 miles while Ralph walks 2 miles. At that rate, how far can Henry walk while Ralph walks 12 miles?

2. The speedometer of a car read 9433 at the beginning of a trip and 10,265 at the close of the trip. Averaging 16 miles per gallon, what was the cost of the gasoline at 22¢ per gallon?

3. A man drove 84 miles in 3 hours. At that rate, how long will it take him to drive 126 miles?

4. Henry can ride his bicycle 10 miles in the time that he could walk $2\frac{1}{2}$ miles. At this rate, how far can he ride in the time that he could walk 6 miles?

5. An ice cream can was $\frac{5}{6}$ full. After 20 dishes were taken out it was still $\frac{1}{6}$ full. How many dishes remained?

6. A piece of wire 2 ft. long is to be divided into 2 pieces so that one piece will be twice as long as the other. Find the length of each piece.

7. If 3 yd. of linen shrinks 2 in., what will be the length of 5 yd. after shrinking?

8. When a merchant makes a margin of twice the cost of an article, what part of the selling price is he making?

9. William is 6 years old and his sister is twice as old. When William is 21, what will be the age of his sister?

10. Mr. Jones made a 250-mile trip. He drove the first 100 miles in 5 hours. If he increased his speed $\frac{1}{4}$ in the remaining distance, how long did it take to make the whole trip?

B

1. If a runner can run 100 yd. in 10 sec., how many inches will he move while he is photographed with a camera that snaps in $\frac{1}{1000}$ sec.?

2. Tom has a square garden plot that requires 100 ft. of fencing to enclose it. Ray has a plot just as large in the shape of a rectangle which is 15 ft. wide. How much fencing is needed to enclose Ray's garden?

3. Mary crossed a lake in 2 hr., swimming half of the time and rowing half of the time. If she can row twice as fast as she can swim, how long would it take her to row the entire distance? To swim the entire distance?

4. John made a trip in 5 hours. By increasing his speed $\frac{1}{4}$, how long would it have taken him?

5. If a novelty is sold at a margin of $\frac{2}{3}$ of the selling price, the cost is what part of the margin?

6. Mr. Smith planned to make a trip in 4 hours, but he had to travel $\frac{1}{4}$ slower than he planned. How long did it take him to make the trip?

7. If a fowl loses $\frac{1}{3}$ in dressing, how many pounds of undressed meat will be necessary to dress 10 pounds?

8. From a sale of 15¢ and 25¢ tickets, there was a return of $52. There were 60 tickets sold at 15¢ each. How many 25¢ tickets were sold?

9. The circumference of a circle is about 3.14 times its diameter. What is the diameter of a circle whose circumference is 8 feet?

10. A man set out on a 300-mile trip. He drove the first 120 miles in 4 hours. To finish the trip in 9 hours, he will need to increase his speed what part of his former rate?

ESTIMATING THE ANSWER TO A PROBLEM

Often in life an approximate answer is all that we need. Also to be able to estimate an answer may help us to discover an error in computation. Whenever it is possible, you should always approximate the answer to see if it is reasonable.

1. When fowl costs 48¢ per pound, about how much will a $5\frac{1}{4}$-lb. fowl cost?

Think, "This is *about* 5 lb. at *nearly* $.50 per pound, hence *about* $2.50."

2. If a boy can ride $3\frac{1}{2}$ mi. in 25 min., how far can he ride in $2\frac{1}{2}$ hr.?

Think, "25 min. is *nearly* $\frac{1}{2}$ hr., so he can ride over 7 mi. per hr. So he can ride *nearly* 18 mi."

3. At $19\frac{1}{2}$ mi. per hour, how far can a man drive in $3\frac{1}{2}$ hr.?

Think, "This is *nearly* $3\frac{1}{2} \times 20$ mi., or *nearly* 70 mi."

4. If a dealer pays $2.45 for a stalk containing 100 bananas, about how much is that per dozen?

Think, "About 8 doz." Then think, "$2.45 ÷ 8 = *about* 30¢."

First estimate the answer, then find the exact answer and see how nearly you estimated:

5. A man raised 57 bu. of potatoes on .3 acre. At this rate, how many bushels will an acre yield?

6. A box of 96 apples cost \$3.75. How much was that for each apple?

7. When averaging 17.8 mi. per gallon of gasoline, how far will $10\frac{1}{2}$ gal. carry a car?

8. When a box of 156 oranges cost \$4.20, how much was that for 6 dozen?

9. When it requires $4\frac{7}{8}$ yd. of material to make a dress, how many dresses can be made from 20 yd. of material?

10. A boy can ride his bicycle at the rate of $6\frac{7}{8}$ mi. per hour. To ride 20 mi., how long will it take him?

11. At an average yield of $39\frac{1}{2}$ bu. of corn per acre, how much will 40 acres yield?

12. There are 7.48 gal. per cubic foot. How many gallons in a tank containing 40 cubic feet?

13. There are 5280 ft. in a mile. If a boy's average step is 25 in., how many steps will he take in going a mile?

14. How many square feet in a rectangle 9.25 ft. long and 6.75 ft. wide?

15. The circumference of a circle is 3.1416 times its diameter. What is the circumference of a circle whose diameter is 14.75 ft.?

16. In making ice cream the recipe said, "Use 1 part cream to $1\frac{1}{2}$ parts whole milk." If $1\frac{1}{2}$ pt. of cream is used, how much milk should be used?

17. A grocer bought 5 crates (160 boxes) of berries. From the first 8 boxes examined he had to throw away 1 box. At this rate how many boxes will be fit to sell?

18. A recipe for salad dressing said, "Use 5 parts of oil to 3 of vinegar." If 6 tablespoonfuls of oil are used, how much vinegar should be used?

19. A farmer used 14 bu. of seed wheat on an 8-acre field. At that rate how much will he need for a field of 50 acres?

20. Mrs. Brown found when making jelly that from 6 pt. of fruit juice and 4 lb. of sugar she got 8 pt. of jelly. How

much of each will she need to make 2 dozen half-pint glasses of jelly?

21. When a car travels $\frac{1}{2}$ mile in $\frac{7}{8}$ of a minute, what is the rate per hour?

22. On a map drawn to a scale of 50 miles to the inch, two cities were represented as being $3\frac{3}{8}$ inches apart. How many miles apart are the cities?

23. During the winter months a family uses an average of $1\frac{3}{4}$ tons of coal each month. At that rate how long will $8\frac{1}{2}$ tons last?

24. If a farmer raised 270 bu. of potatoes from $2\frac{1}{4}$ acres, how many bushels should he get from $4\frac{3}{4}$ acres?

25. If an airplane makes a trip of 268.4 miles in 2 hr. 5 min., what is the rate per hour?

Test in Approximating the Answer

Do not solve these problems. Read the problem carefully, then select the answer given that seems to be the most reasonable. None of the answers is exactly true.

1. If $3\frac{1}{2}$ yd. of lace cost \$2.25, what will 6 yd. cost? \$2.50; \$4; \$6.

2. A baseball team won 16 games and lost 9 games. What part of the whole number of games did it win? $\frac{3}{4}$; $\frac{1}{2}$; $\frac{9}{10}$.

3. If potatoes cost 57¢ a peck, what is the cost per bushel? \$2.50; \$5; \$1.50.

4. A turkey that weighed $8\frac{3}{4}$ lb. cost \$5.25. What was the cost per pound? \$1.50; 60¢; 25¢.

5. If clover hay sells for \$24.50 a ton, what will be the cost of a bale that weighs 250 pounds? \$5; \$10; \$3.

6. If a train runs $\frac{1}{2}$ mile in $\frac{3}{4}$ min., what is the rate per hour? 45 mi.; 60 mi.; 20 mi.

7. Mary and Jane joined a Christmas Club. Mary had twice as much money in the club as Jane, and together they had \$18.75. How much did Mary have? \$6; \$10; \$12.

8. John needed \$3.50 for a football. If he sold papers at a gain of $\frac{9}{10}$¢ on each paper, how many did he need to sell to earn the required amount? 30; 550; 400.

9. If bananas sell 3 for 5¢, what will be the cost of a bunch that contains 115? 70¢; \$2; \$4.

10. A $7\frac{1}{2}$-pound sack of flour lasts a family 2 weeks. At that rate, how long would a barrel of flour last (196 lb.)? 25 wk.; 100 wk.; 50 wk.

11. A certain grade of steel weighs 7.75 times as much as water. If a cubic foot of water weighs 62.5 lb., what is the weight of a cubic foot of steel? 500 lb.; 8 lb.; 4750 lb.

12. If milk sells for \$2.65 per 100 lb., how much per gallon is that, allowing 8.59 lb. per gallon? 35¢; 8¢; 20¢.

13. Mr. Smith drove his car 88.5 miles on 5.4 gallons of gasoline. At that rate, what will it cost for gasoline at 22¢ per gallon when he drives 200 miles? \$26; \$10; \$2.50; \$5.

14. The circumference of a circle is 3.1416 times its diameter. If the circumference of a circle is 12.5 ft., what is its diameter? 37 ft.; 4 ft.; 9 ft.

15. The diagonal of a square is 1.414 times its side. What is the diagonal of a square that has a side of 50 feet? 70 ft.; 35 ft.; 350 ft.

16. A rectangular lot that has an area of 3684 sq. ft. is 40 ft. wide. How much fencing will it take to inclose it? 125 ft.; 500 ft.; 250 ft.

17. A farmer got 50 bu. of potatoes from .3 of an acre. How many bushels should he get from $2\frac{1}{2}$ acres? 150 bu.; 100 bu.; 400 bu.

18. A runner completed a one-mile course in 4 min. 19.4 sec. What was his average rate per minute? 400 yd.; 2500 ft.; $\frac{1}{2}$ mi.

19. A glass jar has a capacity of 1275.5 cubic inches. Allowing 231 cu. in. to the gallon, how many gallons will the jar hold? 6 gal.; 30 gal.; 10 gal.

20. If the length of a flag is 1.9 times the width, what must be the width of a flag that is 14 feet long? 27 ft.; 7 ft.; 10 ft.

Problems Having Needed Facts Missing

Find the necessary facts and solve the following problems:

1. Gettysburg Battlefield includes an area of 25 square miles. How many acres is that?

2. The world's record for the 100 yd. dash is $9\frac{3}{5}$ sec. How many feet per second is that?

3. A farmer has a bin 8 ft. long, 4 ft. wide, and 6 ft. deep. How many bushels of grain will it hold?

4. Mr. Crane bought a piece of land 450 ft. long and 150 ft. wide, at the rate of $300 per acre. Find the cost of the land he bought.

5. A gardener uses 2 tons per acre of a certain kind of fertilizer. How much would he need for a plot 40 ft. wide and 60 ft. long?

6. If 12 oz. of sirloin steak cost 45¢, what is the cost per pound?

7. If hay sells for $18 a ton, what is the value of a load that contains 3200 pounds?

8. A coffee pot that has a capacity of 288 cubic inches will hold how many quarts?

9. A grocer paid $2.25 a bushel for potatoes and sold them at 4 lb. for 23¢. How much did he make on a bushel?

10. An empty 10-gallon can weighs $8\frac{1}{2}$ pounds. When filled with water, how much will it weigh?

Problems Without Numbers

State what process to use.

1. If you know the monthly rent a man pays, how can you find the cost per year?

2. If you know the weekly earnings and expenses of a man, how can you find his yearly savings?

3. If you know the number of hours a fully charged battery will run your radio, and the number of hours per day you use the set, how can you predict when the battery will need recharging?

4. If you know the number of readings taken during the day and the temperature at each reading, how do you find the average or mean temperature for the day?

5. If you know the weight of each football player on the team, how do you find the average weight of each player?

6. If the distance between two cities is known, the time a train left one city, and the time of its arrival at the other, how is the average rate of the train determined?

7. If you are drawing a map to a given scale, and the distance between two cities is known, how would you know how far apart to mark the cities on the map?

8. If you know the number of people that can be served from a quart of ice cream, and the number of people who will be present, how can you tell how many gallons to order?

9. If a merchant knows the cost of goods and the selling price, how does he find what part of the cost the margin is?

10. If you know the number of magazines sold by a boy, the price he received for each, and his total profit, how can you find the cost of each magazine?

A Test in Using Decimals

1. The circumference of a circle is about 3.14 times the diameter. Find the circumference of a circle whose diameter is 5.6 inches.

2. If water is 1.03 times as heavy as cream, find the weight of a gallon of cream, knowing that a gallon of water weighs 8.36 pounds.

3. Find the number of miles per hour a ship is sailing when it makes a speed of 24 knots. (1 knot = 1.15 mi. per hour.)

4 What must be the capacity in cubic feet of a hot water

tank that will hold 75 gallons, if a cubic foot of water weighs 62.5 lb., and a gallon of water weighs 8.36 lb.?

5. If a man drove his car 4358 miles in a season and averaged 16.3 miles per gallon of gasoline, what was the cost of the gasoline at 22¢ per gallon?

6. If butter fat is worth 42¢ a pound, what is the value of 75 lb. of milk that contains .036 butter fat?

7. It took a man 5.6 hours to drive 115 miles. If his speed had been 1.25 times as much, how long would it have taken him?

8. The diagonal of a cube is 1.73 times as long as one of the edges. Find the length of the edge of a cube whose diagonal is 12.45 inches.

9. The diagonal of a square is 1.41 times the length of a side. Find the length of the diagonal of a square whose side is 8.25 inches.

10. A rectangular lot 40.5 ft. wide has an area of 2450.25 square feet. How much fencing is needed to inclose the plot?

A Test in Using Fractions

1. When a coal bin is $\frac{5}{6}$ full, the coal cost \$120. What is the value of the coal when the bin is but $\frac{1}{4}$ full?

2. When a train travels $1\frac{1}{4}$ mi. in $1\frac{1}{2}$ min., what is the rate per hour?

3. The weights of 4 hens were as follows: 5 lb. 6 oz.; 6 lb. 8 oz.; 5 lb. 12 oz.; and 4 lb. 10 oz.

Find the average weight. (Express the ounces as fractional parts of a pound.)

4. If $\frac{3}{4}$ yd. of lace costs 60¢, how much will $5\frac{1}{2}$ yd. cost?

5. When a losing candidate for office received but $\frac{3}{4}$ as many votes as the winning candidate, what part of the total number of votes cast did the losing candidate receive?

6. A man's estate gave $\frac{1}{3}$ of his money to charity, and $\frac{1}{2}$ of the remainder to each of his two children. If each child received \$15,000, what was the value of the estate?

7. If it takes $4\frac{1}{4}$ yd. of cloth to make a dress, how many dresses can be cut from 15 yd. of material? How much cloth will remain?

8. A crate containing 14 hens weighed $83\frac{1}{4}$ lb. The weight of the empty crate was $16\frac{3}{4}$ lb. At 36¢ per lb., what was the value of the hens?

9. Henry spent $\frac{1}{3}$ of his monthly allowance for a football and $\frac{1}{4}$ of the remainder for confections. What part of it did he have left?

10. A man used 5 gallons of gasoline while driving 84 miles. At that rate, how much would he use on a trip of 310 miles?

General Problems

If you find problems in this exercise that you cannot solve at first sight, try to make an analysis of them as suggested at the beginning of this chapter. Teach yourself to rely on your own ability and do not expect too much help from others. It will not help you to reason out problems by having others help you with the difficult ones.

1. If $2\frac{1}{2}$ acres produce 225 bushels of potatoes, how much will $4\frac{1}{2}$ acres produce at the same rate?

2. If $\frac{3}{4}$ yd. of lace costs \$1.20, how much can be bought for \$5?

3. A cubic foot of water weighs 62.5 lb. If lead is 11.3 times as heavy as water, find the weight of 2.6 cu. ft. of lead.

4. A man drove his car 2670 miles one summer and used 178 gallons of gasoline. At an average cost of 22¢ per gallon, what was the cost per mile for gasoline?

5. In 3 years a man drove his car 25,875 miles and averaged 15 miles per gallon of gasoline. He figured that it cost him 7¢ per mile for operating the car. This cost included gasoline at 22¢ per gallon, tires and repairs \$564 and oil at a cost of $\frac{1}{8}$ as much as the total cost of gasoline. The rest of the total cost went to depreciation. What was the amount of the depreciation?

6. Milk weighs 1.03 times as much as water. If 10 gallons of milk weigh 86 lbs., what would be the weight of an equal amount of water?

7. A man earning \$250 per month, spends \$75 for rent, \$60 for food, \$60 for other expenses, and the remainder is saved. What part of his salary is used for each item?

8. If $3\frac{1}{2}$ yd. of material 36 in. wide are required to make a dress, how many yards will be needed if the material is 42 in. wide?

9. A ton of fertilizer is usually put into 12 sacks. How many sacks of fertilizer would be required for $\frac{1}{8}$ of an acre, allowing 2 tons per acre?

10. It is estimated that 8 eggs have the same food value as 12 oz. of steak. Eggs at 40¢ per dozen are as cheap as steak at what price per pound?

11. A tank is $\frac{1}{2}$ full. After 420 gallons had been taken out, it was $\frac{1}{8}$ full. Find the capacity of the tank.

12. If a boat has a speed of 15 miles per hour in still water, how long would it take to go 15 miles downstream when the current is running 5 miles per hour?

13. Using the data in Problem 12, how long would it take to go 25 miles upstream?

14. If a family uses weekly $\frac{3}{4}$ of a 25-pound sack of flour, how long will 6 such sacks last?

15. A boy bought papers at \$1.80 a hundred and sold them at 3¢ each. What was his profit each week if he sold 40 papers daily and got a profit of 3¢ for each Sunday paper?

16. The average yearly consumption of sugar in this country is 83.8 lb. for each person. A soldier is allowed 3.2 oz. daily. How much more or less is that than the yearly average?

17. At a husking contest, a champion husked 28.4 bu. of corn in $1\frac{1}{3}$ hours. How many bushels per hour did he husk?

18. When a merchant sells $\frac{2}{3}$ of his goods for what the whole shipment cost him, his margin is what part of the cost?

19. If a farmer raises 3300 lb. of sweet corn per acre and sells it at $27 a ton, what will be his gain from an 8-acre field if he has to pay $44.80 for labor?

20. The length of the diagonal of a square is 1.41 times the length of a side. Find the distance to second base from home plate on a baseball diamond if the distance between bases is 90 feet.

21. Using the data in Problem 20, find the side of a square whose diagonal is 36 feet.

22. The average yield of potatoes in the United States is 106 bu. per acre. Maine averages 240 bu. per acre. Find the number of acres it will take in the country as a whole to raise as many as Maine raises on 10 acres.

23. If a family uses $\frac{1}{12}$ of a ton of coal a day, how many days will $6\frac{1}{4}$ tons last?

24. A boy sold papers at 3¢ each and made a profit of 75¢ on 60 papers. How much did each paper cost him?

25. At an entertainment $63 was received from the sale of tickets at 25¢ and 50¢ each. If there were 72 of the 25¢ tickets, how many were sold at 50¢ each?

26. Frank bought peaches at $2.50 per bushel and put them up in 2-quart baskets. He sold them at 30¢ per basket. The baskets cost 2¢ each. How much did he make on $5\frac{1}{2}$ bu. that he sold one week?

27. Mary bought $4\frac{1}{2}$ yd. of dress goods at 72¢ per yard, and $1\frac{7}{8}$ yd. of lace at $1.20 per yard. She paid for it with a $10 bill. How much change should she receive?

28. Frank is just 13 years old and weighs 90 lb. The normal increase in weight from 13 yr. to 16 yr. is 5% per year. At this rate, how much should he weigh when 16 years old?

29. Mr. Moore was a traveling salesman. He got a salary of $1800 and $2\frac{1}{2}$% of his sales. He sold $96,500 worth of goods one year. How much did he earn that year?

30. Mrs. Brown and Mrs. Cole had 3 barrels of potatoes shipped to them from a farm. There were $2\frac{3}{4}$ bu. in each barrel. The potatoes cost $3.30 a barrel. The freight and drayage was $2.20 per barrel. Mrs. Brown took 5 bu. of them and Mrs. Cole the rest. How much should each pay?

31. A man drove 132 mi. in 5.5 hr. At this rate, how long will it take him to drive 174 mi.?

32. If a quart of ice cream will serve 6 people, what will the ice cream cost for a party of 48 at $2.40 per gallon?

33. At a special "one-fourth off" sale Nell bought a dress for $11.25. What was the former price?

34. James rode his bicycle $2\frac{1}{4}$ mi. in 18 min. At that rate, how many miles per hour was he riding?

35. A merchant had a special sale in which he marked certain articles down 10% each day from the price of the preceding day. An auto-lunch kit that sold regularly for $36 was sold on the fifth day of the special sale. How much did it cost the buyer?

36. I have a lawn 60 ft. by 40 ft., which a contractor agrees to sod at $3\frac{1}{2}$¢ per square foot. How much will it cost me to sod it?

37. Two boys rolled and took care of Mr. Rose's tennis courts one summer for $48. One boy worked 42 hr. during the summer and the other worked 54 hr. How should they divide the money?

38. If it costs $40 to set a barberry hedge around a lot 60 ft. by 40 ft., how much will it cost at the same rate to set a hedge around a lot 50 ft. square?

39. A farmer used 6 bu. 4 qt. of seed wheat on $3\frac{1}{2}$ acres. At this rate, how many bushels will he use on 10 acres?

40. The butcher charged Nell's mother $1.98 for a $5\frac{1}{2}$-pound beef roast. How much a pound did he charge?

41. Mary's mother paid $54 for a rug 9 ft. by 12 ft. If the price varies according to the size, what would you expect a rug $8\frac{1}{2}$ ft. by 10 ft. of the same quality to cost?

42. If a man rents a house that cost $9000 for $75 a month, what should be the monthly rent of a house that cost $12,000?

43. During the high school baseball season, Frank was at the bat 64 times and made 25 hits. Find his batting average to 3 decimal places.

44. John and Walter contracted to mow a lawn. John mowed $\frac{2}{5}$ of it and received $3.20. How much should Walter receive for his share?

45. What will it cost to plant a hedge on the two longer sides and the front of a lot 50 ft. wide and 120 ft. long, if the plants are set 8 in. apart and each plant costs 12¢?

46. Mary can solve only 3 problems while Jane can solve 4. At that rate, how many problems can Jane solve while Mary solves 12?

47. A certain grade of steel weighs 7.6 times as much as water. What will be the weight of a steel beam that has a volume equal to 5.38 cubic feet?

48. If the wholesale cost of oranges is $1.60 a crate of 120, and the oranges are sold at 30¢ a dozen, what is the margin on 25 crates?

49. The distance from New York to Chicago is 909 miles. If an express train makes the distance in 22 hr. 45 min., what is the average rate per hour?

50. What will it cost for an aërial that is 180 ft. long, if the weight of $8\frac{1}{2}$ ft. of wire is $2\frac{3}{4}$ oz., and wire sells for 75¢ a pound?

Problems Requiring More Careful Analysis

Many of the problems in this exercise are more difficult than those you have been solving. Some of these problems do not occur in the daily needs of arithmetic, but they are given here in order that you may acquire ability to analyze a new situation. Often you will find it profitable to make a diagram in order to find the relationships that need to be discovered. Usually you can find more than one way to solve a problem.

1. If a manufacturer's margin is $\frac{1}{8}$ as much as the cost when selling an article for \$180, what did it cost him to manufacture it?

Note.—The word "profit" is often used for "margin" in such problems, but it is misleading. The difference between the cost and the selling price should be called *margin*, for there is no real profit unless the margin is greater than the expense incurred in selling the article.

2. A margin of $\frac{2}{5}$ of the cost is what part of the selling price?

3. A suit marked "$\frac{1}{5}$ off" sold for \$36. What was the former price?

4. A merchant sold a suit for \$37.50 and made a margin of $\frac{1}{3}$ of the cost. What was the cost of the suit?

5. A real estate agent sold $\frac{2}{3}$ of his lots in a certain tract of land for $\frac{8}{9}$ of the total cost. If he should sell the others at the same rate, his margin would be what part of the cost of the whole tract? What part of the selling price?

6. A merchant marked goods down $\frac{1}{4}$ and still made a margin of $\frac{1}{4}$ of the selling price. They were marked to give a margin of what part of the cost?

7. A boy spent $\frac{3}{8}$ of his money for radio tubes, and then had \$12 remaining. How much had he at first?

8. A certain company gives its agents a commission of $\frac{2}{5}$ of their sales for selling aluminum wares. If an agent's commission amounts to \$250 a month, how much money should he return monthly to the company?

9. A man bought a house and paid $\frac{2}{3}$ of the price in cash. He had yet to pay \$2400. What was the purchase price?

10. A boy sold a bicycle for \$35 at $\frac{7}{8}$ of the cost. What did it cost him?

11. A margin of $\frac{1}{5}$ of the selling price is what part of the cost?

12. In a certain high school, the enrollment of boys is only $\frac{3}{8}$ of the entire enrollment. If there are 320 girls, what is the total enrollment?

13. A and B each owned $\frac{1}{2}$ of a certain business. Later B sold $\frac{1}{4}$ of his share to A. What part of the business did each own after that?

14. In a paper mill where 150 men were employed, 30 men were laid off. A month later there was a reduction of the same fractional part. How many men were employed after the second reduction?

15. Tom bought a used shotgun for \$15. He was able to make a saving of $\frac{1}{3}$ of the price of a new gun by taking the used gun. What was the price of a new one?

16. Frank, being very successful casting for trout, gave away $\frac{1}{4}$ of his fish to one of his neighbors, $\frac{1}{3}$ of the remainder to another neighbor, and then had 16 left for himself. How many did he catch?

17. A man marked goods down $\frac{1}{3}$ and still made a margin of $\frac{1}{5}$ of the cost. They were marked to give a margin of what part of the former price?

18. An Indian trapper can ski 5 miles in 7 minutes. What is his rate per hour?

19. After serving 6 people from a gallon of ice cream, $\frac{3}{4}$ of it remained. How many more can be served?

20. A salesman's commission amounted to $\frac{1}{12}$ of his sales. To earn \$3000 yearly, what would his monthly sales average?

21. If the father's age is now 4 times that of his son, whose age is 10 years, what was the relation of their ages 5 years ago?

22. If about $\frac{3}{4}$ of the weight of wheat goes into flour, how many bushels of wheat are needed to make a barrel of flour weighing 196 pounds?

23. A workman's wages are $\frac{1}{2}$ more than they were four years ago. They are now $60 a week. What were they four years ago?

24. A boy shoveled snow one afternoon from 3:45 until 5:30 at 60¢ an hour. How much did he make that afternoon?

25. If 3 eggs have the same food value as a pint of milk, which is the cheaper and how much, eggs at 60¢ a dozen or milk at 64¢ a gallon?

26. A quart of milk has the same food value as 12 ounces of steak. At that rate, if steak is worth 50¢ a pound, what is the value of a gallon of milk?

27. If a man can pick $\frac{2}{5}$ of his apples in 12 days, how long will it take him to finish the job?

28. John sold his rifle for $10 at a loss of $\frac{1}{6}$ of the cost. What did it cost him?

29. A man sells goods at a margin of $\frac{1}{3}$ of the cost, but his expenses are $\frac{1}{5}$ of the selling price. His gain is what part of the cost?

30. If a hen loses $\frac{1}{3}$ in dressing, how much undressed fowl is necessary for 12 pounds of dressed meat?

31. A boy said, "My wages now are $\frac{1}{4}$ more than they were last year." If he is now getting $20 a week, what were they last year?

32. A man finds that his average mileage per gallon of gasoline now is $\frac{1}{10}$ less than when his car was one year old. If he is getting 13.5 miles per gallon now, what mileage did he get when the car was one year old?

33. When the winter was but half gone, a man had but $\frac{1}{3}$ of his fall supply of coal remaining. At that rate, what part of his fall purchase will he need to buy to last through the winter?

34. If the cost of making a pair of shoes is \$6, at what price must they be sold to make a margin of $\frac{1}{3}$ of the selling price?

35. If goods were marked down $\frac{1}{3}$ and still give a margin of $\frac{1}{5}$ of the cost, they were marked to give a margin of what part of the cost?

36. The enrollment of a room is 40 pupils. If there are $\frac{3}{5}$ as many boys as girls, how many of each are there on the roll?

37. A merchant lists his goods $\frac{1}{2}$ above cost and then gives a discount of $\frac{1}{4}$ of the marked price. His margin is what part of the cost?

38. An article costs \$90 to manufacture. At what price must it be listed to give a margin of $\frac{1}{4}$ of the selling price after allowing a trade discount of $\frac{1}{3}$ of the list price?

39. A merchant received a shipment of goods costing \$250. He marked them $\frac{1}{2}$ above cost, but had to sell them at a discount of "$\frac{1}{4}$ off." What was his margin?

40. A clerk received an increase of \$3 per week, which amounted to $\frac{1}{9}$ of his salary before it was increased. What was his increased salary?

41. If a train runs $\frac{1}{2}$ mi. in 40 sec., what is the rate per hour?

42. A delicatessen store sold 50 one-half pint bottles of cream one day. Express the answer in gallons and quarts.

43. A bin 5 ft. wide and 8 ft. long contains 80 bu. of grain. Allowing .8 bu. to the cubic foot, to what depth is the bin filled?

44. If a coal bin is 8 ft. long, 6 ft. wide, and 5 ft. deep, what will it cost to fill the bin with coal at \$13.75 per ton, allowing 32 cu. ft. to the ton?

45. When an article increases in cost from 15¢ to 20¢, this is an increase of what part of the former price?

46. If an article now sells for \$36, what would the former price have been, using the rate of increase in Problem 45?

47. The formula for mixed feed said, "Use 3 pounds of oats to 5 of corn." How much of each in a mixture of 20 pounds?

48. The distance between two cities on a map is $3\frac{1}{8}$ inches. If the actual distance is 125 miles, to what scale is the map drawn?

49. The contract price for 5 issues of a high school publication is \$720. If the advertisements amount to \$560, how many copies of each publication must be sold at 25¢ each to pay for the paper?

50. In a certain high school the budget for all the school activities during the year amounted to \$1820. The enrollment of the school was 624 pupils, but only $\frac{5}{6}$ of them pledged themselves to contribute to the budget. What was the amount of the annual dues to be paid by those who supported the budget?

Tests in Reasoning

The following exercises are given to see whether or not you are able to analyze a problem and see the relations which exist between the different numbers. Be careful that you apply the right process after you know what is given and what you are to find. Check your computation so as to be sure there are no errors.

Test A

1. If \$8 will buy 5 yards of cloth, how much can be bought for \$10?

2. Mr. Jones drove his car 72 miles in 3 hours. How long will it take him to drive 90 miles when driving only $\frac{3}{4}$ as fast?

3. If a man 6 feet tall casts a shadow 5 feet long, what will be the length of the shadow of a tree that is 30 feet in height?

4. A father is 4 times as old as his son. If the son is now 12 years of age, what was the age of the father 8 years ago?

5. Frank wants to divide a piece of wire into two parts, so that one piece will be just $\frac{3}{4}$ as long as the other. If he divides a piece 14 feet long, what will be the length of each part?

6. If a boy buys papers at 90¢ per 100 and sells them at 2¢ each, how many must he sell to make $2.75?

7. If a train runs 5 mi. in 6 min., what is the rate per hour?

8. A certain square has a perimeter of 60 inches. What will be the perimeter of a rectangle of the same area, if its width is 10 inches?

9. If a boy was at the bat 46 times and made 18 hits, express his batting average to three decimal places.

10. Oranges were selling at the rate of 12 for 25¢. A boy did not know which would be cheaper, to buy 12 for 25¢ or 5 for 12¢. Which was the cheaper and how much?

Test B

1. A snowstorm began at 4:15 A.M. and continued until 2 P.M. If it averaged $1\frac{1}{4}$ in. per hr., what was the total depth of the snow?

2. The Senior Class of a certain high school had $122.75 in the treasury. They planned to have a dance which would cost $185. How many tickets did they need to sell at 75¢ in order to have enough money for the dance?

3. How many bushels of potatoes can be dug from $2\frac{1}{2}$ acres, if .3 acre produces 48 bushels?

4. If a fish loses $\frac{2}{5}$ in dressing, how many pounds of fish will it take to dress 15 pounds?

5. A class decided to plant a garden. They spent $\frac{1}{4}$ of their money for bulbs, $\frac{1}{10}$ of it for markers, and $\frac{1}{5}$ of it for fertilizer. If they then had $4.50 remaining, how much was spent for each item?

6. A hen loses $\frac{1}{4}$ in dressing. What is a 6-pound undressed hen worth if the dressed meat is worth 50¢ a pound?

7. A man could make a trip in 3 hr. 45 min. by increasing his speed $\frac{1}{5}$. Without increasing his speed, how long would it take him?

8. The price for a $7\frac{1}{2}$-pound sack of flour was 51¢. A $12\frac{1}{2}$-pound sack sold for 60¢ at a "special sale." If the regular price per pound was the same as that of the small sack, what was the amount of reduction for the "special sale"?

9. A boy got $\frac{1}{5}$ of all the money he received for selling vegetables for a gardener. If the boy received $15, how much should the gardener receive?

10. Frank can walk 3 mi. per hour and he can ride $5\frac{1}{4}$ mi. per hour. By walking half the distance and riding the rest, how long will it take him to go as far as he can ride in 4 hours?

Test C

1. Mr. Jones noted that one day he burned 8 scuttles of coal. If each scuttle contained 30 pounds, what was the cost of fuel for that day when coal was selling for $14 a ton?

2. The distance between two cities is 150 miles. A man drove one way at an average speed of 20 miles per hour and the other way at 25 miles per hour. What was his average speed for the entire trip?

3. When coffee is sold at 35¢ a pound, it gives a margin of $\frac{1}{6}$ of the cost. If it sells for 40¢ a pound, the margin will be what part of the cost?

4. A recipe calls for 2 pounds of cracked corn to 3 pounds of oats. If a boy wishes to make 25 pounds of the mixture, how much cracked corn and oats will he take?

5. Mary reduced in one month from 150 pounds to 145 pounds. If she reduces at the same rate the following month, what will be her weight at the end of the month?

6. A recipe for preserving calls for 4 pounds of sugar to $6\frac{1}{2}$ pounds of fruit. In a mixture of 21 pounds, how much of each has been used?

7. Using the same recipe, how much sugar would be needed for 39 pounds of fruit?

8. To what depth will 500 bu. of grain fill a bin that is 10 ft. wide and 15 ft. long, allowing .8 bu. to the cubic foot?

9. If a lawn is 40 ft. by 60 ft., to what depth will 800 cu. ft. of top soil cover it?

10. It required 30 days from the time a man began a piece of work until he finished it, but he was idle $\frac{1}{3}$ of the time. Had he been idle but $\frac{1}{4}$ of the time, how long would it have taken him to finish?

Tests in Solving Problems Without a Pencil

Solve these without a pencil:

Test A

1. If $\frac{3}{4}$ of a yard of ribbon costs 60¢, what will be the cost of $2\frac{1}{2}$ yards?

2. If a man can drive his car 40 miles in 2 hours, how long will it take him to drive the same distance if he drives just $\frac{4}{5}$ as fast?

3. Mary is $\frac{3}{4}$ as old as Jane. If Mary is 12 years old, what will be Jane's age in 4 years?

4. If a man can drive 50 miles in $2\frac{1}{2}$ hours, how long will it take him to drive 150 miles?

5. If it cost $1000 an hour for a company to "broadcast" as a means of advertising, what will it cost a company that broadcasts weekly two programs of 45 minutes each?

6. John made scores of 70, 90, and 80 on arithmetic tests. What was his average for the three tests?

7. If apples sell 3 for 5¢, how many can be bought for 50¢?

8. If a square has a perimeter of 48 feet, what is its area?

9. If a runner can run 100 yards in 10 seconds, how many feet per second is that?

10. A man bought a radio set for \$150. If this was $\frac{2}{3}$ of its former price, what was the former price?

Test B

1. If oranges sell 3 for 5¢, how many can be bought for 75¢?

2. If candy sells for \$1 a box of $2\frac{1}{2}$ pounds, how much per pound is that?

3. One season a football player carried the ball 1164 yards. If he averaged 6 yards for each time he carried the ball, how often did he carry it?

4. A man put 7 quarts of alcohol in the radiator of his car. If alcohol sold for 80¢ a gallon, what did it cost him?

5. If it takes 9 gallons of gasoline to drive 150 miles, how much will he need to drive 250 miles?

6. If a sweater was bought at a "$\frac{1}{3}$ off" sale for \$8, what was the former price?

7. A square has an area of 225 square feet. What is its perimeter?

8. A board 8 feet long is to be sawed into two pieces, so that one piece is to be 3 times as long as the other. What will be the length of each piece?

9. When $\frac{1}{2}$ dozen of eggs costs 30¢, how many eggs can be bought for \$1.50?

10. What will be the milk bill of a family for the month of December, if milk sells for 15¢ a quart and 2 quarts are used daily?

Test C

1. If oranges sell for 40¢ a dozen, find the cost of 9 oranges.

2. John and Frank start on a hike and go in opposite directions. If John walks 4 miles per hour, and Frank 5 miles per hour, how far apart will they be in 3 hours?

3. Potatoes sell for 50¢ a peck. At this rate find the cost of a sack that contains $2\frac{1}{2}$ bushels.

4. A fruit dealer bought bananas at 75¢ a bunch of 120 bananas each. If he sold them 3 for 10¢, what was his margin?

5. If paper sells at 35¢ for 100 pounds, how much will a boy receive for 150 pounds?

6. John can read $1\frac{1}{2}$ times as fast as Henry. How many pages can Henry read while John reads 24 pages?

7. Mary took a spelling test and her paper was graded 84. If there were 4 words misspelled, how many words were on the test?

8. A dairyman got, on an average, 60 gallons of milk daily. If he filled 100 quart bottles and put the rest in pint bottles, how many pint bottles did he fill?

9. A man's weekly salary was increased $\frac{1}{6}$. If his increase amounted to $5, what was his new salary?

10. A sack contained $2\frac{1}{2}$ bushels of peanuts. If these were put up into packages of one pint each, what was the value of the peanuts at 10¢ a package?

Test in Telling the Processes to Apply

1. A man knows the distance he drove on a certain amount of gasoline. Knowing the amount of gasoline in the tank of his car, how can he tell how far it will carry him at the same rate?

2. If the amount of fertilizer that should be used to the acre is known, how can you determine the amount to use on a rectangular garden whose dimensions are known?

3. If a salesman has a fixed yearly salary plus a commission of a certain fractional part of his sales, how can you find his

yearly income, knowing his salary, his sales, and his rate of commission?

4. A boy mowed a fractional part of a lawn in a given number of hours. At the same rate, how can he find how long it will take him to finish it?

5. In order to get a margin of a certain fractional part of the cost, how can a merchant determine the selling price from the cost and the rate of margin?

6. When you know the fractional part of a mile that an automobile will run in a certain number of minutes, how can you find the rate per hour?

7. Knowing a boy's weight at the beginning and the end of a year, how can you find what he will weigh at the end of the next year if he makes the same rate of gain?

8. At a certain school play, the adults paid twice as much for a ticket as the children did. If you know the total amount received for tickets, the number of adult tickets sold, and the price at which they were sold, how can you determine the number of children's tickets that were sold?

9. If a merchant makes a margin of a certain fractional part of the selling price, how can you find what part of the cost is margin?

10. If you know the length of a trip, the average rate of miles per hour covered in going, and the average rate in returning, how can you find the rate averaged on the entire trip?

CHAPTER IV

SQUARES AND SQUARE ROOT WITH APPLICATIONS

You know that the area of a square can be found by multiplying the length of one side by itself. Thus, if the side of a square is 4 in., its area is 16 sq. in. On the other hand, if the area of a square is 81 sq. in., you know that the length of one side can be found by finding two equal numbers which, when multiplied, will give 81. Thus, since $9 \times 9 = 81$, 9 in. is the side of the square.

The product of two equal numbers is called the **square** of one of the numbers. Thus, 36 is the square of 6.

Instead of writing $6 \times 6 = 36$, we may write $6^2 = 36$. **6^2 is read "6 squared."**

The **2** is called the **exponent** and **6**, the **base.**

The square root of a number is one of its two equal factors.

Corresponding to the signs for addition, subtraction, multiplication, and division, we have the sign $\sqrt{}$ for square root. Thus

$\sqrt{81} = 9$ is read, "the square root of 81 is 9."

Exercises

1. Give the squares of all whole numbers from 1 to 20 inclusive.

2. Give the square root of: 4, 121, 64, 49, 169, 225, 1, 196, 9, 144.

Find the value of the following:

3. 50^2 **4.** 70^2 **5.** 40^2 **6.** 80^2 **7.** 150^2

8. If a number ends in one zero, in how many zeros will its square end?

9. Is it always true that the square of a given number will always end in twice as many zeros as the given number?

10. Tell how to find the square root of a number ending in an even number of zeros like 8100.

11. Give the square root of: 4900, 12100, 2500, 3600.

FINDING THE SQUARE ROOT BY FACTORING

$$\begin{array}{r|r} 2 & 1764 \\ \hline 2 & 882 \\ \hline 3 & 441 \\ \hline 7 & 147 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

$2 \times 3 \times 7 = 42$

Here we cannot see by inspection that $42 \times 42 = 1764$. *So we begin dividing by any factor that we see, until the number is broken up into small factors. We can then see that the number is made up of two 2's, two 3's, and two 7's. Hence* $2 \times 3 \times 7$ *will give the square root. To check it, find* 42×42.

Find the square root by factoring, and check:

12. 9216	**15.** 396,900	**18.** 176,400
13. 11,025	**16.** 360,000	**19.** 331,776
14. 194,481	**17.** 117,649	**20.** 275,625

SQUARING A NUMBER ENDING IN 5

To square a number ending in 5, multiply the tens' digit by the next higher digit and annex 25 to the product.

Thus $25^2 = 2 \times 3$ with 25 annexed, or 625; $35^2 = 3 \times 4$ with 25 annexed, or 1225.

Find the squares of: 45, 55, 65, 75, 85, 95.

FINDING APPROXIMATE SQUARE ROOTS

You have found the square roots of a few numbers which have *exact* square roots. Although most numbers have no exact square root, we can find two equal factors whose product will *nearly* give the required number and thus can find the *approximate square root.*

Thus, if you wish to find the approximate square root of 5, we can find by trial that two equal factors whose product is nearly equal to 5 are 2.2×2.2. Their product is 4.84, which, of course, is too small. Since $2.3 \times 2.3 = 5.29$, 2.3 is too large. So we know that the square root of 5 lies between 2.2 and 2.3.

Again, if we try 2.23 and 2.24 we find that 2.23 squared is 4.9729 while 2.24 squared is 5.0176. Hence the square root of 5 lies between 2.23 and 2.24.

A better way of approximating the square root is shown below.

1. Find the square root of 500.

$$\begin{array}{rl}
 & \;22.727 \\
22 & \overline{)\,500.000} \\
 & \underline{44} \\
 & 60 \\
 & \underline{44} \\
 & 160 \\
 & \underline{154} \\
 & 60 \\
 & \underline{44} \\
 & 16
\end{array}$$

Ans. = 22.363

You know that $20^2 = 400$, *and that* $30^2 = 900$. *Hence* $\sqrt{500}$ *lies between* 20 *and* 30. *Since* 500 *is nearer to* 400 *than to* 900, $\sqrt{500}$ *is nearer* 20 *than* 30. *So try* 22. *Then by dividing you find that the approximate square root of* 500 *lies between* 22 *and* 22.727. *The average of the two numbers is a good approximation or,*

$$\frac{22 + 22.727}{2} = 22.363.$$

2. Find the approximate square root of 180.

$180 \div 13 = 13.846$ $\frac{1}{2}(13 + 13.846) = 13.423$ Ans. $= 13.423$	*Remembering that* $13^2 = 169$, *and that* $14^2 = 196$, *we try* 13 *as divisor. The quotient,* 13.846, *shows that* 13 *is too small and we take as the approximate root the average between* 13 *and* 13.846.

Of course the answer will be more nearly the true value if your first guess is nearly correct. Had you taken 15 *as a divisor instead of* 13 *your result would have been* 13.5, *which is not so nearly correct as* 13.423. *Dividing* 180 *again by* 13.5 *we obtain* 13.333 *as a quotient. The average between* 13.5 *and* 13.333 *is* 13.417.

Learn the squares of the numbers from 1 to 20.

Find the approximate square root of:

1. 75	**4.** 98	**7.** 116	**10.** 165	**13.** 200
2. 50	**5.** 105	**8.** 138	**11.** 108	**14.** 216
3. 91	**6.** 120	**9.** 150	**12.** 185	**15.** 242

Find the approximate square root of 8375.

$$\begin{array}{r} 91.032 \\ 92\,\overline{)\,8375.000} \\ \underline{828} \\ 95 \\ \underline{92} \\ 300 \\ \underline{276} \\ 24 \end{array}$$

Recall that $90^2 = 8100$, *and that* $95^2 = 9025$.

Observe that 8375 *is much nearer* 8100 *than* 9025. *So try* 92 *for divisor. The quotient* (91.032) *shows that* 92 *is nearly right, but too large. Therefore take the average of* 92 *and* 91.032, *or* 91.516. *Check by squaring* 91.516.

Always check your result in square root by squaring it. If the square is not very nearly the given number, take the root found and divide again and find the average between the divisor and quotient.

USING A TABLE OF SQUARES

If you have a table of squares, you can find the approximate square root of any number with but one division.

A TABLE OF SQUARES

Number	Square	Number	Square	Number	Square
16	256	46	2116	76	5776
17	289	47	2209	77	5929
18	324	48	2304	78	6084
19	361	49	2401	79	6241
20	400	50	2500	80	6400
21	441	51	2601	81	6561
22	484	52	2704	82	6724
23	529	53	2809	83	6889
24	576	54	2916	84	7056
25	625	55	3025	85	7225
26	676	56	3136	86	7396
27	729	57	3249	87	7569
28	784	58	3364	88	7744
29	841	59	3481	89	7921
30	900	60	3600	90	8100
31	961	61	3721	91	8281
32	1024	62	3844	92	8464
33	1089	63	3969	93	8649
34	1156	64	4096	94	8836
35	1225	65	4225	95	9025
36	1296	66	4356	96	9216
37	1369	67	4489	97	9409
38	1444	68	4624	98	9604
39	1521	69	4761	99	9801
40	1600	70	4900	100	10000
41	1681	71	5041	101	10201
42	1764	72	5184	102	10404
43	1849	73	5329	103	10609
44	1936	74	5476	104	10816
45	2025	75	5625	105	11025

Find the approximate square root and check:

1. 985	**6.** 8045	**11.** 26.45	**16.** 136.4
2. 1125	**7.** 6538	**12.** 38.24	**17.** 151.8
3. 1720	**8.** 9438	**13.** 51.16	**18.** 218.9
4. 2645	**9.** 8741	**14.** 66.38	**19.** 684.3
5. 5163	**10.** 6783	**15.** 82.41	**20.** 976.5

A SECOND METHOD OF FINDING SQUARE ROOT

There is another way of finding square root which is often used.

1. Find the square root of 1296.

```
      12'96)36
       9
66    |396
      |396
```

By inspection you can see that the square root is going to be between 30 *and* 40, *so you know that there must be two figures in your quotient.*

Beginning with the units' figure, divide the number into groups of two figures. Now find the largest square in the group at the left. In this case it is 9. *Subtract this from* 12 *and you have a remainder of* 3. *The square root of* 9 *or* 3 *is put in the quotient as the first figure of the root.*

Next, bring down the second group, 96, *and annex it to the remainder, making* 396.

Multiply 3 *(tens) by* 2, *and write the product* 6 *at the left of* 396 *for the new divisor (called the trial divisor). (This is really* 60 *but the zero is not annexed as it will be replaced by another figure.)*

Now divide 396 *by* 60. *This gives a quotient of* 6. *Write* 6 *in the root and annex* 6 *to the divisor, making* 66. *Now multiply* 66 *by* 6 *and write it under* 396. *Since there is no remainder,* 36 *is the exact square root.*

2. Find the square root of 2.7889.

	$2.7889 \times 10000 = 27{,}889.$
	2′78′89)167
	1
26	1 78
	1 56
327	2289
	2289

Find the square root of 27,889 *in the usual manner. Then, since* $\sqrt{10000} = 100$, *the answer must be divided by* 100 *to properly place the decimal point. Hence the answer is* 1.67.

3. Find the square root of 3 to three decimals.

	3′00′00′00.) 1732
	1
27	200
	189
343	1100
	1029
3462	7100
	6924

Since we must find three decimals in the root, find the square root of 3,000,000. *Then point off three decimal places in the answer.*

Hence the answer is 1.732.

TO FIND THE SQUARE ROOT OF A NUMBER

1. Separate the number into groups of two figures, beginning at the right or units' place.

2. Find the greatest square in the left group and write its square root as the first figure of the root.

3. Subtract this square from the left group and to the remainder annex the next group.

4. Double the part of the root already obtained for a trial divisor.

5. **Divide by this trial divisor.**

6. **Annex the quotient to the root and also to the divisor.**

7. **Multiply the complete divisor by the new figure in the root, and subtract.**

8. **If there is another group left, proceed as before, doubling the part of the root already obtained for the new trial divisor.**

9. **If the number has a decimal point, move the decimal point an even number of places to the right to get a whole number. Then, in the root found, place the decimal point half the number of places to the left that it was first moved.**

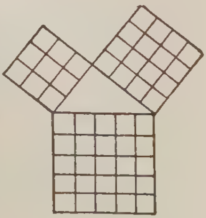
Find the square root of the following:

1. 5776	**4.** 8836	**7.** 123,904
2. 4624	**5.** 80,089	**8.** 106,929
3. 5329	**6.** 67,081	**9.** 133,225

Find to the nearest hundredth the square root of the following:

10. 78	**14.** 78.42	**18.** .865
11. 720	**15.** 36.56	**19.** .42
12. 816	**16.** .3968	**20.** .3
13. 537	**17.** .7432	**21.** .563

THE PYTHAGOREAN THEOREM



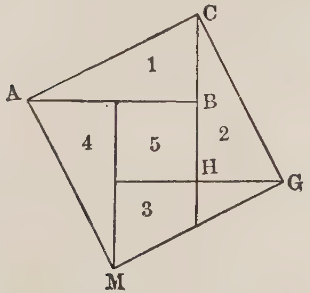
A right triangle is a triangle of which one angle is a right angle. The side opposite the right angle is called the **hypotenuse,** and the other sides are called the **legs.** By drawing a right triangle whose legs are 3 in. and 4 in., respectively, it will be seen that the hypotenuse is just 5 in., and that the area of the square on the hypotenuse equals the sum of the areas of the squares on the two legs.

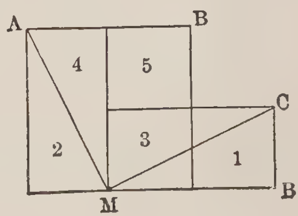
This important truth was proved by Pythagoras about 500 B. C. to be true of any right triangle. That is,

The square on the hypotenuse of any right triangle is equal to the sum of the squares on the other two sides.

NOTE.—Carpenters make use of this fact in laying out the foundation of a building when they want the walls at right angles to each other. Starting at one corner, a line 8 ft. long is taken in one direction along which the foundation is to be laid. Starting from the same corner, another line 6 ft. long is fastened to the end of the first line and moved about until a 10 ft. rod will just reach the outer extremities of the two lines.

The truth of the Pythagorean theorem may be seen by drawing, or cutting from cardboard, figures like the following:





Let ABC be the right triangle. The square on the hypotenuse AC is equal to the four triangles, 1, 2, 3, and 4, and the small square, 5. Now put 1 and 2 in the position of the figure at the right, and the figure is equal to a square on AB and one on CB.

APPLICATIONS OF SQUARE ROOT AND THE PYTHAGOREAN THEOREM

NOTE.—These applications make use only of measurement facts learned in the grades, but may be omitted until Chapter V has been taken.

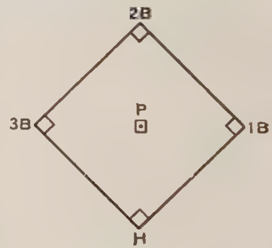
1. One leg of a right triangle is 40 ft. and the other is 36 ft. What is the length of the hypotenuse?

2. If the hypotenuse of a right triangle is 85 ft. and one leg is 51 ft., what is the length of the other leg?

3. A ladder is 26 ft. long. It is placed so that it exactly reaches to a window 24 ft. high. How far from the foot of the building is the ladder set?

4. Some boys wished to lay out a baseball diamond. They knew that the distance between each base is 90 ft., but they did not know how to get right angles. Show how they could be sure to get right angles. (See note, page 121.)

5. What is the horizontal distance a catcher must throw a ball to get a man sliding to second base, if the catcher is standing at home plate?



6. The pitcher's box is 60 ft. from home plate. How far is the pitcher's box from second base?

7. What is the length of the throw from first base to third base?

8. If the center fielder is playing 90 ft. back of second base, how far does he need to throw the ball to return it to the home plate?

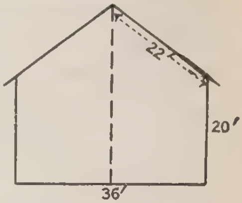
9. The perimeter of a rectangle is 120 ft. If the length is twice the width, what is the length of a diagonal (the distance between opposite corners)?

10. Some high school students were decorating a gymnasium, 76 ft. long and 50 ft. wide, for a dance. They wished to run streamers from the opposite corners. Allowing 8 ft. for sagging in each streamer, find the length of the two streamers.

11. How far is a place 12 miles east of you from one 15 miles south of you?

12. One city is 25 miles northeast of another city. After driving 12 miles on the straight road connecting these two cities, Mr. Jones saw a sign which said, "Detour 5 miles east and then turn north." How much farther did Mr. Jones need to drive because of the detour?

13. A telephone pole is 36 ft. high. How much cable will it take to fasten the top of the pole to an anchor 15 ft. from the foot of the pole, allowing 2 ft. for fastening the cable?



14. The dimensions of a barn are the same as those given in the diagram. Find the distance from the foundation to the ridge of the roof.

15. How much fencing will it take to inclose a square garden that has an area of 2025 square feet?

16. To find the approximate height of a kite when 400 ft. of string had been let out, a second boy stood directly under the kite and then found that he was 180 ft. from the boy holding the string. Not considering the sag in the string nor the height of the boy holding it, find the approximate height of the kite.

17. If the base of an isosceles triangle (a triangle having two equal sides) is 10 in. and the height is 8 in., what is the area of the triangle?

NOTE.—The altitude of an isosceles triangle divides the base into two equal parts.

18. If the base of an isosceles triangle is 12 in. and the altitude is 10 in., what is the length of the equal sides?

19. Find the area of an equilateral triangle each of whose sides is 10 inches.

20. An aërial for a radio extended from a tree 60 ft. high to a window 8 ft. above the ground. If the tree was 150 ft. from the window but on the same horizontal plane with it, how much wire was needed for the aërial?

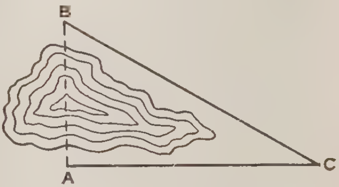
21. A room is 24 ft. wide, 30 ft. long, and 12 ft. high. What is the length of the diagonal from one corner on the floor to the opposite corner at the ceiling?

22. To get to his school by the road, John had to walk 60 rd. north and 75 rd. east. If he could have walked in a

straight line direct from his home to school, what distance could he have saved?

23. If a rectangular field has a diagonal of 90 rd. and the length is 54 rd., what is the area of the field?

24. To find the distance from a point A to a point B on the opposite side of a lake, some boys measured from A to a point C in a line at right angles with line AB and measured AC and CB. The distance from A to C was 260 yd. and from C to B was 300 yd. Find the distance from A to B.



FURTHER APPLICATIONS OF SQUARE ROOT

1. The surface of a square table contains 2704 square inches. What must be the length of one side?

2. Find the dimensions of a rectangle twice as long as it is wide that contains 8978 square inches.

NOTE.—The rectangle will make two squares. The area of each of these will be one-half that of the rectangle. Make a diagram to show this.

3. A circle has an area of 314 square inches. What is the radius of the circle?

4. A factory is supplied with water by a pipe 4 in. in diameter. If twice as much water is needed, what must be the diameter of the new pipe?

NOTE.—To have twice the carrying capacity, the area of a cross section must be twice as great as that of the original pipe.

5. A town is supplied with water by two mains, one 8 in. in diameter and the other 10 in. What must be the diameter of one pipe which will supply the same amount of water?

6. What is the diameter of a pail 8 in. high that will hold a gallon?

7. A man wished to have a hot-water tank to hold 100 gallons and be 5 ft. high. What will be the diameter of this tank?

8. A farmer has a pile of grain in the shape of a cone 3 ft. high and containing 60 bu. Allowing .8 bu. to the cubic foot, what is the area of the floor space taken up by the grain? To the nearest foot, what is the diameter of the pile of grain?

9. Frank has a rectangular garden 38 ft. wide and 96 ft. long. If he replaces this by a square garden of the same area, what must be its side?

10. How much less fencing is needed to inclose a square garden having the same area as a rectangular one 120 ft. wide and 300 ft. long?

11. The area of a triangle is sometimes expressed by the formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where A = area, s = one-half the sum of the three sides, and a, b, and c are the lengths of the three sides. From the formula, state the rule for finding the area of the triangle when the three sides are given.

12. Using the formula given in Ex. 11, find the area of a triangle if the sides are 8 in., 15 in., and 17 in.

13. Find the area of a triangular piece of ground, if the sides are 24 rd., 30 rd., and 48 rd.

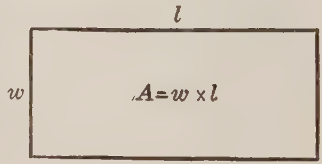
14. Find the area of a triangle, if its sides are 12 ft., 8 ft., and 14 ft.

CHAPTER V

AREAS, VOLUMES, AND OTHER MEASURES

THE AREA OF A RECTANGLE

You have studied areas before and know that by the area of any figure we mean the number of square units contained in the figure. Thus you know that



The area of a rectangle is equal to the product of the length by the width.

This same fact may be expressed in a shorter form by using letters. Thus

$$A = l \times w, \text{ or } A = lw$$

Such a statement is called a formula and simply states that "the area is equal to the product of the length and width."

NOTE.—The length and width must always be expressed in the same units.

TABLE OF SQUARE MEASURE

144 sq. in.	=	1 sq. ft.
9 sq. ft.	=	1 sq. yd.
$272\frac{1}{4}$ sq. ft.	=	1 sq. rd.
$30\frac{1}{4}$ sq. yd.	=	1 sq. rd.
160 sq. rd.	=	1 acre (A.)
1 sq. mi.	=	640 A.

Exercises

1. Draw a rectangle 4 in. wide and 5 in. long. By dividing it into smaller squares each 1 in. by 1 in., show that the area of the rectangle is 20 sq. in.

2. If you do not remember the table, how can you determine the number of square inches in a square foot?

3. Show how you can find the number of square feet in a square rod.

4. Prove that there are 640 A. in a square mile.

5. How much will it cost to build a concrete walk 4 ft. wide and 75 ft. long at $2.50 per square yard?

6. Find the cost of plastering the walls and ceiling of a room 12 ft. wide, 16 ft. long, and $8\frac{1}{2}$ ft. high, at 85¢ per square yard, no allowance being made for doors and windows.

7. Find the area of a walk 3 ft. 6 in. wide which surrounds a rectangular grass plot 18 ft. wide and 30 ft. long. (The walk is not a part of the plot.)

8. What will it cost to sod a front lawn 25 ft. by 60 ft. at 6¢ per square foot?

9. Find the cost of covering a kitchen floor 9 ft. by 12 ft. with linoleum at $2.25 per square yard.

10. A man has a lot with a frontage of 60 ft. There is a strip 6 ft. wide between the sidewalk and curbing that he wishes to sod. Deducting the area of a walk 4 ft. wide extending from the sidewalk to the curbing, find the cost at 8¢ per square foot.

FINDING A MISSING DIMENSION

A field 10 rd. wide contains 1 A. How long is it?

In solving this problem you think

$$10 \times ? = 160.$$

You have learned that

When the product of two numbers is given and one of the

numbers is known, the other number can be found by dividing the product by the given number.

Or you can solve the same problem by using the formula $A = lw$.

Since the width w is 10, and the area A is 160 sq. rd., you have

$$10\,l = 160.$$

This expression is called an **equation.** By using a letter to represent the missing number we frequently make the work easier.

Exercises

Find the value of the letter in the following:

1. $3l = 15$	**4.** $4y = 30$	**7.** $5.2\,x = 26$
2. $6w = 48$	**5.** $3\frac{1}{3}\,x = 10$	**8.** $4\,x = 26.4$
3. $4\frac{1}{2}\,x = 36$	**6.** $8\,x = 35$	**9.** $12\,x = 0$

10. If the area and one dimension of a rectangle are given, how is the other dimension found?

11. John has a garden 40 ft. square. Henry has a garden just as large in the shape of a rectangle which is 25 ft. wide. How long is Henry's garden?

12. A classroom should have 18 sq. ft. of floor space for each pupil. If a room seating 40 pupils is 24 ft. wide, what should be the length of the room?

13. The amount of glass surface admitting light to a classroom should be $\frac{1}{4}$ of the floor area. How much glass surface should there be in the classroom mentioned in Ex. 12?

14. A cloakroom for a classroom should have about $\frac{1}{5}$ as much floor space as the classroom. What should be the width of the cloakroom in Ex. 12 to provide the required amount of space? (The length of the cloakroom is to be the same as the width of the classroom.)

15. A playground should provide 30 sq. ft. of surface area for each pupil. How wide should a playground be to accommodate 450 pupils, if it is rectangular in shape and is 250 ft. long?

16. If the length of a rectangle is doubled and the width remains unchanged, what will be the effect upon the area?

17. If the width of a rectangle is doubled and the length remains unchanged, what will be the effect upon the area?

18. If both the length and width are doubled, what effect will it have on the area?

19. If one dimension is made three times as great and the other remains unchanged, what effect will it have on the area?

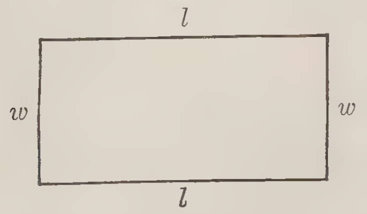
20. If both dimensions are made three times as great, what will be the effect on the area?

THE PERIMETER OF A RECTANGLE

By the **perimeter** of a rectangle is meant the distance around it.

Show that if the length of a rectangle is l and its width is w its perimeter is

$$P = l + w + l + w = 2l + 2w, \text{ or}$$
$$P = 2(l + w).$$



THE AREA OF A PARALLELOGRAM

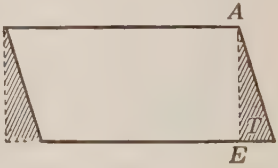
A **parallelogram** is a figure with four straight sides in which the opposite sides are parallel. The rectangle is a special

kind of parallelogram in which all of the angles are right angles. Any side of a parallelogram is called the **base** and the distance from that side to the opposite side is called the **altitude.**


A PARALLELOGRAM

It is impossible to find the area of a parallelogram by dividing it up into squares as you can with the rectangle.

Instead, cut a parallelogram from paper and draw a perpendicular from one corner to the opposite side as the line AE is drawn in the figure. Cut off the triangle T and place it at the other end of the parallelogram as shown.



1. What kind of figure is thus formed?

2. Compare the base and altitude of the original parallelogram with the length and width of the rectangle.

3. What is the area of the rectangle?

4. How then, would you find the area of the parallelogram?

Thus we see that

The area of a parallelogram is equal to the product of its base and altitude.

Or, $A = hb$, where A is the area, h is the altitude, and b is the base.

Exercises

1. The sides of a rectangle are 4 in. and 6 in. The sides of a parallelogram are the same. Are the areas of the two figures the same? Show why your answer is correct.

2. Draw a parallelogram whose base is 6 in. and whose altitude is 4 in. Find its area. What other figure having the same dimensions will have the same area?

3. If a parallelogram has a base of 5 in. and an altitude of 3 in., what is its area?

4. What is the base of a parallelogram whose altitude is 18 in. and whose area is 99 sq. in.?

5. Find the altitude of a parallelogram whose area is 35 sq. ft. and whose base is 3 ft. 6 in.

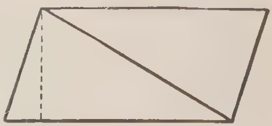
6. Show that if the perimeter of a parallelogram is P, the length is l and the width is w, then

$$P = 2(l + w)$$

THE AREA OF A TRIANGLE

A **triangle** is a figure bounded by three straight sides. Any one of the three sides may be considered the **base,** and the distance from the opposite vertex to this side is called the **altitude.**

1. From cardboard or paper construct two triangles exactly alike and place them as in the figure. What kind of figure is formed by the two triangles?



2. If the base of each triangle is 5 in. and the altitude 4 in., what is the base and altitude of the parallelogram formed by the two? What is its area? What, then, is the area of each triangle?

Thus you see that

The area of a triangle is equal to half the product of the base by the altitude.

Or, $A = \frac{hb}{2}$ when A is the area, h the altitude and b the base.

Exercises

Find the areas of triangles whose dimensions are:

1. h, 6 in.; b, 12 in.
2. h, 4 ft.; b, 9 ft.
3. h, 20 rd.; b, 34 rd.
4. h, 7.7; b, 16.3

5. A barn 36 ft. by 54 ft. is 20 ft. high to the eaves. The two gables are each 36 ft. wide and 15 ft. high. At $1.75 per hundred sq. ft., find the cost of painting the barn.

6. A triangular plot of ground is 8.5 rd. along one side. The distance from the opposite corner to this side is 6.4 rd. Find the value of the plot, if an acre is worth $350.

7. A boy has a sail-boat with a triangular sail that is 14 ft. high and 8 ft. wide. What is the area of the sail?

8. If the altitude of a triangle is doubled and the base is unchanged, how is the area changed?

9. If the base of a triangle is doubled and the altitude remains unchanged, how is the area changed?

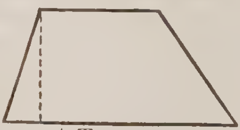
10. If the base and the altitude of a triangle are both doubled, how does it affect the area?

THE AREA OF A TRAPEZOID

A **trapezoid** is a figure with four straight sides only two of which are parallel.

The two parallel sides are called the **bases** of the trapezoid.

The distance between the two bases is called the **altitude.**



A Trapezoid

1. Cut from cardboard or paper two trapezoids exactly alike. Place them as in the figure. What is the shape of the figure thus formed?

2. If the upper and lower bases of the trapezoid are 3 in. and 4 in., respectively, and the altitude is 5 in., what are the dimensions of the parallelogram formed by the two? What is its area?

3. What is the area of a trapezoid whose dimensions are those given in Ex. 2?

Thus we have

The area of a trapezoid is half the product of the altitude by the sum of the two bases.

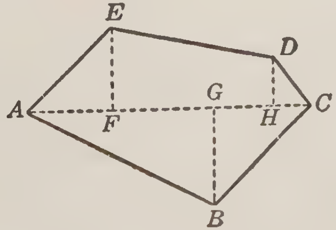
Or, $A = \frac{h(b + b')}{2}$, where A is the area, h the altitude and b and b' the two bases.

NOTE.—b' is read "b prime."

EXERCISES

Find the area of the trapezoids whose dimensions are:

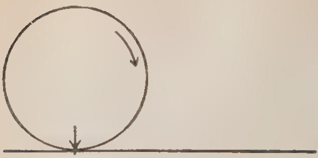
1. $h = 6$ in.; $b = 12$ in.; $b' = 8$ in.
2. $h = 5$ ft.; $b = 10$ ft.; $b' = 6$ ft.
3. $h = 12$ rd.; $b = 16$ rd.; $b' = 10$ rd.
4. $h = 6.2$; $b = 12.4$; $b' = 7.5$.
5. $h = 4\frac{1}{2}$; $b = 20$; $b' = 16\frac{1}{4}$.
6. $h = 6.4$; $b = 8$; $b' = 5\frac{3}{8}$.



7. Find the area of the field shown if $AC = 30$ rd., $BG = 10$ rd., $AF = 8$ rd., $EF = 8$ rd., $DH = 6$ rd., and $HC = 4$ rd.

THE CIRCUMFERENCE OF A CIRCLE

Cut out of cardboard a circle whose diameter is $3\frac{1}{2}$ in. In order to find the length of the circumference of this circle mark a point on the circle and also mark a point on a straight line drawn on your paper. Then, placing the two points



together, roll the circle along the line until the point on the circle is again on the line. Mark this point. The distance between the two points on the line is the **circumference** of the circle.

Now find the ratio between the circumference and the diameter of the circle. Repeat the experiment with a circle whose diameter is 7 in. You should get the same result in both cases. That is, the circumference of the circle is a little more than 3 times the diameter. An approximate value is $3\frac{1}{7}$. This ratio is represented by the Greek letter π (called *pi*). A more exact value is 3.14 or 3.1416. Thus,

The circumference of a circle is π times the diameter, or 2π times the radius.

The formula is $C = \pi d$ or $C = 2\pi r$.

Exercises

1. If the diameter of a circle is 10 ft., what is its circumference? Use $\pi = 3.14$ and then use $\pi = 3.1416$. Find the difference between the two answers.

2. What is the length of the equator, if the diameter of the earth is 8000 mi.? Use both values of π and find the difference between the two answers.

3. State when it is advisable to use $\pi = 3.1416$ and when 3.14 is sufficient to insure accuracy.

4. If the radius of a circle is 6 ft., what is the circumference?

5. An athletic field is to have a circular running track $\frac{1}{4}$ mile long. What should be the length of the string (radius) used in laying out the track?

6. How many revolutions will an automobile wheel 30 inches in diameter make in going a mile?

7. If an automobile wheel 34 inches in diameter makes 150

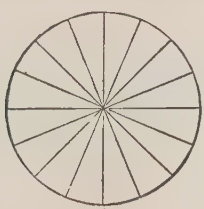
revolutions per minute, what is the rate of the car in miles per hour?

8. A running track has two parallel sides each forming a 100 yd. straight-away. The ends are semi-circular and have a diameter of 92 ft. What is the distance around the track?

9. Using the data in Ex. 8, if the track is 10 ft. wide, how much farther does the runner on the outside go than the inside runner?

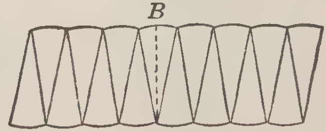
FINDING THE AREA OF A CIRCLE

In getting the formula for the area of a triangle, we compared the area of the triangle with the area of a parallelogram. We also compared the area of a trapezoid with the area of a parallelogram. In a similar way we shall compare the area of a circle with the area of a parallelogram.



A

B



Divide a circle into 16 or more equal sectors as shown in the figure A. Then cut it out and fit the sectors together as shown in figure B. The figure formed is *nearly* a parallelogram. Its base is half the circumference and its altitude is equal to the radius of the circle. Hence we infer a fact proved in other courses in mathematics:

The area of a circle is equal to the product of the radius by half the circumference.

Or, $A = \frac{1}{2}Cr$, when A is the area, C the circumference and r the radius.

Since we know that $C = 2\pi r$ we can say $A = \frac{1}{2}\,Cr = \frac{1}{2} \times 2\pi r \times r = \pi rr$.

This is usually written $A = \pi r^2$, where r^2 means $r \times r$. (r^2 is read "r square".)

Find the area of a circle when:

1. The radius is 6 in.
2. The diameter is 10 ft.
3. The diameter is 15 rd.
4. The radius is 4.8 rd.
5. Compare the areas of two circles, their diameters being 10 ft. and 20 ft., respectively.
6. When the steam pressure in a cylinder is 120 lb. per square inch, what is the total pressure on a 10-inch piston?
7. A 4-foot walk surrounds a circular fountain 25 ft. in diameter. At $2.75 per sq. yd., find the cost of laying the walk.
8. A small town is supplied with water by a 6-inch main and an 8-inch main. Find the diameter of a pipe to contain just as much water as both mains.
9. A circle is inscribed in a square (the circumference touches all four sides of the square). If the area of the square is 400, what is the area of the circle?
10. If a cake 8 in. in diameter sells for 50¢, what should be the price of a cake 12 in. in diameter, if both cakes have the same thickness and the same quality?
11. Show that if the diameter of one circle is 4 times that of another, its area is 16 times as great.
12. Which of two barns having the same height will have the greater capacity: a rectangular barn 36 ft. by 80 ft., or a circular barn having the same perimeter as the other? What is the difference in the floor areas of the two?
13. Doubling the radius of a circle has what effect upon the circumference?
14. Doubling the radius has what effect upon the area of a circle?

15. If the radius of one circle is three times that of another, how do their circumferences compare? Their areas?

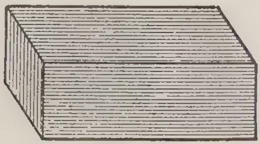
16. If the area of one circle is sixteen times that of another, how do their diameters compare?

17. Show that the areas of two circles have the same ratio as the squares of their radii.

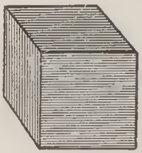
18. Show that when the ratio between the areas or the circumferences of two circles is found, it is a saving of time not to multiply by the numerical value of π.

THE VOLUMES AND SURFACES OF RECTANGULAR SOLIDS

A solid such as a box or a room whose faces are all rectangles is called a **rectangular prism.** When all the faces of a rectangular prism are squares, the solid is called a **cube.**



RECTANGULAR PRISM



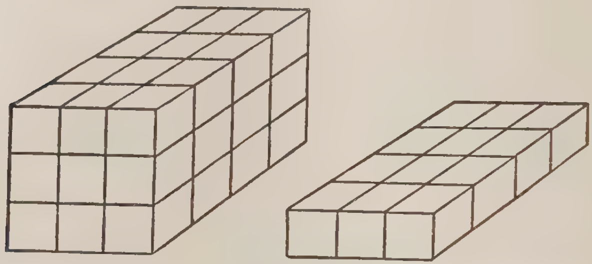
CUBE

FINDING THE VOLUME OF A RECTANGULAR PRISM

You know that, in order to find the area of a surface, you must find how many times the surface contains a square whose side is one unit, which is called the **unit** of area.

In the same way the **volume of a solid** is found by finding how many times it contains the unit of volume, which is a cube. Thus, to find the cubic contents of a solid in cubic inches we find how many times the solid contains a cube each of whose sides is 1 inch.

1. The box shown is 3 in. wide and 4 in. long. How many 1-inch cubes can be packed in one layer?



2. If the box is 3 in. high, how many layers will there be? Then what will be the cubic contents or volume of the box?

3. How many 1-inch cubes will it take to fill a box 6 in. long, 5 in. wide, and 4 in. deep?

DIMENSIONS OF A RECTANGULAR PRISM

The length, width, and height of a rectangular prism are called its **dimensions.** The height is sometimes called the altitude.

Any side of the prism may be called its base.

The exercises above show us that

The volume of a rectangular prism is equal to the product of its three dimensions.

Or we may say

The volume of a rectangular prism is equal to the product of the area of its base by its altitude.

In formulas we have

$V = lwh$ and $V = Bh$ ($V =$ volume, $l =$ length, $w =$ width, $h =$ height, $B =$ area of base).

MEASURES OF VOLUME AND CAPACITY

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet (cu. ft.) = 1 cubic yard (cu. yd.)
1 gallon contains 231 cubic inches
1 bushel contains 2150.42 cubic inches

Exercises

1. A classroom 23 ft. by 26 ft. by 12 ft. will provide enough air for 35 pupils. How many cubic feet per pupil is provided?

2. The aquarium for a biology classroom in a high school should be about 2 ft. by 3 ft. by 15 in. Find the number of gallons the aquarium will hold.

3. A farmer has a grain bin 8 ft. by 6 ft. by 6 ft. Allowing .8 bu. per cu. ft., how many bushels of grain can be stored in the bin?

4. What is the weight of a block of stone which is 8 ft. long, 2 ft. 6 in. wide, and 1 ft. 6 in. thick, if 1 cu. ft. of stone weighs 165 pounds?

5. The excavation for a house is 24 ft. wide, 30 ft. long, and 7 ft. deep. At 40¢ per cu. yd., find the cost of excavating.

6. The swimming pool at a certain college is 100 ft. long, 36 ft. wide, and has an average depth of $7\frac{1}{2}$ ft. How many gallons of water does it take to fill the pool, allowing 7.48 gal. per cubic foot?

7. A refrigerator will hold a piece of ice 18 in. long, 16 in. wide, and 9 in. thick. Allowing 57 lb. to the cu. ft., find the cost of filling the refrigerator when ice is selling at 65¢ per 100 lb.

8. A coal bin is 12 ft. long, 8 ft. wide, and 6 ft. deep. How many tons of coal will it hold, allowing 35 cu. ft. per ton?

9. The surface of a pond contains $1\frac{1}{2}$ acres. When ice is frozen to a depth of 12 in., how many tons of ice may be cut from it, allowing 57 lb. per cubic foot?

10. If one dimension of a rectangular prism is doubled, how does it affect the volume?

11. If two dimensions of a rectangular prism are doubled, how does it affect the volume?

12. If all three dimensions are doubled, how does it affect the volume?

13. If a block of ice in the shape of a cube 1 ft. long weighs 57 lb., what will be the weight of a cube 2 ft. long?

14. If the volume and two dimensions of a prism are given, how is the third dimension found?

15. If the volume and the altitude of a prism are given, how may the area of the base be found?

FINDING MISSING DIMENSIONS

1. A prism 3 ft. wide and 5 ft. long has a volume of 45 cu. ft. What is the height?

2. If a cubic foot of coal weighs 63 lb., to what depth will 8 tons fill a bin that is 6 ft. wide and 10 ft. long?

3. There are approximately $7\frac{1}{2}$ gal. to the cubic foot. To what depth will 75,000 gal. of water fill a pool that is 30 ft. wide and 60 ft. long?

4. A dump cart used for hauling dirt holds $1\frac{1}{3}$ cu. yd. per load. What is the area of the cart bed if the dirt is 2 ft. deep in each load?

5. A farmer wished to build a bin to hold 600 bu. of grain. If the area of the base of the bin is to be 120 sq. ft., what must be the depth of the bin?

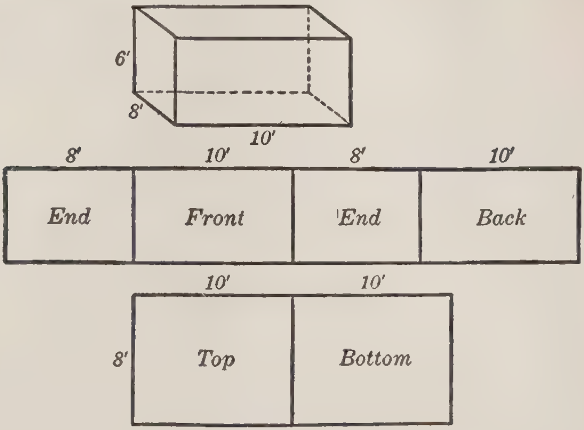
6. A classroom 24 ft. wide and 30 ft. long has an enrollment of 38 pupils. What must be the height of the ceiling to allow 218 cu. ft. of air for each pupil?

7. From these problems you have seen that

$$h = \frac{V}{B};\ B = \frac{V}{h};\ l = \frac{V}{wh};\ h = \frac{V}{wl};\ w = \frac{V}{hl}.$$

Tell in words what each formula means.

THE SURFACE OF A RECTANGULAR PRISM



1. How many square feet in the entire surface of the prism shown in the picture?

2. What is the area of the top and bottom? (They are sometimes called the two bases.)

3. What is the *lateral area* of the prism—that is, the area of the two ends and the front and back?

4. Using l for length, w for width, and h for height, write a formula for the area of the two bases of a prism; write a formula for its lateral area.

5. Explain why the formula $A = 2\,lw + 2\,hw + 2\,hl$ expresses the area of the entire surface of the prism.

6. Since every term in the formula in Ex. 5 is multiplied by 2, we can add the areas lw, hw, and hl and then multiply by 2. Thus we have $A = 2(lw + hw + hl)$.

This step is called **factoring the expression.**

7. If L is the lateral area of a prism, show that $L = 2h(l + w)$.

8. If A is the total area of a prism, show that $A = L + 2B$, where B is the area of the base.

When the numerical values of the letters are substituted in place of the letters, we say that the formula is **evaluated.**

Evaluate the following formulas:

9. $V = lwh$ when $l = 6$, $w = 5\frac{1}{2}$, and $h = 4$.
10. $P = 2(l + w)$ when $l = 10$, $w = 6$.
11. $A = lw$ when $l = 14$, $w = 6.5$.
12. $A = 2(lw + lh + wh)$ when $l = 8$, $w = 5$, and $h = 7$.
13. $A = \frac{h}{2}(b + b')$ when $h = 18$, $b = 7$, and $b' = 3$.
14. $A = \pi r^2$ when $r = 7$ and $\pi = 3\frac{1}{7}$.
15. $C = 2\pi r$ when $r = 2\frac{1}{3}$ and $\pi = 3\frac{1}{7}$.
16. $V = Bh$ when $B = 36$ and $h = 3\frac{1}{6}$.
17. Tell what each of the formulas in Ex. 9-16 means.

MEASURING LUMBER

The unit of measure used in measuring lumber is called a **board foot.**

A board foot is a board 1 foot wide, 1 foot long, and 1 inch thick.

NOTE.—A fractional part of an inch in thickness is counted as an inch when measuring lumber less than 1 inch in thickness.

Lumber is usually sold by the thousand (M) board feet. Thus "$50 per M" means $50 per 1000 board feet.

EXERCISES

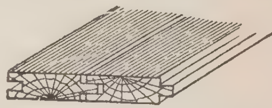
1. How many board feet in a piece of lumber 12 ft. long, 15 in. wide, and 1 in. thick?

2. Find the number of board feet in a piece of lumber 15 ft. long, 8 in. wide, and $\frac{7}{8}$ in. thick.

3. Find the number of board feet in a piece of lumber 12 ft. long, 4 in. wide, and 2 in. thick.

4. How much lumber is there in a post 4 in. by 4 in., if it is 15 ft. long?

5. Hardwood flooring is called 3-inch flooring when it is made from lumber which is three inches wide. In making the flooring there is a waste of $\frac{3}{4}$ of an inch in planing and in cutting the "tongue and groove." Then how wide a strip of floor is covered by each 3-inch board?



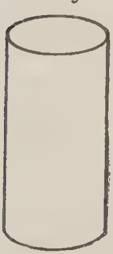
6. It is seen in Ex. 5 that each piece 3 in. wide will cover $2\frac{1}{4}$ in. of floor. Show that the area of the lumber that will be used is $\frac{1}{3}$ greater than the area of the floor to be laid.

7. How much lumber will be needed to floor 300 sq. ft. with 3-inch flooring? To floor 450 sq. ft? To floor 1500 sq. ft.?

FINDING THE VOLUME OF A CYLINDER

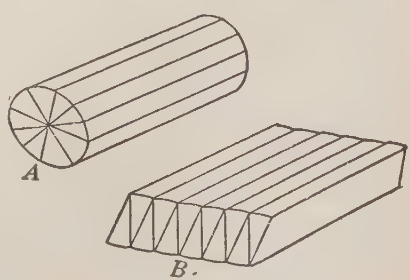
Another solid whose volume and surface we shall study is the cylinder.

A **cylinder** is a solid with a curved surface and two circular bases, like the illustration.



The distance between the bases is called the **altitude.** Many fruits and vegetables are packed in cylindrical cans.

A cylinder may be divided as shown in the figure and formed into a solid closely resembling a prism, from which we infer a fact proved in higher mathematics that



The volume of a cylinder is equal to the product of the area of the base by the altitude.

$V = Bh$. Since the area of the base B of a cylinder is $B = \pi r^2$, we also have $V = \pi r^2 h$.

Exercises

1. A hot-water tank 1 ft. in diameter and 5 ft. long will hold how many gallons of water? ($7\frac{1}{2}$ gal. = 1 cu. ft.)

2. An artesian well is 145 ft. deep and has a diameter of 6 in. If water rises to within 15 ft. of the top, how many gallons of water does it contain?

3. A water main is 6 in. in diameter. If the water runs through at the rate of 80 ft. per minute, how many gallons flow through the pipe in a minute?

4. A cylindrical silo is 12 ft. in diameter and 30 ft. high. Find the number of tons required to fill the silo, allowing 42.5 lb. to the cubic foot.

5. If corn will yield 12 tons of ensilage per acre, how many acres will be required to fill the silo in Ex. 4?

6. A farmer feeds 25 lb. of ensilage to each cow per day and has a herd of 30 cows. How many days will he be able to feed his herd from a silo that is 12 ft. in diameter and 30 ft. high filled with ensilage, allowing 42.5 lb. to the cubic foot?

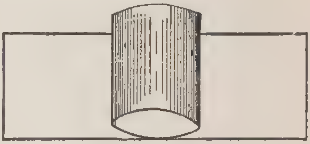
7. When a body is placed under water in a cylindrical pail 30 in. in diameter, the level of the water rises 6 in. What is the volume of the body?

8. In a certain city in the wheat region there is a grain elevator consisting of 45 cylindrical towers. If each tower is 20 ft. in diameter and 50 ft. high, how many bushels of grain can be stored in the elevator, allowing .8 bu. to the cubic foot?

9. Doubling the radius of the base of a cylinder has what effect on the volume?

10. Doubling the altitude of a cylinder has what effect upon the volume?

FINDING THE SURFACE OF A CYLINDER



The **total area** of a cylinder consists of the area of its two circular bases and of the area of its curved surface, called the **lateral area.**

1. Roll a strip of paper just as wide as the height of a cylindrical can about the can, as shown in the figure, using just enough to cover the lateral area. Now, unrolling it, describe the shape and the dimensions of the paper used. State a rule for finding the lateral area of a cylinder.

2. When A represents the lateral area of a right circular cylinder whose radius is r and whose height is h, give in words the relation expressed by the formula

$$A = 2\pi rh.$$

3. Two cylindrical columns in the front of a house are 15 ft. long and 18 in. in diameter. Find the cost of painting the columns at 2¢ per square foot.

4. A cylindrical stack is 150 ft. high and 4 ft. in diameter. Find the cost of painting it at $2.25 a hundred square feet.

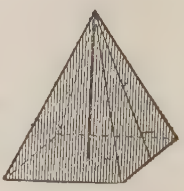
5. A room is heated by 8 steam pipes, each 18 ft. long and 2 in. in diameter. How much heating surface is provided for the room?

MEASURING THE PYRAMID AND THE CONE

A **pyramid** is a solid whose base is any kind of a polygon and whose other faces are triangles meeting at a point called the **vertex.** A **regular square** pyramid has a square for its

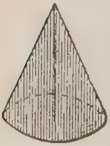
base and all of its other faces are isosceles triangles. It is the only kind of a pyramid discussed in this book.

The altitude of one of the triangular faces of such a pyramid is called its **slant height.**



It is evident from the figure that the area of the surface of a pyramid can be found by finding the areas of its faces and adding the results. The sum of the areas of the triangular faces meeting at the vertex of the pyramid is called the **lateral area** of the pyramid.

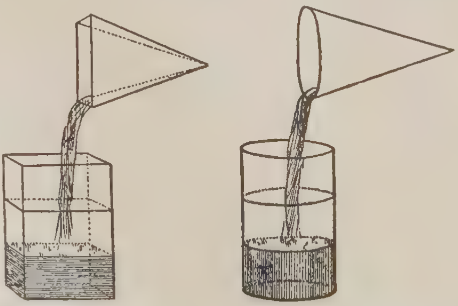
The illustration shows a figure called a **cone.** The **base** of a cone is a circle and its curved surface (called the lateral surface) tapers uniformly to a point called the **vertex.** The height of the vertex above the base is called the **altitude** of the cone. The distance from the vertex to any point in the circumference of the base is called the **slant height** of the cone.



The area of the curved surface of the cone is called the **lateral area.**

The lateral area of a cone is equal to the product of half the circumference of the base by the slant height.

Or $A = \pi r l$, where r is the radius of the base and l is the slant height.



It can be proved in higher mathematics or shown experimentally as in the figures that a pyramid contains one-third as much as a prism whose base and altitude are the same as the base and altitude of the pyramid. The same

relation is true between the cone and cylinder. Thus,

The volume of a pyramid or cone is equal to one third the product of the area of the base by the altitude.

Or $V = \frac{1}{3} Bh$.

Since the base of a cone is a circle whose area B is πr^2, we have, putting πr^2 in place of B,

$V = \frac{1}{3} Bh = \frac{1}{3}\pi r^2 h.$

Exercises

1. If the base of a pyramid is a square 6 in. on a side, and the altitude is 8 in., what is the volume?

2. The diameter of a cone is 4 ft. and the height is 3 ft. Find the number of cubic feet in the volume.

3. A conical pile of wheat is 8 ft. in diameter and 5 ft. high. How many bushels in the pile? (.8 cu. ft. = 1 bu.)

4. How much will it cost to water-proof a conical tent 20 ft. in diameter having a slant height of 15 ft. at 25¢ per square yard?

5. Find the lateral surface of a monument in the form of a regular square pyramid, if each side of the base is 4 ft. and the slant height is 8 ft.

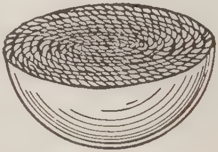
6. A pile of sand is conical in shape, being 5 ft. high and 10 ft. in diameter. How many cubic yards in the pile?

7. A haystack has the shape of a cone. Find the area to be covered with canvas, if the stack is 20 ft. in diameter and the slant height is 18 ft.

MEASURING THE SPHERE

The next figure that we shall study, the ball or sphere, is also of great importance. A **sphere** is a surface all of whose points are equally distant from the center. This distance from the center is called the **radius** of the sphere. The **diameter** of a sphere is twice the radius.

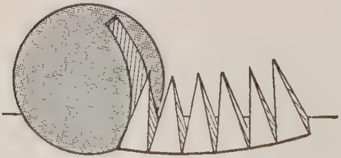
If you cut straight through the center of a sphere two hemispheres are formed. The circle made by the cut is called a **great circle** of the sphere.



Archimedes, a Greek mathematician who lived 225 B.C., was greatly excited when he discovered that the entire area of a sphere is equal to the areas of four great circles, or

$$A = 4\pi r^2.$$

It is seen in the figure that the volume of a sphere can be divided into a number of solids each of whose vertices lies



at the center of the sphere. The sum of the areas of the bases of these solids is equal to the area of the sphere. If each of these solids had flat bases they would be pyramids.

From this we can infer a truth which is proved in higher mathematics that

The volume of a sphere is the same as the volume of a pyramid whose base is equal to the area of the sphere and whose altitude is equal to its radius.

Hence, if V is the volume of a sphere, A the area, and r the radius

$V = \frac{1}{3}rA$. Since $A = 4\pi r^2$ we have

$V = \frac{1}{3}r \times 4\pi r^2 = \frac{4}{3}\pi r^3$.

NOTE.—r^3 means $r \times r \times r$ and is read "r cubed."

Exercises

1. Find the surface of a sphere whose radius is 10 in.; of one whose radius is 16 ft.

2. The radius of the earth is approximately 4000 mi. How many square miles in its surface?

3. What is the volume of a sphere 10 ft. in diameter?

4. What is the volume of a sphere whose radius is 8 in.?

5. If a cubic foot of steel weighs 460 lb., what will be the weight of a steel ball 2 in. in radius?

6. A washbowl in the shape of a hemisphere 12 in. in diameter will hold how many gallons? (231 cu. in. = 1 gal.)

7. The ball on the top of St. Paul's Cathedral in London is 6 ft. in diameter. How much will it cost to gild it at 15¢ per square inch?

8. A spherical hollow iron shell is 1 in. thick and the outer diameter is 6 in. If the iron weighs 450 lb. per cubic foot, find the weight of the shell.

9. Find the ratio between the volume of a sphere 2 in. in diameter and the volume of one 4 in. in diameter.

Note.—Since π occurs in each product, in comparing the areas or the volumes of two spheres, you need not multiply by the numerical value of π.

10. Compare the volume of an orange 3 in. in diameter with that of one 4 in. in diameter.

11. If you double the radius of a sphere how is its volume changed? How is it changed if you make the radius three times as great?

12. If you double the radius of a sphere, what effect will it have on the surface?

13. If you treble the diameter of a sphere, what effect will it have on the surface?

SOME THINGS LEARNED IN THIS CHAPTER

You have become familiar with using formulas and with substituting values for the letters in formulas. In these

exercises, find the value of each of the letters that is not known. In every case tell what the formula means and what kind of a figure it measures.

1. $A = lw$. Find l when $A = 40$ and $w = 5$.
2. $A = bh$. Find h when $A = 36$ and $b = 9$.
3. $A = \frac{bh}{2}$. Find b when $h = 4$ and $A = 16$.
4. $A = \frac{h}{2}(b + b')$. Find b' if $A = 80$, $h = 8$, and $b = 12$.
5. $C = 2\pi r$. Find C when r is 6.
6. $A = \pi r^2$. Find A when $r = 4$.
7. $V = \pi r^2 h$. Find h if $V = 216$ and $r = 4$.
8. $A = 2\pi rh$. Find r if $h = 6$ and $A = 75$.
9. $V = lwh$. Find w when $l = 6$, $V = 144$, and $h = 8$.
10. $V = \frac{Bh}{3}$. Find V when $B = 24$ and $h = 5$.
11. $V = \frac{\pi r^2 h}{3}$. Find V when $r = 5$ and $h = 6$.
12. $A = 4\pi r^2$. Find A when $r = 6$.
13. $V = \frac{4}{3}\pi r^3$. Find V when $r = 2$.

True and False Test

If a statement is true, mark it +; if it is false, mark it −:

1. If a square and a rectangle have equal areas, they have equal perimeters.

2. When the area of a square is given, the length of one side may be found by dividing the number of units in the area by 4.

3. If the length of a rectangle is twice the width, the width may be found by dividing the perimeter by 6.

4. If the sides of a parallelogram are each the same as the sides of a rectangle, the area of the two figures are the same.

5. If a triangle and a parallelogram have equal bases and equal altitudes, the area of the triangle is equal to half that of the parallelogram.

6. Area is always expressed in square units.

7. The volume of a cylinder is equal to one third the product of the area of the base by the altitude.

8. A trapezoid never has its two parallel sides equal.

9. If the diameter of a circle is doubled, the circumference is doubled.

10. If the diameter of a circle is doubled, the area is doubled.

11. The ratio of the circumference and the diameter is always the same in all circles.

12. If two circles have equal circumferences, their areas are equal.

13. The ratio of the areas of two circles is the same as the ratio of the squares of their diameters.

14. If each of the three dimensions of a prism is doubled, the volume will be multiplied by 6.

15. A square whose side is half that of another square has one fourth the area of the larger square.

16. If the value of the radius is given, the volume of a cylinder can be found.

17. A board $\frac{1}{2}$ in. thick will have the same number of board feet as a board 1 in. thick, assuming that the other dimensions are the same.

18. If a pyramid and a prism have equal bases and equal altitudes, their volumes are equal.

19. The radius of one sphere is 1 in. and that of another is 3 in. The larger sphere has 27 times the volume of the smaller one.

20. The radius of one sphere is 1 in. and that of another is 3 in. The larger sphere has 9 times the area of the smaller one.

Test in Matching Formulas

In Column II you will find figures whose areas and volumes you have studied. In Column III you will find formulas for the figures given in Column II. In Column I write the number of the formula in Column III that should go with the figure in Column II.

Thus, 11 in Column III goes with 1 in Column II. Therefore write 11 in Column I.

I		II		III
11	1.	Area of a circle	1.	$\frac{h(b+b')}{2}$
	2.	Volume of a prism	2.	$\frac{1}{3}Bh$
	3.	Surface of a sphere	3.	$\frac{4}{3}\pi r^3$
	4.	Area of a parallelogram	4.	$6a^2$
	5.	Volume of a cone	5.	a^3
	6.	Area of a trapezoid	6.	$2\pi rh$
	7.	Volume of a sphere	7.	$4\pi r^2$
	8.	Volume of a pyramid	8.	$\frac{\pi r^2 h}{3}$
	9.	Volume of a cylinder	9.	Bh
	10.	Volume of a cube	10.	lwh
	11.	Lateral surface of a cylinder	11.	πr^2
	12.	Surface of a cube	12.	$\pi r^2 h$

THE METRIC SYSTEM; OTHER MEASURES

You have found that it is generally easier to compute with decimals than it is with common fractions because the denominator of a decimal fraction is a power of 10. A system of linear measurement which expresses all of its fractional parts

in a decimal form is much easier than the present English system.

The metric system expresses all of its fractional parts of units in such a form, hence it is easier to compute and measure by this system.

The metric system originated in France. During the French Revolution the National Assembly (1790) appointed a commission of the Academy of Sciences to study the matter. This commission selected as the unit of measure one ten-millionth of the distance from the equator to the North Pole and called it the **meter,** "the measure." This was calculated to be about 39.37 inches, or about $3\frac{1}{3}$ inches longer than the yard.

The report of the commission was adopted in 1799, and since that time it has been adopted in nearly all countries except England and the United States. In the scientific field the metric system is used almost entirely, but in other fields it has not been adopted so widely.

THE METRIC TABLE OF LENGTH

10 millimeters (mm)	=	1 centimeter (cm)
100 centimeters	=	1 meter (m)
1000 meters	=	1 kilometer (km)

THE ENGLISH LINEAR MEASURE

12 inches (in.)	=	1 foot (ft.)
3 feet	=	1 yard (yd.)
$5\frac{1}{2}$ yards	=	1 rod (rd.)
320 rods 5280 feet	=	1 mile

1. Using the fact that 39.37 in. = 1 meter, complete the table given below. Carry out the division to three decimal places.

1 millimeter	= ......in.		1 in.	= ......cm.
1 centimeter	= ......in.		1 ft.	= ......cm.
1 meter	= ......in.			= ......m.
	= ......ft.		1 yd.	= ......m.
	= ......yd.		1 rd.	= ......m.
1 kilometer	= ......ft.		1 mi.	= ......m.
	= ......yd.			= ......km.
	= ......mi.			

2. The distance between two cities in France is 64 kilometers. How many miles is that?

3. During the World War the troops occupying a certain sector advanced 3.7 kilometers. How many miles is that?

4. The distance from Chicago to Denver is 1034 miles. How long will it take a mail-plane to cover the distance if the plane is averaging 150 kilometers per hour?

5. Which of two runners is going the faster, the one who runs the 100-yd. dash in 10 seconds, or the runner who covers the 100-meter dash in $11\frac{1}{5}$ seconds? How much difference in the distance covered is there in one second of time? Express the answer in feet.

6. In a recent altitude climb, an aviator reached a height of 8326 meters. How many miles is that?

7. Find the volume of a cylindrical block of wood that has a diameter of 4.5 cm. and an altitude of 7 cm.

8. Find the area of a board that is 65 cm. wide and $2\frac{1}{2}$ m. long.

9. Measure your desk and see what the dimensions are in the metric unit.

10. Find the length of your classroom in feet. Change the result to meters.

MEASURING TEMPERATURE

Temperature is measured by an instrument called a **thermometer.** It consists of a glass bulb and a small hollow stem. The bulb is filled with mercury or some colored fluid. The bulb is placed in boiling water and the height of the mercury is marked. Then the bulb is put in freezing water and again the level of the fluid is marked. These two points are the ones from which all graduations are made. (A thermometer is said to be graduated to a scale when some unit is accepted between the freezing and boiling points.) There are two different scales that have been adopted for graduating thermometers. The one most generally used in recording temperatures about the home is the **Fahrenheit** scale. The other scale is used for recording temperatures in all scientific work and is called the **Centigrade** scale.

In the Fahrenheit scale the boiling point is 212° and the freezing point is 32°. From this you see that a Fahrenheit thermometer has 180° or graduations between the freezing and the boiling points. The freezing point on the Centigrade thermometer is marked 0° and the boiling point is marked 100°. On this scale there are 100° or graduations between the freezing and the boiling points.

It is quite evident that one degree on the one scale would not be equivalent to one degree on the other.

1° C means 1 degree Centigrade scale.

1° F means 1 degree Fahrenheit scale.

The formulas connecting the two scales are:

$$C = \frac{5}{9}(F - 32) \text{ and } F = \frac{9}{5}C + 32$$

1. Tell what advantage the Centigrade scale might have over the Fahrenheit scale for scientific measurement.

2. What other system of measurement used for scientific work has its units of measure based on 10 or a power of 10?

3. 1° F equals how many degrees C?

4. 1° C equals how many degress F?

5. 9° F equals how many degrees C?

6. 36° C equals how many degress F?

7. Walter knew that 1° F equalled $\frac{5}{9}$° C. One day when the reading of the house thermometer (F) was 77° he thought that he would find the Centigrade thermometer registering about 43°. Instead, he found it at 25°. Find where he made a mistake in his reasoning.

Change to the scale indicated:

8. 46° F to C.

9. 58° F to C.

10. 106° F to C.

11. 10° F to C.

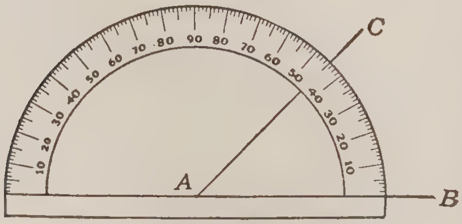
12. 15° C to F.

13. 45° C to F.

14. 81° C to F.

15. 9° below zero C to F.

MEASUREMENT BY SCALE DRAWING

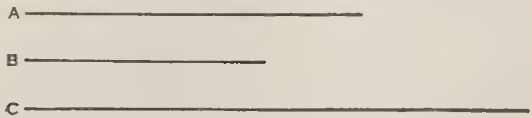


PROTRACTOR

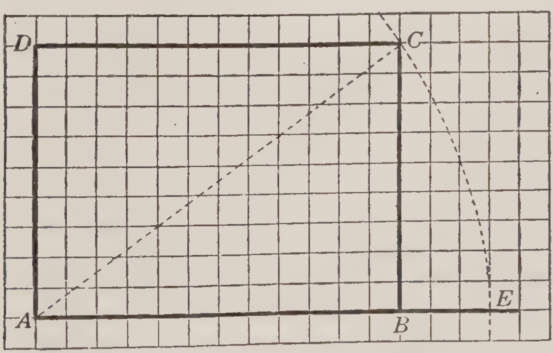
It is much easier to draw to a scale if paper lined into squares is used. This form of paper is called **graph paper.** If you have graph paper and a **protractor** you are able to make many measurements which would otherwise be hard to make. A protractor is semi-circular in shape and is graduated in degrees. It is used to measure angles.

Exercises

1. Measure these lines and tell what distance each represents. Scale 1″ = 10′.



2. Mr. Jones owned a lot 50 ft. by 125 ft. He wished to build a house on it 24 ft. wide and 30 ft. long. He wanted to place the house 20 ft. back from the property line and wished to allow 10 ft. on the one side of the house for a driveway. If the 24 ft. dimension faced the front of the lot, draw to scale the location of the house on the lot.



3. *ABCD* is a plat or map of a rectanglar field 27 rd. wide and 36 rd. long. To what scale was it drawn? That is, what length is represented by the side of each small square?

4. With compasses, and with center at *A* and with radius *AC*, draw an arc cutting *AB* extended in *E*. Find the length of *AC* by measuring *AE*.

5. See if this is the same result that you would get by using the Pythagorean theorem.

6. Frank rode his bicycle 6 mi. north from his home, then 5 mi. west. Using squared paper, draw a plan of the route he took and find how far from home he was. Use a convenient scale. Check your result by using the Pythagorean theorem.

7. Turn to a map in your geography and see what scale is used. If the scale is 1″ = 500 mi., would your measurement need to be very accurate in order to get the actual distance between two places on the map? Can you tell why this is true?

8. If 1″ = 500 ft. on one scale and 1″ = 50 ft. on another scale, where would an error in measurement make the greater difference? How much greater will this error be for the one scale than for the other?

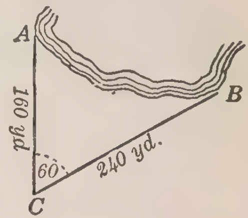
9. An athletic field of a certain high school is in the shape of a trapezoid. One of the parallel sides is 415 ft. long and the other is 475 ft. long. At one side a street 260 ft. long connects the two parallel sides and is perpendicular to them. Find the length of the other side.

10. Using the data in Ex. 9, draw to scale a baseball diamond which can be laid out on this field, assuming that the home plate is 50 ft. from the street. Allow a distance of 120 ft. back of each base for the outfield.

11. Make a scale drawing of your school ground. Estimate what you consider the length of the boundary lines. Then measure the distances and see if your estimate was a good one. See how far from the boundary lines your school is located and place it in the proper position on your drawing. See if you can approximate the amount of playground you have. There should be about 30 sq. ft. for each pupil. Does your recreation ground provide this much space for each pupil in your school?

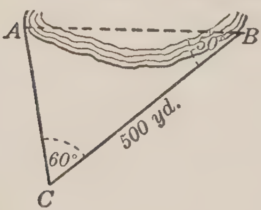
12. Two boys swam from a point on one shore of a lake to a

point on the opposite shore and wished to know the distance. They found a point C from which they could measure to both points (A and B in the figure). At this point they found the angle ACB to be 60° by using a field protractor. AC was measured and found to be 160 yd., and CB was found to be 240 yd.



Use squared paper and a protractor and draw a plat to some scale and find the distance from A to B.

13. Walter found a way to measure the distance AB by measuring but one length. From B he sighted a tree, called C in the plan, and measured the angle ABC, which was 50°. He then measured to C and found it to be 500 yd. At C he measured the angle ACB and found it to be 60°.

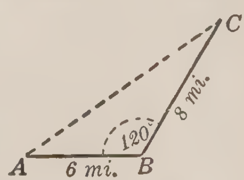


Draw a plan to some scale and find the distance from A to B.

14. To find the height of a tall church spire, Walter measured the angle between a horizontal line and the line of sight and found it to be 60°. He then walked 100 ft. nearer and found that an angle measured in the same way was 80°.

Draw a plan to some scale and find the height of the spire.

15. Frank rode from A to B, 6 mi.; then from B to C, 8 mi. At B angle ABC was 120°. If he could ride directly back in a straight line from C to A, how many miles could he save?



16. When a distance of 150 mi. is represented on a map by $1\frac{1}{2}$ in., what distance is represented by $3\frac{1}{4}$ in.?

17. When a distance of 200 mi. is represented by $1\frac{1}{2}$ in., what length will represent 500 mi.?

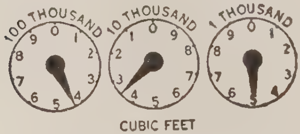
18. When a map is drawn to a scale of 1″ = 200 mi., what distance is represented by an error of $\frac{1}{32}$ of an inch?

19. When a map is drawn to a scale of 1″ = 20′, what distance is represented by an error of $\frac{1}{32}$ of an inch?

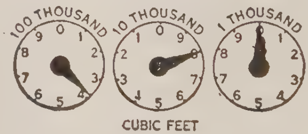
20. State when scale drawing is reasonably accurate and when its results give only a rough approximation to the real distance.

MEASURING AND BUYING GAS

The cost of gas is usually computed on the price per 1000 cu. ft. (per M). It is measured by a **gas meter** which shows the number of cubic feet used.



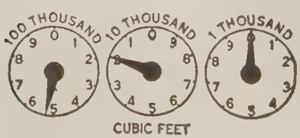
The reading is 43,500 cu. ft.

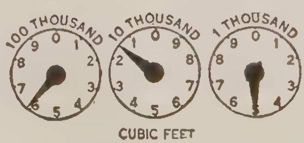


The reading is 38,000 cu. ft.

1. The reading on my gas meter on Feb. 26 was 42,500 cu. ft. On March 27 it was 44,000 cu. ft. At $1.10 per M, what was the amount of my bill?

2. The present reading of my gas meter is 78,800 cu. ft. A month ago the reading was 75,200 cu. ft. At 90¢ per M, find the cost of gas for the month.





3. Read and find the cost of gas consumed at 90¢ per M.

4. A certain type of hot water heater consumes 90 cu. ft. of

gas per hour. If it is used $1\frac{1}{2}$ hr. during the day, what will the gas cost at \$1.10 per M?

5. One burner in a kitchen range, turned medium height, consumes 35 cu. ft. per hour. If Mrs. Brown has an average of four burners going for $1\frac{1}{2}$ hr. in getting dinner, find the cost of the gas used at \$1 per M.

6. When the cost of gas was reduced from \$1.10 per M to 90¢ per M, what was the yearly saving to a family that used an average of 7500 cu. ft. per month?

MEASURING AND BUYING ELECTRICITY

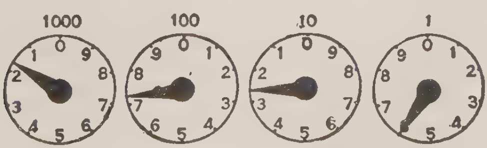
The **unit of measure** in measuring electricity is the **watt hour.** 1000 watt hours make a **kilowatt hour.**

The cost of electricity is usually computed on the *kilowatt hour*, usually called merely **kilowatt.**

It is measured by an **electric meter** which shows the number of kilowatts used. The meter has four dials as in the figure.

Each division on the right-hand dial denotes 1 kilowatt; the next to the left of it denotes 10 kilowatts; the next, 100 kilowatts; and the next, 1000. Hence the meter is read as easily as any four-figured number.

Thus the reading of the following dial is 1726 kilowatt hours.



KILOWATT HOURS

1. A month ago the reading of my meter was 6480 K.W.H. (kilowatt hours). The reading now is 6945 K.W.H. At 10¢ per kilowatt hour, find the cost of electricity for the month.

2. The April reading of a meter was 2865 K.W.H. The May

reading was 3090 K.W.H. Find the cost of the electricity used at 9¢ per kilowatt hour.

3. Check the following bill:

March	25	1930	2972 kilowatt hours	
Feb.	20	1930	2935 kilowatt hours	
			37 kilowatt hours at 10¢	$3.70

4. Find the cost at 10¢ per kilowatt hour: April reading, 1732; May reading, 1761.

5. A family using six 25-watt lights (lamps consuming 25 watts per hour) is using how many watts per hour? What decimal part of a kilowatt? What is the cost at 10¢ per kilowatt?

6. A family that uses, on an average, ten 40-watt lights for 5 hours each evening during the winter months is paying how much per day for light at 10¢ per kilowatt hour?

7. If the assembly hall in your school uses forty 60-watt lights for 45 minutes each day, what is the daily cost at 10¢ per kilowatt hour?

8. A 6-pound electric iron uses 550 watts per hour. Find the cost per week at 10¢ per kilowatt hour when the iron is used $5\frac{1}{2}$ hours.

9. A vacuum cleaner using 150 watts an hour will cost how much per hour to run when electricity costs 10¢ per kilowatt?

10. A 12-inch electric fan using 60 watts per hour is used 10 hours per day. Find the cost per month (30 da.) at 10¢ per K.W.H.

11. If an electric coffee percolator (450 watts) is used 40 min. per day, and an electric toaster (600 watts) is used 20 min. per day, find the weekly expense for electricity at 9¢ per K.W.H.

12. The carbon filament lights consume $3\frac{1}{3}$ watts hourly per candle power, and the tungsten lamp consumes but $1\frac{1}{4}$ watts

per candle power. If a building uses 2400 candle power of light, find the saving per hour by using the tungsten lamps, allowing 8¢ per K.W.H.

13. Electric motors consume 746 watts of energy per hour for each horse power. If a shop uses a 1-horse-power motor 8 hours daily, what is the cost at 8½¢ per K.W.H. of the electricity used?

The following rate is charged by one of the large electric lighting corporations in New Jersey.

ELECTRIC LIGHTING RATE

Electric Minimum Charge, $1.00 per month.

The first	20 K.W.H. per month	9¢ per K.W.H.
The next	30 K.W.H. per month	8¢ per K.W.H.
The next	450 K.W.H. per month	7¢ per K.W.H.
The next	500 K.W.H. per month	6¢ per K.W.H.
The next	1000 K.W.H. per month	5¢ per K.W.H.
The next	8000 K.W.H. per month	4¢ per K.W.H.
All over	10,000 K.W.H. per month	3¢ per K.W.H.

14. What will it cost a family in New Jersey during the month of August for light if 7 K.W.H. is consumed?

15. If a factory uses on an average of 15,650 K.W.H. each month, what will be the yearly cost of the current?

16. A boy has his radio attached to the electric lighting system of his home. He found that it consumes about 50 watts per hour. If he uses the radio about 3 hours each day, how much extra cost is added to his family's light bill each month of 30 days, if the rate is 9¢ per K.W.H.?

17. A large power company in Chicago in 1900 was serving 16,000 customers who used an average of 16 K.W.H. per month. The same company in 1924 was serving 755,000 customers at an average of 845 K.W.H. per year. Find the per cent of increase in the consumption of electricity.

18. Bring an electric light bill from your home and see how the rate compares with the rate given in New Jersey.

THE AVERAGE AND THE MEDIAN

You know that the **average** of a series of measures is found by adding them and dividing by the number of measures in the series. Thus, if the weights of 5 boys are 93 lb., 97 lb., 103 lb., 105 lb., and 107 lb., the average is

$$\frac{93+97+103+105+107}{5} = 101, \text{ the number of lb.}$$

This means that if each boy weighs 101 lb., the combined weight of the five boys is the same.

When a series of measures is arranged in order, the middle one is called the **median.** Because the median is easy to find it is frequently used instead of the average.

Very likely you have used Standard Drills in the four fundamental operations in arithmetic. You found whether you were above the standard or below it for each operation. This standard is found by giving a test to many pupils in each grade. Then the mark of the middle student is taken as what should be expected for a median score. Many colleges wish to know whether you stand above or below the middle of your class in order to determine your fitness to enter college.

Exercises

1. The 5 highest scores made on a test were 100, 87, 95, 98, and 96. What was the median score? The average?

2. The weights of a football team's players were as follows: 156 lb., 192 lb., 160 lb., 153 lb., 203 lb., 191 lb., 166 lb., 187 lb., 148 lb., 138 lb., and 175 lb. What was the median weight? The average weight?

3. The ages of 5 boys were 12 yr. 6 mo., 15 yr. 3 mo., 12 yr. 9 mo., 14 yr. 3 mo., and 14 yr. 6 mo. Find the median age; the average age.

4. Are the median and the average ever the same? If so, give a problem where they are.

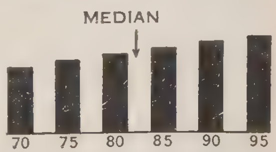
5. In a class of 27 pupils in arithmetic, the scores made were as follows:

2 pupils received 100	3 pupils received 78
4 pupils received 95	3 pupils received 75
4 pupils received 90	2 pupils received 70
4 pupils received 80	1 pupil received 87
3 pupils received 92	1 pupil received 60

Find the median score and the average score.

You will notice that the median score in each example given above could be found accurately as there was an odd number of cases given each time. When there is an even number of cases there are two middle scores and the median is taken as $\frac{1}{2}$ the sum of the two middle scores.

6. The scores on a recent test in history were 75, 70, 85, 90, 80 and 95. Find the median. From this you see that the median is midway between 80 and 85.



7. The weights of four people are 140 lb., 144 lb., 136 lb., and 152 lb. Find the median.

8. The next test you have in arithmetic, find what the median score is for the class; also find the average score. See which you think is a better measure by which to judge your scores.

Review Exercises on Areas and Volumes

1. The perimeter of a rectangle is 120 ft. If the length is 40 ft., what is the area?

2. The length of a rectangle is twice the width. What are the dimensions, if the perimeter is 300 ft.?

3. A triangle has an area of 60 sq. in. If the base is 10 in., what is the altitude?

4. A parallelogram has a base of 12 ft. and an altitude of 6 ft. What is the area?

5. A lot in the shape of a rectangle has an area of 5400 sq. ft. If the lot is 120 ft. deep, what is the width?

6. In making a radio set a boy wishes to wrap a cylinder whose diameter is 4 in. with 75 turns of wire. How many feet of wire will be required to wrap the cylinder?

7. A 30-in. automobile wheel will make how many revolutions in going $\frac{1}{4}$ mile?

8. A circular fountain 20 ft. in diameter has a 4 ft. walk around it. Find the cost of laying the walk at $3.25 per square yard.

9. The athletic field at a certain high school contains 9.4 A. If a quarter-mile circular running track is laid out, what part of the whole field is enclosed by the track?

10. How do the areas and circumferences of two circles compare if the diameter of one circle is twice that of the other?

11. Find the weight of a steel beam if it is 8 ft. long, 4 in. wide, and has an average thickness of 2 in., allowing 460 lb. to the cubic foot.

12. A monument is in the shape of a rectangular prism. If it is 12 ft. high, 6 ft. wide, and 2 ft. thick, how much surface is there to be polished?

13. A bin is 8 ft. long and 6 ft. wide. If it contains 300 bu., to what depth is it filled?

14. Find the surface of a tennis roller, if it is 3 ft. in diameter and 4 ft. wide.

15. A cylindrical tank is 2 ft. long and 10 in. in diameter. How many gallons will it hold?

16. Find the volume of a pyramid, if each side of the square base is 4 ft. and the height is 6 ft.

17. A conical pile of grain is 6 ft. in diameter and 4 ft. high. How many bushels in the pile?

18. What is the surface of a sphere 7 in. in diameter?

19. Find the weight of a steel ball 4 in. in diameter if steel weighs 490 lb. to the cubic foot.

20. If the weight of a steel ball 2 in. in diameter is known, how many times that amount will be the weight of a steel ball 8 in. in diameter?

REVIEW OF OTHER MEASURES

1. If an airplane is 2000 meters high, how many feet high is it?

2. When a train is running at the rate of 45 miles per hour, how many kilometers is that?

3. When a thermometer at your house is reading 68° F, what would be the reading on a Centigrade thermometer?

4. A thermometer reading 25° C would have what reading Fahrenheit?

5. Draw to scale a rectangular lot that is 60 ft. by 125 ft.

6. If a map is drawn to a scale of 1″ = 50 mi., what is the distance on the map between two cities that are 460 mi. apart?

7. An error of $\frac{1}{32}$ of an inch in measuring a distance that is drawn to a scale 1″ = 150 mi. will make the actual distance how much too small or too great?

8. Mr. Brown's gas meter on Mar. 1st read 42,560. On Apr. 1st it read 45,690. At $1.20 per M, find the cost of the gas consumed during the month of March.

9. Mr. Brown's electric meter for the same period of time read 8650 and 9120, respectively. At 9¢ per K.W.H., find the cost of the electricity consumed during the month of March.

10. The heights of the members of the basketball team were 5 ft. 9 in., 6 ft. 2 in., 5 ft. 10 in., 6 ft., and 5 ft. 8 in. Find the median height and also the average height.

11. Counting 10 for each problem in this exercise, find the median score and also the average score made by the members of your class.

CHAPTER VI

PER CENT AND SOME OF ITS APPLICATIONS

1. You have learned that

$$a.\ \$8 \div \$4 = 2;$$
$$b.\ \$8 \div \$8 = 1;$$
$$c.\ \$8 \div \$16 = \tfrac{1}{2} \text{ or } .5.$$

a states that \$8 is ——— times as large as \$4.
b states that \$8 is just as large as ———.
c states that \$8 is ——— as large as \$16.

I. When both dividend and divisor are like numbers, the quotient is always the relation (ratio) of the dividend to the divisor.

II. The relation of one number to another may be a whole number, a fraction, a decimal, or a mixed number. But these numbers may always be read as so many hundredths or per cent (%).

Thus,
$$2 = \tfrac{200}{100} = 200\%$$
$$1 = \tfrac{100}{100} = 100\%$$
$$\tfrac{1}{2} = \tfrac{50}{100} = 50\%$$

Per cent is only another name and notation for hundredths.

2. *Express as per cent:*

A	*B*	*C*	*D*	*E*
.07	.125	1.35	2.3	$\frac{1}{2}$
.12	.045	2.40	4.5	$\frac{3}{4}$
.36	.005	1.75	1.2	$\frac{4}{5}$

3. *Express as a per cent of one dollar:*

5¢, 8¢, 12¢, 15¢, 20¢, 35¢.

4. In a spelling test of 100 words, give the per cent right of:

Mary, 96 right.	Walter, 87 right.
Frank, 91 right.	Harry, 83 right.
Helen, 90 right.	Nell, 76 right.

5. In a test of 50 words, tell the per cent made by:

John, 49 right.	Frank, 41 right.
Ruth, 46 right.	Elizabeth, 38 right.
Lucile, 43 right.	Ralph, 32 right.

Exercises in Per Cent

"Per cent of" means "hundredths of."

So 4% of \$100 means .04 × \$100; and ½% of \$100 means .005 × \$100.

Give without a pencil:

1.	2.	3.
5% of \$100	20% of 100 mi.	100% of 150
6% of 150 ft.	40% of 20 gal.	150% of 40
8% of 200 mi.	50% of 40 yd.	200% of 80

In changing from per cent to decimals, be sure that the number of decimal places is two more when the per cent sign is dropped.

In changing from decimals to per cent, be sure that the number of decimal places is two less when the per cent sign is used.

ILLUSTRATION

5% = .05	.03 = 3%
12% = .12	.15 = 15%
100% = 1	2 = 200%
225% = 2.25	3.45 = 345%

EXERCISES

Express as decimals:

1.	2.	3.	4.	5.
23%	7%	135%	$3\frac{1}{2}$%	1.3%
48%	3%	250%	$6\frac{1}{4}$%	2.6%
54%	8%	175%	$1\frac{3}{4}$%	5.8%
90%	1%	850%	$\frac{1}{8}$%	.2%

Change the decimals to per cent and the per cent to decimals:

6.	7.	8.	9.	10.
48%	1.28	3%	.135	$15\frac{1}{2}$%
4.8%	135%	.07%	$7\frac{1}{2}$%	$\frac{1}{2}$%
.43	2.4	$3\frac{1}{2}$%	.045	.005

FINDING WHAT PER CENT ONE NUMBER IS OF ANOTHER

1. Our team played 24 games and won 18 of them. We won what per cent of the games played?

$$\begin{array}{r@{\,}l} & .75 = 75\% \\ 24\,) & \overline{18.00} \\ & 16.8 \\ & \overline{1.20} \\ & 1.20 \end{array}$$

Think, "I am to compare 18 *with* 24*, which means that I am to divide* 18 *by* 24*."*

Then think, ".75 = 75%*."*

2. Frank has a flock of 70 hens. Donald has 40. Frank's flock is what per cent of Donald's?

$\frac{70}{40} = 1\frac{3}{4} = 175\%$	*Think, "I am to compare 70 with 40." Then think, "70 = $1\frac{3}{4}$ times 40." Then think, "$1\frac{3}{4}$ = 1.75 = 175%."*

3. In Problem 2, Donald's flock is what per cent of Frank's?

$\frac{40}{70} = .57\frac{1}{7} = 57\frac{1}{7}\%$	*How can you tell in all such problems which number is the divisor and which the dividend?*

4. One year Chicago's baseball team played 90 games and won 55 of them. It won what per cent of the games played?

5. If there are 180 days in the school year and you are absent 12 of them, you are present what per cent of the time?

6. A young man's salary increased in three years from $15 per week to $75. This was an increase of what per cent?

7. The value of a city lot increased from $24 per front foot to $100 per front foot. This was an increase of what per cent?

8. When an article selling for $4.80 gave a profit of $1.20, the profit was what per cent of the selling price? What per cent of the cost?

9. Last year our school enrollment was 480. This year it is 552. That is an increase of what per cent?

10. The price of an automobile was decreased from $1625 to $1350. That was a decrease of what per cent?

11. In a school of 640 pupils 312 are boys. What per cent are boys and what per cent are girls?

12. A man has a house that cost him $15,000 and he rents it for $125 per month. He has another that cost him $25,000

that he rents for \$200 per month. Which gives him the larger yearly per cent on his investment and how much?

13. In 15 years, a city increased in population from 10,000 to 160,000. What per cent of increase is that?

14. It is predicted that this same city in 2 years will increase its present population 60,000. What per cent of increase will that be?

FINDING A PER CENT OF A NUMBER

1. Walter sold \$280 worth of vegetables for a gardener and got 35% of it for his work. How much did he earn?

\$280 .35	*Think,* "35% *of* \$280 *means* .35 × \$280."

2. One year a cow gave 13,246 lb. of milk that tested 3.58% butter fat. Find how much butter fat she produced during the year.

13246 .0358	*Think,* "3.58% *of* 13,246 *means* .0358 × 13,246."

3. An agent sold Mr. Brown's farm for him for \$11,500 and charged $3\frac{1}{2}$% of the price as his fee. How much did he get for selling it?

\$11500 .035	*Remember that* $3\frac{1}{2}\% = .035$.

4. A man wants to save 20% of his salary to apply on a home. To do this he must save how much from a salary of $3600 per year?

5. A teacher getting a salary of $1600 was given an increase of 15%. How much did she then get?

6. A man bought a radio set listed at $269 at a 10% discount. How much did it cost him?

7. A car selling at $2150, but having been used three months for demonstration purposes, is offered at a reduction of 15%. What will it cost?

8. One year a farmer's corn crop averaged 36.5 bu. per acre. He thinks that more care in the selection of seed and fertilization should increase it 20%. If so, how much would the average be?

9. A young man earning $35 per week estimated that he should use it as follows: Room and board, 40%; clothing and laundry, 18%; recreation and incidentals, 12%; advancement such as books, night school, etc., 15%; saving the balance. Find how much this will allow per year for each.

10. At a certain high school 65% of the enrollment is girls. If there are 520 pupils enrolled, how many boys and how many girls are there?

11. A boy at college is given an allowance of $25 a week for a period of 40 weeks. If his tuition amounts to 20% of his allowance, room and board 36% of it, books 5% of it, and the remainder for incidentals, find the amount spent for each.

SPECIAL PER CENT

It is easier to find the following per cents of a number by changing them to fractions instead of to decimals. Hence learn,

$50\% = \frac{1}{2}$	$37\frac{1}{2}\% = \frac{3}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$
$25\% = \frac{1}{4}$	$62\frac{1}{2}\% = \frac{5}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$
$12\frac{1}{2}\% = \frac{1}{8}$	$87\frac{1}{2}\% = \frac{7}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$

The following are also easier to use as fractions, but they seldom occur:

$14\frac{2}{7}\% = \frac{1}{7}$	$42\frac{6}{7}\% = \frac{3}{7}$	$11\frac{1}{9}\% = \frac{1}{9}$
$28\frac{4}{7}\% = \frac{2}{7}$	$83\frac{1}{3}\% = \frac{5}{6}$	$22\frac{2}{9}\% = \frac{2}{9}$

Exercises in Using Special Per Cents

Find without a pencil:

1. 50% of \$360
2. 25% of 480 mi.
3. $12\frac{1}{2}\%$ of 96 ft.
4. $33\frac{1}{3}\%$ of \$480
5. $66\frac{2}{3}\%$ of 72 bu.
6. $16\frac{2}{3}\%$ of \$120
7. $37\frac{1}{2}\%$ of 24 in.
8. $62\frac{1}{2}\%$ of \$40
9. $87\frac{1}{2}\%$ of 32 lb.
10. Frank earned \$16 and saved \$8 of it. What per cent did he save?
11. Helen had \$12 and spent \$3 of it. What per cent did she spend?
12. Ruth worked 16 problems and had 2 of them wrong. What per cent did she have wrong?
13. A ball team played 8 games and won 3 of them. What per cent did it win?
14. A hockey team played 12 games and won 8 of them. What per cent did it win?
15. A man started on a 300-mile trip. When he had driven 100 mi., what per cent of the trip had he driven?

Per Cents Greater Than 100%

1. How many times a number is 200% of it?
2. 300% of a number is how many times the number?

Tell how many times a number:

3. 500% of it is
4. 150% of it is
5. 250% of it is
6. 125% of it is
7. $233\frac{1}{3}$% of it is
8. 375% of it is
9. 1000% of it is
10. 1500% of it is
11. $166\frac{2}{3}$% of it is

Find:

12. 200% of \$80
13. 300% of 90 mi.
14. 500% of \$25
15. 1000% of 80
16. 125% of \$60
17. $133\frac{1}{3}$% of \$90
18. 150% of 80 mi.
19. $166\frac{2}{3}$% of 24 ft.
20. $137\frac{1}{2}$% of \$40
21. $112\frac{1}{2}$% of 16 lb.
22. 275% of 12 in.
23. 325% of 16 mi.

24. An article increased $1\frac{1}{2}$ times its former price. This was an increase of what per cent?

25. The population of a town is 3 times what it was 20 years ago. This is an increase of what per cent?

26. When a merchant doubles his money on a certain novelty, what per cent of the cost is he making?

27. When an article costs $2\frac{1}{3}$ times what it formerly cost, the increase in cost is what per cent?

28. In December eggs cost 4 times what they did in April. This was an increase of what per cent?

29. A man said, "My living expenses are twice what they were ten years ago." What per cent had they increased?

Practice Exercises

1. 3% of \$200 = ——.
2. $5\frac{1}{2}$% of \$400 = ——.
3. 15% of 60 mi. = ——.
4. 25% of \$2.00 = ——.
5. $\frac{1}{2}$% of \$600 = ——.
6. 120% of \$50 = ——.
7. .3% of 1000 = ——.
8. 250% of 40¢ = ——.
9. $1\frac{1}{2}$% of 600 ft. = ——.
10. 150% of \$200 = ——.
11. 15% of \$200 = ——.
12. 1.5% of \$200 = ——.
13. 16 = ——% of 32.
14. 24 = ——% of 80.
15. 42 = ——% of 70.
16. 4.2 = ——% of 70.

17. 4.2 = ——% of 700.
18. 18 = ——% of 90.
19. 9 = ——% of 900.
20. 4 = ——% of 8.
21. 4 = ——% of 80.
22. 4 = ——% of 800.
23. 90 = ——% of 45.
24. 70 = ——% of 35.
25. \$150 = ——% of \$100.
26. \$200 = ——% of \$50.
27. 120 = ——% of 80.
28. $\frac{1}{4}$% of \$60 = ——.
29. 140% of \$25 = ——.
30. \$24 is what % of \$30?
31. \$30 is what % of \$80?
32. \$30 is what % of \$20?
33. \$50 = —% more than \$40.
34. 60 = ——% less than 80.
35. 40 = ——% of 50.
36. 40 = ——% more than 32.
37. 40 = ——% less than 50.
38. 20% more than 60 = ——.
39. 20% less than 50 = ——.
40. 20% of 80 = ——.
41. 15% of \$90 = ——.
42. 15% more than \$90 = ——.
43. 15% less than \$90 = ——.
44. \$3 = ——% of \$200.
45. \$5 = ——% of \$400.
46. \$2 = ——% of \$400.
47. \$1.50 = ——% of \$200.
48. \$2.50 = ——% of \$500.
49. 1¢ = ——% of \$1.
50. 5¢ = ——% of \$10.

A Multiple Choice Percentage Test

Some of the answers given here do not belong in the series. State which are correct and those which do not belong there.

1. $10\% = \frac{1}{10}$; $\frac{10}{1000}$; .1; $\frac{1}{5}$; .01.
2. $25\% = .25$; $\frac{1}{4}$; .025; $\frac{25}{100}$; $2\frac{1}{2}$.
3. $33\frac{1}{3}\% = \frac{1}{3}$; $\frac{10}{300}$; $\frac{100}{300}$; .0333; 3.333.
4. $37\frac{1}{2}\% = 37.5$; .0375; $\frac{3}{8}$; $\frac{5}{16}$; .375; $\frac{75}{200}$.
5. $50\% = .5$; $\frac{1}{2}$; .05; 0.50; $\frac{50}{100}$.
6. $66\frac{2}{3}\% = \frac{2}{3}$; $.06\frac{2}{3}$; $\frac{200}{600}$; .666; $6\frac{2}{3}$.
7. $\frac{1}{2}\% = .5$; .5%; $\frac{50}{100}$; .05; $.00\frac{1}{2}\%$; .005%.
8. $\frac{3}{4} = 62\frac{1}{2}\%$; 75%; 75; .75; $\frac{75}{100}$.
9. $1 = 100\%$; .1; 100; $\frac{10}{100}$; .01.
10. $2\frac{1}{2} = 250\%$; 2.5%; 250; 2.5; $\frac{500.00}{200.00}$.

FINDING A NUMBER WHEN A PER CENT OF IT IS KNOWN

1. A boy finds that he averages 75% as many marketable chickens as he sets eggs. If he wishes to raise 375 marketable chickens, how many eggs should he set?

> *From the problem we get the statement:*
>
> *75% of the eggs set = 375.*
>
> *Written as a decimal, and using the sign of multiplication, this means:*
>
> .75 × *no. of eggs* = 375.

This is the same problem as $4 \times ? = 12$, $5 \times ? = 20$, or $7 \times ? = 56$. That is, the product of two factors and one of the factors are known, and we are to find the other.

You know from the meaning of division that

If the product of two factors and one of the factors are known, the other factor may be found by dividing the product by the known factor.

Hence you know that the solution of $.75 \times ? = 375$ is

$$.75\overline{)375}$$

2. A woman found that fruit lost 40% in canning. To have 24 qt. of canned fruit she must use how much raw fruit?

> *Since 40% is lost, 60% of it remains. Hence 60% of the raw fruit = 24 qt.*
>
> *Written as a decimal, this means*
>
> $.6 \times ? = 24.$
>
> *What is the solution?*

3. A boy sold his bicycle for \$36. He told a friend that he got 20% more than he paid for it. From this find how much he paid for it.

> *Since \$36 was 20% more than he paid, it was 120% of what he paid. Hence we get the statement:*
>
> 120% *of the cost* = \$36.
>
> *This means*
>
> 1.2 × *the cost* = \$36.
>
> *What is the solution?*

4. When furniture was selling at a "special 20% discount sale," a chair cost \$48. Find the regular price.

5. Some pupils were earning money to buy a radio. When they had earned \$96, the teacher said, "This is 80% of the amount we need for the radio selected by your committee." Find the price of the one selected.

6. Frank's father has a fish market. The father asked Frank to find how much undressed fish he should order to make 360 lb. of dressed fish, knowing that there would be a loss of 40% in dressing. Find the answer.

7. On an automobile trip, a family drove 120 mi. before noon. The father said, "We have made 60% of the trip." Find how much farther they had to go.

8. An advertisement said, "We will completely furnish your 6-room house for \$1200. This is a saving of 20% on the regular price by the piece." Find the saving over buying by the piece.

9. A boy weighing 72 lb. was told that he was 10% underweight. How much should he have weighed?

10. A younger boy weighing 75 lb. was told that he was 8% overweight. How much should he have weighed?

KINDS OF PROBLEMS IN PER CENT

From your study so far you have noticed that there are three distinct types of problems to be met in dealing with per cent. These are the only different types there are. They may be illustrated by the following problems:

1. Frank sold \$58.50 worth of brushes and got 40% of it as his commission for selling. How much did he earn?

$$\begin{array}{r} \$58.50 \\ .40 \\ \hline \$23.4000 \end{array}$$

You know that 40% *of* \$58.50 *means* .40 × \$58.50.

To solve Problem 1 you had to know:

I. *How to express a per cent as a decimal;* and

II. *How to multiply by a decimal.*

2. Walter sold \$75.60 worth of aluminum ware and got a commission of \$26.46 for selling it. Find the rate of commission.

$$\begin{array}{r} .35 = 35\% \\ \$75.60\overline{)\$26.4600} \\ 22680 \\ \hline 37800 \\ 37800 \\ \hline \end{array}$$

You are to find the ratio of the commission to the amount sold. Hence the commission is divided by the amount sold and the quotient expressed as per cent.

In Problem 2 you had to know that:

I. *To compare one number with another, you divide the number compared by the number with which it is compared;* and

II. *To express any number as a per cent, move the decimal point two places to the right and use the sign of per cent.*

3. John is selling brushes on a 40% commission. How many dollars' worth will he have to sell to make $30 per week?

.4)$30.0 $7 5	*You know that $30 is the product of his sales multiplied by .4. So $30 ÷ .4 must give his sales.*

In Problem 3 you had to know:

I. *If the product of two factors and one of them are known, the other factor may be found by dividing the product by the known factor;* and

II. *How to divide by a decimal.*

4. A merchant paid $28 for a suit. For how much will he have to sell it to make a margin of 30% of the selling price?

.7)$28.0 $4 0	*Think, "Since 30% of the selling price is to be margin, the rest of it, or 70% of it, must equal the cost." That is, .7 × the selling price = $28. So the solution is $28 ÷ .7.*

5. It is estimated that chickens lose 35% in dressing. A crate of live chickens weighing 68 lb. will make how many pounds of dressed chickens?

6. Using the datum of Problem 5 (35% lost in dressing), how many pounds of live chickens will it take to make 130 lb. of dressed chickens? Prove your answer.

7. The milk of a certain cow tests 3.8% butter fat. How much of her milk will it take to make 100 lb. of butter fat?

8. How much butter fat in 1200 lb. of milk testing 3.75% butter fat?

9. One season the football team of a high school won 6 games and lost 4. What per cent of the games played did it lose?

10. It is estimated that the average yearly earnings, during their entire earning period, is \$950 more for high school graduates than for those not entering high school. How much money invested at 5% a year would it take to earn \$950 yearly?

11. A girl having a yearly allowance of \$160 for clothes saves 35% of it by making her own dresses. Find how much of it she can save in 4 years.

12. It is estimated that the average cost of food for a family of five is \$720 per year. If $8\frac{1}{3}\%$ of this is wasted, find the yearly waste.

13. A dealer estimated that in cutting "filet de sole" from flounders there was a loss of 60%. How many pounds of "filet" will he expect to get from 240 lb. of flounders? How many pounds of flounders should he buy if he wants 240 lb. of "filet"?

14. Frank's father paid \$15,000 for a small house, which he rents at \$125 per month. The yearly rent is what per cent of the cost of the house? If the yearly upkeep, taxes, and insurance amount to \$450 per year, his net income per year is what per cent of the cost of the house?

15. Frank said, "I earned 20% more this year than last year." If he earned \$60 per month this year, how much did he earn last year?

16. Walter's father has an income of \$3000 per year. He estimates that rent will cost 30% of this amount and that food for the year will cost \$100 per month. What per cent will be left for other things?

17. A man noticed that he averaged 25% more mileage the second year he drove his car than the first year. If he is getting 20 miles per gallon the second year, what was his average the first year?

18. A radio that usually sold for \$175 was put on a "special sale" for \$135. This was a discount of what per cent?

19. During the "January clearance sale" velour portieres that sold for $21.50 were reduced to $16.95. What was the per cent of saving by buying at the reduced rate?

20. An automobile agent sold a car for a margin of 15% of the selling price. If this was a margin of $300, what was the selling price of the car?

21. A house costing $25,000 is rented so as to give a return of 12% on the investment. What is the yearly rent?

22. The figures given below represent the average number of days schools were open in the United States, and the average number of days the schools were used by pupils of school age. Find the per cent of educational opportunity used for each of the periods given.

Year	Days School Was Open	Days Used	Per Cent
(a) 1876	133	52	?
(b) 1886	130	57	?
(c) 1896	141	66	?
(d) 1906	151	74	?
(e) 1916	160	91	?
(f) 1926	170	114	?

23. Using the data in Problem 22, what was the per cent of increase in the number of days school was open from 1886 to 1926?

24. In a certain milk corporation, milk is of a standard grade if it contains 3.5% butter fat. For every .1 per cent over or under this grade, 4¢ is added to or subtracted from the price of standard grade milk, which is $2.80 per 100 pounds. If a farmer sold 5500 lb. of milk in the month of March, and his milk tested 3.8% butter fat, what did he receive for it?

25. Using the data in Problem 24, which would be the better and how much, to have 4000 lb. that will test 4% butter fat, or 5000 lb. that will test 3% butter fat?

26. A dairyman had 30 cows from which he sold in the month of June 11,500 lb. of milk that tested 3.7% butter fat. Using the data in Problem 24, what was the average value of the milk produced by each cow during the month?

Percentage Problems with Large Numbers

You have learned to picture large numbers by graphs so as to show their relation to each other, and to express the approximate relation as fractions. It is sometimes necessary to express more exact relations as per cents. The following problems give practice in using large numbers.

1. Of the 30,091,500 men engaged in all occupations in a recent year, 734,000 were coal miners. What per cent were coal miners?

$$\begin{array}{r@{}l}
 & .024 = 2.4\% \\ \cline{2-2}
30{,}091{,}500) & 734{,}000.000 \\
 & 601\ 830\ 00 \\ \cline{2-2}
 & 132\ 170\ 000 \\
 & 120\ 366\ 000 \\ \cline{2-2}
 & 11\ 804\ 000
\end{array}$$

To use all the figures makes a lot of useless work.

$$\begin{array}{r@{}l}
 & .024 = 2.4\% \\ \cline{2-2}
3009) & 73.400 \\
 & 60\ 18 \\ \cline{2-2}
 & 13\ 220 \\
 & 12\ 036 \\ \cline{2-2}
 & 1\ 184
\end{array}$$

To cut off all but four or five of the figures of the divisor and a corresponding number in the dividend, the quotient is not usually affected when but two or three figures are required.

2. One year the entire cotton crop of the world was 20,250,000 bales, of which the United States produced 12,200,000 bales. The United States produced what per cent of the entire crop?

$2025\overline{)1220.000}$	*Show why these figures will suffice to give the right quotient.*

3. Kansas is the leading wheat-producing state. One year she produced 144,500,000 bu. of the entire 746,800,000 bu. raised that year. Kansas produced what per cent of the entire crop?

4. The year that Kansas produced 144,500,000 bu. of wheat, North Dakota ranked second, producing 61,800,000 bu. The Kansas crop was what per cent of the North Dakota crop?

$618\overline{)1445.000}$	*Show why* 61,800,000 *was the divisor.* *Also show why* $1445 \div 618$ *will give the right result.*

5. Iowa and Illinois lead as corn-producing states. One year, of the total production of 3,045,400,000 bu., Iowa produced 444,900,000 bu., and Illinois produced 297,580,000 bu. Find what per cent of the crop each of these two states produced.

6. Iowa and Illinois lead as hog-raising states. In a recent year Iowa raised 10,416,000 and Illinois raised 5,438 000. (*a*) Compare the number raised by Iowa with that of Illinois. (*b*) Compare the number raised by Illinois with that of Iowa.

7. Texas and Iowa lead in the production of cattle. In a recent year Texas produced 5,480,000 and Iowa 4,360,000. Compare each with the other, making two problems as in Problem 6.

8. Louisiana leads in the production of rice. She raised 22,450,000 bu. of the 48,250,000 bu. raised in the United States one year. She raised what per cent of the crop?

9. The total oil production in the world in a recent year was 1,013,139,000 barrels of 42 gal. each. Of this amount, the United States produced 707,265,000 barrels of 42 gal. each. What per cent of the world's supply of oil did the United States produce?

10. In a recent year the value of the gold mined in the United States was $51,912,000, and the value of the silver mined was $43,540,000. What was the ratio in per cent of the value of the gold to the silver? What was the ratio in per cent of the silver to the gold?

11. In 1890 the number of students enrolled in the public high schools in the United States was 202,963. In 1900 the enrollment was 519,291. What was the per cent of increase?

12. From 1900 to 1910 the enrollment in the high schools increased from 519,291 to 915,061. What was the per cent of increase?

13. From 1910 to 1920 the enrollment in the high schools increased from 915,061 to 2,119,389. What was the per cent of increase?

14. Using the data in Problems 11 to 13 inclusive, what was the average yearly increase in the enrollment from 1890 to 1920?

15. During which decade was there the greatest percentage increase in the enrollment?

Practice Exercises

Give without a pencil:

1. 1% of 800
2. 2% of 600
3. $\frac{1}{2}\%$ of 400
4. 5% of 300
5. 1% of 250
6. 4% of 350
7. 10% of 175
8. 20% of 750
9. $\frac{1}{4}\%$ of 800
10. $1\frac{1}{2}\%$ of 200
11. 100% of 45
12. 200% of 60
13. 300% of 25
14. 500% of 20
15. 150% of 80
16. 125% of 60
17. 175% of 40
18. 250% of 20
19. 225% of 60
20. 350% of 12
21. $\frac{1}{2}\%$ of \$8000
22. $\frac{1}{4}\%$ of \$1200
23. $\frac{1}{3}\%$ of \$1500
24. $\frac{1}{8}\%$ of \$2400
25. $\frac{1}{5}\%$ of \$3000
26. $1\frac{1}{2}\%$ of \$2000
27. $2\frac{1}{4}\%$ of \$4000
28. $1\frac{1}{3}\%$ of \$6000
29. $.2\%$ of \$2000
30. $.4\%$ of \$2500

Express as decimals:

31. 6.3%
32. 3.25%
33. 4.5%
34. 16.5%
35. 2.25%
36. 175%
37. $5\frac{1}{2}\%$
38. $4\frac{1}{4}\%$
39. $3\frac{1}{8}\%$
40. $\frac{5}{8}\%$
41. $\frac{3}{4}\%$
42. $\frac{2}{5}\%$

Express as per cent:

43. .625
44. .035
45. .0175
46. .315
47. .003
48. .0015
49. .75
50. .205
51. 1.35
52. 2.5
53. 1.125
54. 1.005

Find:

55. 25% of 450
56. $\frac{1}{4}\%$ of 640
57. 150% of 375
58. 1.5% of 350
59. 125% of 750
60. $\frac{1}{2}\%$ of 175
61. 350% of 225
62. $66\frac{2}{3}\%$ of 835
63. $16\frac{2}{3}\%$ of 358

Give without a pencil:

64. 4 is 50% of what?
65. 8 is 25% of what?
66. 6 is 10% of what?
67. 6 is 3% of what?
68. 16 is $33\frac{1}{3}\%$ of what?
69. 80 is 20% of what?
70. 12 is $12\frac{1}{2}\%$ of what?
71. 16 is 2% of what?
72. 5 is $16\frac{2}{3}\%$ of what?
73. 24 is 200% of what?

Find the answer:

74. 450 is 50% more than what number?
75. 600 is 25% more than what number?
76. 1200 is 200% more than what number?
77. 600 is 25% less than what number?
78. 400 is 20% less than what number?
79. 65 is 5% of what number?
80. 135 is 15% of what number?

Find the per cents:

81. $3 is what per cent of $15?
82. $6 is what per cent of $300?
83. 8 ft. is what per cent of 4000 ft.?
84. 350 mi. is what per cent of 175 mi.?
85. $360 is what per cent less than $480?
86. $360 is what per cent more than $240?

Fill the blanks:

87. 3% of 60 = $\frac{1}{2}$% of ——.
88. 5% of —— = $\frac{1}{4}$% of 80.
89. ——% of 80 = 50% of 120.
90. 15% of 60 = ——% of 240.
91. $\frac{1}{2}$% of 90 = 5% of ——.
92. .2% of 400 = 1% of ——.
93. .5% of 800 = 2% of ——.
94. 100% of 75 = ——% of 37.5.
95. 200% of 12 = 50% of ——.
96. 1% of 1600 = 100% of ——.
97. 25% of 480 = ——% of 60.
98. 50% of 640 = ——% of 80.
99. $\frac{1}{2}$% of 64 = 50% of ——.
100. 25% of 12 = $\frac{1}{4}$% of ——.

True and False Test

Tell which are true and which are false:

1. $.03 = 3\%$.
2. $45\% = .45$.
3. $3.5\% = .35$.
4. $.135 = 135\%$.
5. $\frac{1}{2}\% = .05$.
6. $7\% = .7$.
7. $200 = 200\%$.
8. $\frac{1}{2} = 50\%$.
9. $.5\% = .05$.
10. $225\% = 2\frac{1}{4}$.
11. $1000\% = 10$.
12. $\frac{3}{4}\% = .75$.
13. $10\% = .1$.
14. $1.5 = 150\%$.
15. $\frac{2}{3} = .66\frac{2}{3}\%$.
16. $175\% = 1\frac{3}{4}$.
17. $1.45 = 145\%$.
18. $2\frac{1}{8} = 2.125\%$.
19. $40\% = \frac{2}{5}$.
20. $\frac{5}{8} = .62\frac{1}{2}\%$.

Percentage Test

1. $\frac{1}{2}\%$ of a number is what part of it?
2. $\frac{1}{5}$ of a number is what per cent of it?
3. 1% of $.05$ is what?
4. 100% of 3.5 is what?
5. $\frac{1}{4}\%$ of 200 is what?
6. 200 is what per cent of 100?
7. 2 is what per cent of 400?
8. Papers costing 3¢ sell for 5¢. What per cent of the selling price is made?
9. An increase of 10% increased the speed of a car 3 miles per hour. What was the first speed?
10. When 5 out of 1000 men are college graduates, what per cent are college graduates?
11. One cent is what per cent of one dollar?
12. Find one-half per cent of two dollars.
13. Change $\frac{1}{3}\%$ to a common fraction.
14. Change $\frac{7}{5}$ to per cent.
15. Five cents is five per cent of what?
16. One cent is what per cent of two dollars?

17. 75% of a number is 6. What is the number?
18. 110% of a number is 220. What is the number?
19. $5\frac{1}{2}$% of a number is 44. What is the number?
20. $\frac{1}{5}$ of 1% of anything is what part of it?

HOUSEHOLD EXPENSES: THE FAMILY BUDGET

A **family budget** is a division of the income into proposed expenditures for food, shelter, clothing, operating, advancement, savings, and so on.

1. A family having an income of $3600 proposed to spend 25% of it for food. This would allow how much per week? Per day?

2. A family having a $2400 income proposed to spend but 20% of it for rent. That will allow how much per month?

3. How much will "15% for clothing" allow a family of four having a $2800 income? How much will it allow each?

4. A family having an income of $300 per month planned a monthly budget as follows: Shelter, $40; food, $90; clothing, $45; operating, $30; advancement, $15, incidentals, $20. What per cent of the income is planned for each item, and what per cent of it do they plan to save?

ESTIMATED EXPENSES FOR A FAMILY OF FOUR

Annual Income	Rent	Food	Clothing	Incidentals	Savings
$2000	20%	35%	15%	25%	5%
$2500	20%	30%	15%	20%	15%
$3000	20%	25%	15%	20%	20%

NOTE.—These estimates necessarily vary in different localities. If you can make a better estimate for your locality, do so and use it in problems.

5. Find how much a man having a $3000 income should spend for rent each month if he follows the budget table given above.

6. The budget given above would allow how much for clothing when the income is $2000 per year?

7. Find how much per week the budget allows for food for a family with an income of $2500 per year.

8. A family having an income of $3000 per year paid $60 a month for rent. By what per cent did that exceed the budget table?

9. The National Industrial Conference Board in 1926 made a study of the cost of living in New York City. The investigation showed that an industrial worker's family of five required a minimum of $1894 a year; an office, or "white collar," worker's family of the same size required a minimum of $2153 a year. How much per week must each family of five, belonging to one of the two classes given, have to live in New York City?

10. The yearly budget was found to be as follows:

	Manual Worker's Family	Office Worker's Family
Housing	$ 384.00	$ 480.00
Fuel, light	87.50	107.80
Food	762.32	762.32
Clothing	248.62	290.63
Sundries	411.31	512.20
Total	$1893.75	$2152.95

Each item in the budget is what per cent of the total budget?

11. The Labor Bureau, using the minimum health and decency budget of the United States Department of Labor, would set an annual total of $2330 as a minimum for each of the two classes given. What per cent under this amount is each budget given in Problem 10?

12. For a family to live to a standard as set in Problem 11, what should be the minimum weekly earnings of that family?

13. The average income in 1926 of all people in the United States who were gainfully employed was \$2010. The amount required in Problem 11 is what per cent above the average income of each wage earner?

14. In 1926 there were 44,600,000 gainfully employed out of a population estimated at 115,000,000. What per cent of the population was gainfully employed?

DISCOUNT SALES

To attract trade, to dispose of an overstock, or to sell goods out of season, merchants often have "special sales," in which a reduction, or a **discount,** from the former price is given.

1. At a "20% discount sale" Frank bought a suit that had been selling for \$25 at the regular price. How much did it cost him?

2. At the end of the summer season, Ralph bought a \$4.80 tennis racket at a discount of 10%. How much did it cost him?

3. Mrs. Brown saw an advertisement that said, "Our regular \$48 dinner sets for \$40." What rate of discount was the store giving?

4. Mrs. Young wants to furnish a new house. She finds that it will cost her \$1285 at regular prices. If she waits for the "August sale," she can get a $12\frac{1}{2}$% discount. How much can she save?

5. Mrs. Young saw in the evening paper, "A 30% discount on all floor samples of furniture. But one piece of a kind." For what could she buy a dining room set that had sold for \$275?

6. Frank saw that a dealer was allowing a $12\frac{1}{2}$% discount on all radio sets for five days only. Find the reduced price of sets costing regularly \$84; \$115; \$220; and \$278.

7. A furniture store had a "January sale," in which they gave a 20% discount from former prices. Find the sale price

of the following: a \$175 bedroom set; a \$125 dining-room set; a \$215 living-room set.

8. Find some "special sale" advertised in the daily papers and bring to the class for discussion. Make problems from the data. Can you give a reason for the discount?

9. A merchant advertised a reduction in the price of rugs. Find upon which rug he gave the largest rate of discount: (a) Axminster, regular price \$46, now \$38.95. (b) Wilton Velvets, regular price \$59.75, now \$49.75. (c) Wool velvets, regular price \$37.50, now \$29.75.

10. When a merchant buys an article at a 20% discount and sells it at the marked price, he is making a margin of what part of the cost? Use any list price to solve the problem.

11. When a merchant buys an article at a reduction of $33\frac{1}{3}\%$ and sells it at the list price, his margin is what per cent of the cost? Make a diagram of this and show that your answer is correct.

12. When a merchant buys an article at a 25% discount and sells it at the list price, his margin is what per cent of the cost? Make a diagram and show your answer is correct.

13. When a merchant gets a discount of 40% and sells at the list price, his margin is what per cent of the cost? Use any list price to show your answer is correct.

14. If an article is bought at a 50% discount and sold at the list price, the margin is what per cent of the cost? Make a diagram to show your answer is correct.

SUCCESSIVE DISCOUNTS

It is customary among some wholesalers, when a price is lowered, to give a second discount instead of changing the first discount. Thus a bill of "\$600 less 25% and 10%" means that the \$600 is reduced 25%, leaving \$450, and that the \$450 is reduced 10%, leaving \$405.

1. Find the cost of goods listed \$540, less 10% and 15%.

2. A bill of goods was listed \$1250, less 10% and 25%. Find the cost.

3. Show that the cost would have been the same in Problem 2, if the discounts had been 25% and 10%.

4. Show that discounts of 10% and 10% are not the same as a 20% discount.

5. Show that discounts of 10% and 15% are not the same as a 25% discount.

6. Show that a bill of goods listed for \$900 less 20% and 10%, would be the same as a discount of 28%.

7. A bill of goods listed at \$260 was discounted 10% and 15%. Find the net price.

8. A radio listed for \$240 was given a 10% discount for each of three consecutive days. What was the final reduced price?

9. A billing clerk should have billed goods listed at \$1250 at 20% and 10%; but, not knowing the difference, he billed them at a 30% discount. How much did his firm lose by his mistake?

10. To what single discount are 20% and 10% equal?

$$\begin{array}{r} 100\% \\ \underline{20\%} \\ 80\% \\ \underline{8\%} \\ 72\% \end{array} \qquad \begin{array}{r} 100\% \\ \underline{72\%} \\ 28\% \end{array}$$

Or,

$$20\% + 8\% = 28\%$$

20% off leaves 80% of the list price; 10% of 80% is 8%, leaving 72%, the net cost. This is 28% less than the list price. Hence, the discount is 28%. Or, think, 20%, the first discount, and 8% the second, equals 28%.

A fact used by some is,

The single discount equal to two successive discounts is their sum less their product.

Thus in Problem 10, the result is,

$$20\% + 10\% - 2\% = 28\%$$

11. A merchant bought a bill of goods listed at \$1500 less 30% and 10% and sold them at the list price. What per cent of the cost was his margin?

12. If a merchant gets a discount of 25% from the list price of an article, what discount can he give from the same list price to make a margin of 20% of the cost to him?

$$\begin{array}{r} 100\% \\ \underline{25\%} \\ 75\% \\ \underline{.20} \\ 15.00\% \\ \underline{75\%} \\ 90\% \end{array} \qquad \begin{array}{r} 100\% \\ \underline{90\%} \\ 10\% \end{array}$$

Since no particular list price is given, think, "25% from the list price leaves 75% of it." Then think, "20% of 75% is 15%. So he can sell the goods at 90% of the list price, which is a discount of 10%."

13. When a merchant gets a discount of 40%, what discount can he give from the same list price and make a margin of 25% of the cost to him?

14. If a merchant gets a discount of 20% and sells at the list price, what per cent of the cost is his margin?

15. A merchant got a discount of 40% from the list price of an article and sold it at a discount of 20% from the same list price. What per cent of the cost was his margin?

$$\begin{array}{rl} 100\% & \\ \underline{40\%} & \\ 60\%, & \text{cost.} \end{array}$$

$$\begin{array}{rl} 100\% & \\ \underline{20\%} & \\ 80\%, & \text{selling price.} \end{array}$$

$$20 \div 60 = \tfrac{1}{3} = 33\tfrac{1}{3}\%$$

Think, "The merchant paid 60% of the list price and sold it for 80% of the list price. So he made 20% of the list price." Then think, "I am to compare 20% with 60%, or 20 with 60."

16. In Problem 15 what was the rate of gain on the selling price?

17. Goods bought at 10% below the list price and sold at 20% above the list price give a margin of what per cent of the cost?

18. If a dealer gets a discount of 40% and 10%, what discount can he give to make a margin of 30% of the cost?

19. If a merchant gets an order of goods discounted 15% and 10% and sells them at the list price, his margin is what per cent of the cost?

20. After a bill had been discounted 20% and 10%, it amounted to $360. What was the original amount of the bill?

COMMISSION

Commission is entire or part payment for service rendered by an individual or a company for buying or selling a certain article. It is usually a certain per cent of the value of the articles bought or sold. This payment is sometimes called **brokerage** instead of commission, especially in case of selling stocks or bonds.

Commission is determined from the money value of the transaction, except in the case of stocks and bonds. That is, the commission for buying is determined from the cost, and the commission for selling is determined from the selling price.

When a quantity of goods is delivered to a commission merchant it is called a **consignment.**

The person sending the consignment is called the **consignor.**

The person to whom the consignment is sent is called the **consignee.**

After the expenses and commission have been deducted, the amount left is called the **proceeds.**

1. John took subscriptions for magazines for a commission of 30%. If his subscriptions amounted to \$24, what was his commission?

2. Henry sold aluminum ware during the summer to pay his expenses through college. If his commission was 40% and he sold \$1275 worth of aluminum ware, what was his share of the sales?

3. An agent received a commission of 35% for selling vacuum cleaners. What was his commission on each sale, if the cleaner sold for \$55?

4. A man worked in a shoe department of a large store. He received a salary of \$24 a week and a commission of $2\frac{1}{2}\%$ of his sales. If his sales amounted to \$450 a week, what were his weekly earnings?

5. Frank sold automobile accessories on a 40% commission. In order to make \$60 a week, how much should his sales total?

6. A man worked on a salary of \$125 a month plus a commission. He sold automobile tires on a 5% commission. How much must his sales be in order for him to double his regular salary?

7. A fruit dealer ordered a carload of oranges from Florida. He had to pay 75¢ a box and the commission merchant's fee of 5% of the cost of the oranges. The freight amounted to \$325. What was the total cost per box, if there were 950 boxes on the car?

8. A farmer sent a coop of chickens to a commission merchant to sell. There were 24 chickens and they averaged $3\frac{3}{4}$ lb. each. The merchant sold them at 38¢ a pound, but he charged a commission of 5%. What were the proceeds of the sale?

9. A grain dealer sold wheat at a commission of $\frac{1}{2}\%$. One month he earned \$500. How many bushels did he sell, if wheat was selling at \$1.50 a bushel?

10. An auctioneer sold household goods on a 10% commission and live stock at a 2% commission. At one sale the household articles amounted to $860, and the live stock amounted to $3850. What was the amount of the auctioneer's commission?

11. A collector's commission was 6% of the bills. If he earned $60 a week, what was the amount of the bills he collected?

12. A real estate agent charged a commission of $2\frac{1}{2}$% of the value of the property sold. If he sold a house for $12,500, what was his fee?

13. A merchant paid $49.20 to have a bill collected. If the rate of commission was 3%, what was the amount of the bill?

14. An agent charged $150 to sell a piece of property for $7500. What was his rate of commission?

15. A man was earning $55 a week. His salary was $35 and his commission was 2% of his sales. What did his sales total for the week?

16. An architect for a school building charges 5% of the cost of the building. What was the cost of a school building, if the architect's fee amounted to $32,500?

17. A Teacher's Agency charges a commission of 5% of the first year's salary of a teacher in order to get her a position. If her contract called for a salary of $1400, what was the agency fee?

18. Ray was paid 60¢ for each magazine subscription that he sold for $2. What was his rate of commission?

19. Frank got a circular saying, "Our agents are earning from $60 to $90 a week." The commission paid was 25%. The sales must have ranged to what amounts?

20. A commission merchant's rate was 5%. After he took out his commission, the proceeds amounted to $712.50. What was the amount of the sales?

PROFIT AND LOSS

In order to do business a merchant must sell goods for more than they cost him. The amount above cost that he gets is called **margin.** After paying for rent, labor, heating, lighting, advertising, taxes, insurance, etc., called the **overheads,** from the margin, the part left is called the **profit** or the **gain.** If the overheads exceed the margin, there is a **loss.**

1. A merchant paid $37.50 for a table. At what price must he sell it to make a margin of $33\frac{1}{3}\%$ of the cost?

2. A pair of shoes that cost $5 were sold at a 25% margin on the cost. What was the selling price?

3. A chair that costs $12.50 is sold for $18.50. The margin is what per cent of the cost? Of the selling price?

4. A wholesale dealer bought oranges at $1.50 per box and sold them at a margin of $33\frac{1}{3}\%$. His margin was what per cent of the selling price?

5. Select any cost price you wish and find what per cent of the selling price is a margin of 50% of the cost?

6. In the same way find what per cent of the selling price is a margin of $33\frac{1}{3}\%$ of the cost.

7. In the same way find what per cent of the selling price is a margin of 25% of the cost.

8. A merchant bought an article for $6.75 and sold it at a $33\frac{1}{3}\%$ margin on the cost. The overheads were 10% of the selling price. What was his profit?

9. An automobile agent sold a car costing $1275 for $1595. The overheads amounted to 12% of the selling price. The profit was what per cent of the cost?

10. Take any cost that you wish and show that when selling at a margin of 25% of the cost with overheads 16% of the sales, a merchant is making a profit of only 5% of the cost.

11. In the same way, show that if a merchant's margin is 20% of the cost and his overheads 10% of the sales, his profit is but 8% of the cost.

Sometimes the margin is figured from the selling price, and sometimes from the cost. The problem must state which is to be used in finding the rate of the margin.

12. At what must an article costing \$5 be sold to give a margin of 20% of the selling price?

.8)\$5.00 \$6.25	*Since 20% of the selling price is margin, 80% of it must be cost, or \$5. Then it is known that 80% of the S.P. = \$5.* *Or, .8 × S.P. = \$5. Hence, \$5 ÷ .8 will give the selling price.*

13. A table costing \$24.50 was sold at a margin of 25% of the selling price. What was the selling price?

14. What must be the selling price of a pair of shoes costing \$7.50, if they are sold at a margin of $33\frac{1}{3}$% of the selling price?

15. A merchant sold a suit for \$60 at a margin of 50% of the cost. What did the suit cost him?

1.5)\$60.00 \$40.00	*Think, "If 50% of the cost is margin, then the selling price will be 150% of the cost." Now you see that 150% of the cost =* \$60. *Or,* 1.5 × *cost* = \$60. *Hence,* \$60 ÷ 1.5 *will give the cost.*

16. A car is sold for \$1560 by a dealer at a margin of 25% of the cost. What did the car cost him?

17. A fruit dealer bought a carload of oranges for $1286.25 from a commission merchant, who sold them at a margin of 5% of the cost. What did the oranges cost the commission merchant?

18. A man sold a building lot for $5400 at a margin of 20% of the cost. What did it cost him?

19. A man bought a city lot for $2100 which he wished to sell at a margin of 30% of the selling price. At what price must he sell it?

20. An agent sold a car on a 5% commission of the factory price of the car. If his commission amounted to $24.75, what was the factory price of the car?

21. A fruit dealer ordered a carload of 1200 boxes of oranges at $1.50 a box. The freight was $300, there was a loss of 20% of them due to freezing, and the overheads amounted to 25% of the sales. The loss was what per cent of the cost if sold at $2.00 per box?

22. A dealer sold two used cars for $300 each. On one he made a profit of 20% of the cost and on the other he lost 20% of the cost. Did he gain or lose, and by what amount?

23. A dealer bought 1000 bushels of potatoes at 90¢ per bushel. He sold $\frac{3}{4}$ of them at a margin of 30% of the cost and the remainder at cost. The margin on the whole lot was what per cent of the cost?

24. A dealer bought 1000 bushels of apples at 90¢ per bushel. He sold $\frac{3}{4}$ of them at $1.30 per bushel and the remainder at $1 per bushel. The cost of selling was 18¢ per bushel. His profit was what per cent of his sales price?

25. A fruit dealer bought a load of peaches at 90¢ per basket. He sold $\frac{3}{4}$ of them at $1.30 per basket and the remainder at $1 per basket. The cost of selling was 18¢ per basket. The profit was what per cent of the selling price of the whole shipment?

26. Show without using any given cost that 25% of the cost and 20% of the selling price give the same margin.

125% of cost = S.P. $.20$ 25.00% of cost = margin.	*Think, "When sold at a margin of 25% of the cost, the selling price is 125% of the cost." Then think, "20% of this is 25% of the cost."*
Or, 20% of S.P. = margin 80% of S.P. = cost $20 \div 80 = 25\%$	*Or think, "If 20% of the selling price is margin, the 80% remaining must be the cost." Then think, "Comparing 20% of the selling price with 80% of it, the result is 25%."*

27. A margin of 25% of the selling price is equal to what per cent of the cost?

28. A margin of 40% of the cost is equal to what per cent of the selling price?

29. An article bought at 10% below the list price and sold at 20% above it, will give a margin of what per cent of the cost?

30. A merchant bought a suit for \$24. He wished to make a margin of $33\frac{1}{3}\%$ of the cost. At what price must he mark it in order to give a discount of 25% of the marked price?

31. A man bought a lot for \$1500 on which he built a house costing \$5600. Grading the lawn and laying sidewalks cost \$300. If he sold the place two years later for \$10,000, his margin was what per cent of the cost?

32. A suit costing \$20 was sold at a margin of 35% of the cost. Find the selling price.

33. A suit costing \$19.50 was sold at a margin of 35% of the selling price. What was the selling price?

34. A margin of 30% of the cost is what per cent of the selling price?

35. A margin of 15% of the selling price is what per cent of the cost?

36. A margin of 30% of the cost and overheads at 10% of the selling price, give a profit of what per cent of the cost?

37. A margin of 20% of the selling price and overheads at $12\frac{1}{2}$% of the selling price, give a profit of what per cent of the cost?

MARKING BY A CODE

Did you ever buy a suit or a dress that had on it a tag mark that looked like this one?

$$\frac{ahln}{\$37.50}$$

If you did, very likely you did not know the meaning of *ahln*. This was a code mark used by the merchant in order to put the cost price on the tag without having the customer know the cost to the merchant.

One merchant uses this key:

C	**H**	**A**	**R**	**L**	**E**	**S**	**T**	**O**	**N**
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**0**

From this it is seen that

C H L N represents \$12.50.
L E E R represents \$56.64.
H R O L represents \$24.95.

Using the key given above, give the cost prices of articles marked as follows:

1. *C A L N.*
2. *R H S O.*
3. *A S L N.*
4. *H T O A.*
5. *S E R L.*
6. *O N S L.*

Find the per cent of margin on the cost of articles sold as marked below, if there are two decimal places in each code price:

7. $\dfrac{C\ A\ L}{\$2.50}$
8. $\dfrac{T\ A\ N}{\$14.25}$
9. $\dfrac{C\ T\ C\ L}{\$27.45}$
10. $\dfrac{A\ R\ O\ N}{\$37.50}$
11. $\dfrac{H\ A\ R\ L}{\$32.75}$
12. $\dfrac{E\ L\ A\ N}{O\ N\ H\ L}$

Find what the marked price should be, determining the margin on the cost.

13. *H R O* (margin 150%)
14. *A A L* (margin 60%)
15. *E S L* (margin 40%)
16. *A C E O* (margin 35%)
17. *H H L N* (margin $33\frac{1}{3}$%)
18. *R L O N* (margin 50%)

BORROWING AND LOANING MONEY

You know that if you use property belonging to another, you pay for the use of it. Thus, if one lives in a house owned by another person, he pays rent for it. If you use money borrowed from another person, you must pay him for the use of it. What you pay for the use of it is called **interest.** The amount paid depends upon the amount borrowed and the time you have it. The amount paid is a certain per cent of the amount borrowed. This is called the **rate of interest.**

The amount borrowed is called the **principal.** A very common rate of interest in most sections of the country is 6% of the principal for a year's use of it, or "6% per year."

1. What is the interest of \$500 for 1 yr. at 6%?
2. What is the interest of \$800 for $\frac{1}{2}$ yr. at 6%?
3. What is the interest of \$600 for 1 yr. at $5\frac{1}{2}$%?

4. A man borrowed \$800 and paid it back in 96 days with interest at 6%. Find how much he paid back.

$$\frac{\overset{16}{96}}{\underset{\underset{10}{60}}{360}} \times \frac{6}{100} \times \$800 = \$12.80$$

$$\$800 + \$12.80 = \$812.80$$

When interest is paid for a certain number of days less than a year, the time is a number of 360ths of a year.

The solution should be written as shown here.

5. A man borrowed \$1200 and paid it back in 84 days with interest at 5%. Find the amount paid back.

The interest plus the principal is called the *amount.*

Find the interest and amount of:

6. \$500 at 6% for $\frac{1}{2}$ yr.
7. \$1800 at $5\frac{1}{2}$% for 1 yr.
8. \$900 at 5% for $\frac{1}{2}$ yr.
9. \$1600 at 6% for $\frac{3}{4}$ yr.
10. \$1200 at $5\frac{1}{2}$% for $\frac{1}{4}$ yr.
11. \$800 at 6% for 115 da.
12. \$1500 at 5% for 120 da.
13. \$2000 at $5\frac{1}{2}$% for 96 da.
14. \$2400 at 6% for 80 da.
15. \$3600 at 5% for 102 da.

Without a pencil give the interest of:

16. \$2000 at 6% for 1 yr.
17. \$1000 at $5\frac{1}{2}$% for 1 yr.
18. \$800 at 6% for $\frac{1}{2}$ yr.
19. \$4000 at 5% for $\frac{1}{2}$ yr.
20. \$3000 at 6% for 1 yr.
21. \$2500 at 4% for 1 yr.
22. \$1000 at 6% for $\frac{1}{2}$ yr.
23. \$2000 at 6% for $\frac{1}{4}$ yr.
24. \$800 at 5% for $\frac{1}{4}$ yr.
25. \$1200 at 5% for $\frac{1}{2}$ yr.
26. \$1000 at 6% for 60 da.
27. \$2000 at 5% for 90 da.
28. \$5000 at 6% for 90 da.
29. \$2000 at 6% for 30 da.
30. \$4000 at 6% for 90 da.
31. \$1200 at 5% for 180 da
32. \$500 at 6% for 120 da.
33. \$600 at 6% for 120 da.
34. \$2000 at 5% for 30 da.
35. \$2500 at 4% for 90 da.

PROMISSORY NOTES

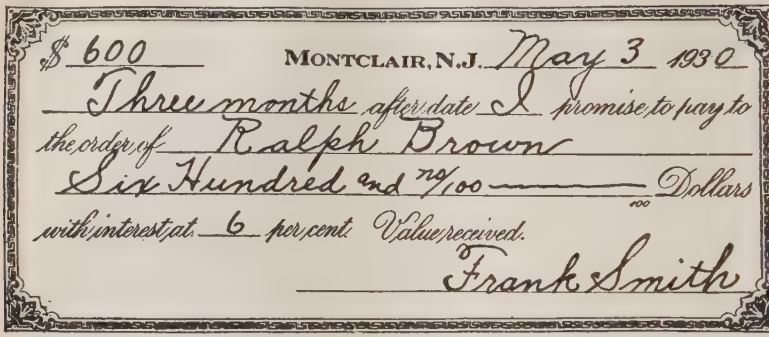
When one loans money, he must have some evidence of the loan; that is, something to show that the loan was made, how much was loaned, when it is to be paid, and the amount of interest to be paid.

This is shown in a written statement called a **promissory note.** The simplest form of a promissory note is shown on the next page.

The actual number of days intervening is used as the basis for computing the interest charge, if the period is less than 1 year. A legal year consists of 360 days.

1. Frank Smith needed \$600 to open business. He borrowed it from Ralph Brown at 6% for 3 months. How much did Mr. Smith pay for the use of the money?

2. This is the form of promissory note that Mr. Smith gave Mr. Brown as evidence of the loan:


$600 MONTCLAIR, N.J. May 3 1930
Three months after date I promise to pay to
the order of Ralph Brown
Six Hundred and no/100 Dollars
with interest at 6 per cent. Value received.
Frank Smith

a. When was the note due? That is, when must Mr. Smith pay it?

b. Who held the note until paid?

c. What is the principal or face of the note?

d. What does "with interest at 6%" mean?

e. How much will Mr. Smith pay Mr. Brown when the note is due?

f. Who gets the note when it is paid? What do you think that he will do with it?

3. Mr. Harris borrowed $800 of Mr. Holmes at 6% on May 3 and paid it back with interest on August 15. How much did he pay Mr. Holmes?

$$\frac{104}{\underset{60}{\cancel{360}}} \times \frac{\cancel{6}}{\cancel{100}} \times \overset{\$8}{\cancel{\$800}} = \frac{\$832}{60}$$

$$= \$13.87$$

$$\begin{array}{r} \$800 \\ 13.87 \\ \hline \$813.87 \end{array}$$

It is 104 da. from May 3 to Aug. 15. For time less than 1 yr., 360 da. is an "interest year." So he owes interest for $\frac{104}{360}$ yr. The interest is $13.87. *The amount due is* $813.87.

Find the interest and amount of:

4. \$750 at 6% for 6 mo.
5. \$850 at 6% for 115 da.
6. \$960 at 5% for 8 mo.
7. \$1200 at $5\frac{1}{2}\%$ for 4 mo.
8. \$1500 at 6% for 80 da.
9. \$600 at 5% for 124 da.
10. \$2000 at 5% for 2 mo.
11. \$1500 at 7% for 15 da.
12. \$900 at $4\frac{1}{2}\%$ for 1 yr.
13. \$650 at 6% for 6 mo.
14. \$1700 at 5% for 36 da.
15. \$4000 at 5% for 140 da.
16. \$2500 at 6% for 2 mo.
17. \$200 at 6% for 96 da.
18. \$150 at 5% for 15 da.
19. \$90 at 6% for 60 da.

SECURITIES

When loaning money, one wants to be sure that he will get it back. So besides the promissory note signed by the borrower, some **security** is usually required. Sometimes two or more sign the note. Any one of those who sign the note becomes responsible for the payment. This is called **personal security.**

Sometimes the borrower makes over a certain kind of property to the one of whom the loan is obtained. This is called a **mortgage.** The property does not thus belong to the one holding the mortgage, but the mortgage gives the holder a right to sell the property and pay himself if the note is not paid when due.

SAVINGS BANKS AND INTEREST

The usual rates of interest on loans are from $5\frac{1}{2}\%$ to 6%, or even 7% in some places. But one having but a few dollars from weekly or monthly savings cannot find people to whom he can safely loan these small sums at any of these rates. So **savings banks** are organized, in which people may make small deposits, which the banks loan in larger sums at these rates. The depositors are paid a smaller rate, usually $3\frac{1}{2}\%$ or 4%, on their savings, and the difference between what the

bank gets on loans and what it pays its depositors is its margin for conducting the business.

1. If you have $100 on deposit in a savings bank that pays 4% and loans at 6%, how much per year does the bank make from your deposit?

2. Savings banks usually pay interest every six months. Instead of sending you the interest when due it is added to your account and begins drawing interest. If you have $200 in a savings bank paying 4%, how much will be added to your account in 6 months?

3. If you have $800 in a savings bank paying $3\frac{1}{2}$%, how much more per year could you get by taking it out and loaning it on a note and security at 6%.

Find the interest for 6 *months of:*

4. $90 at 4%
5. $120 at 4%
6. $80 at $3\frac{1}{2}$%
7. $200 at $3\frac{1}{2}$%
8. $500 at 4%
9. $600 at $3\frac{1}{2}$%
10. $800 at 4%
11. $1200 at $3\frac{1}{2}$%

12. How much more per year will $500 earn when loaned at 6% than when deposited in a savings bank at $3\frac{1}{2}$%?

13. How much more per year will $1200 earn at 5% than in a savings bank that pays $3\frac{1}{2}$%?

INTEREST PAID COMMERCIAL BANKS

Commercial banks, sometimes called **banks of deposit,** are institutions in which money is deposited for safekeeping and where it is paid out on an order, called a **check,** signed by the depositor. For the work of caring for the money and paying it out on order, the bank gets the interest on the money of its depositors that it loans. It keeps but from 10% to 20% of the deposits in the bank and the rest is loaned out on

security. Such banks loan for short periods, ranging from a few days up to 3 or 6 months. The interest is taken out at the time the loan is made and is called **bank discount.** The difference between the face of the note and the interest is called the **proceeds.**

1. Mr. Brown borrowed $1200 from a bank on Apr. 4, 1930, for 2 mo. at 6%. Find the bank discount and the proceeds.

$$\tfrac{61}{360} \times \tfrac{6}{100} \times \$1200 = \$12.20$$

$$\$1200 - \$12.20 = \$1187.80.$$

The note has 61 *days to run. The discount is* $12.20.

The proceeds are $1187.20. *Only* $1200 *is paid back when the note is due, for the interest was paid in advance.*

2. Find the discount and proceeds at 6% on a note of $1800 to run 3 mo. from June 10.

3. Find the discount and proceeds at 6% on a note of $2400 to run 4 mo. from July 15.

Find the discount and proceeds:

4. $600 at 6% for 2 mo. from Apr. 10.
5. $900 at 6% for 3 mo. from May 18.
6. $1500 at 6% for 4 mo. from June 17.
7. $1000 at $5\frac{1}{2}$% for 2 mo. from May 20.
8. $2400 at $5\frac{1}{2}$% for 3 mo. from Aug. 10.
9. $300 at 6% for 118 da.
10. $900 at 6% for 10 da.

MAKING AN INTEREST TABLE

When a man has to compute interest often, he usually makes or buys an **interest table** from which he can find the interest.

1. Rule a form like the one below and finish the table to $9000.

SIMPLE INTEREST TABLE—RATE 6%

DAYS	$1000	$2000	$3000	$4000	$5000	$6000	$7000	$8000	$9000
10	$ 1.6667	$ 3.3333	$ 5.0000	$ 6.6667	$ 8.3333	$10.0000	$11.6667	$13.3333	
20	3.3333	6.6667	10.0000	13.3333	16.6667	20.0000	23.3333		
30	5.0000	10.0000	15.0000	20.0000	25.0000	30.0000			
40	6.6667	13.3333	20.0000	26.6667	33.3333				
50	8.3333	16.6667	25.0000	33.3333					
60	10.0000	20.0000	30.0000						
70	11.6667	23.3333							
80	13.3333	26.6667							
90	15.0000	30.0000							

2. In making the table, show that when you found or knew the interest on $1000 at 6% for 10 da., you could get all others by a single multiplication or addition.

3. From the part of the table shown here, how can you tell without a pencil the interest of $200 at 6% for 50 da.?

4. What is the interest of $3000 at 6% for 100 da.?

5. What is the interest of $2000 for 9 da. at 6%?

6. What is the interest of $700 for 10 da. at 6%?

7. What is the interest of $20,000 for 100 da. at 6%?

8. What is the interest of $50,000 for 3 da. at 6%?

9. What is the interest of $30 for 200 da. at 6%?

10. What is the interest of $10,000 for 90 da. at 6%?

11. Make a similar table for a 5% rate.

12. Make a similar table for a 4% rate.

USING AN INTEREST TABLE

1. Using the 6% table of the last section find the interest of \$3450 for 40 da.

\$20.0000 2.6667 .3333 \$23.0000	*Think, "Int. for \$3000 is \$20; Int. for \$400 is $\frac{1}{10}$ of \$26.6667; Int. for \$50 is $\frac{1}{100}$ of \$33.3333." Then add.*

2. Check the answer to Problem 1 by finding it by the regular method.

Using the table, find the interest at 6% of:

3. \$6530 for 30 da.
4. \$4250 for 40 da.
5. \$12,000 for 70 da.
6. \$23,000 for 50 da.
7. \$65,700 for 20 da.
8. \$32,400 for 10 da.
9. \$6850 for 100 da.
10. \$7500 for 200 da.
11. Find the interest of \$5000 at 6% for 43 da.

\$33.33 2.50 \$35.83	*Why was \$2.50 taken for the interest for 3 days?*

Find the interest at 6% of:

12. \$12,000 for 76 da.
13. \$24,000 for 54 da.
14. \$2100 for 83 da.
15. \$1600 for 12 da.

Percentage Test

If you are not able to get all of these correct, study very carefully the ones which you miss.

Express as per cent:

1. .16
2. .03
3. 1.23
4. 6
5. .008
6. .4
7. 3.275
8. .0075
9. 3.2
10. $\frac{2}{5}$
11. $\frac{7}{5}$
12. $2\frac{1}{8}$

Express the per cents as decimals:

13. 12%
14. 5%
15. 120%
16. 400%
17. .3%
18. .35%
19. $\frac{1}{2}$%
20. $5\frac{1}{4}$%
21. $16\frac{1}{2}$%
22. .002%
23. $\frac{1}{5}$ of 1%
24. Find 6% of $220.
25. Find 150% of 8.
26. 12 is what % of 16?
27. Find $\frac{1}{2}$% of $4.
28. 1¢ is what % of $2?
29. $2 is what % of 1¢?
30. 20% more than 60 is ———.
31. 50 is ———% more than 40.
32. 10 is ———% less than 15.
33. $\frac{1}{4}$% of $40 is ———.
34. 20 is ———% of 2000.
35. $\frac{1}{4}$ is ———% of $\frac{1}{2}$.
36. $\frac{1}{2}$ is ———% of $\frac{1}{8}$.
37. $\frac{1}{3}$ is ———% of $\frac{1}{3}$.
38. $\frac{1}{8}$ is 50% of ———.

Oral Tests in Per Cent

These problems should be solved without using a pencil.

Test A

1. 6 is what per cent of 12?
2. Find 15% of $70.
3. If 25% of a number is 4, what is the number?
4. Express $\frac{1}{2}$% as a decimal.
5. If a bill of goods marked $30 is discounted 15%, what is the cost price?

6. A suit that was marked \$40 was sold for \$35. What per cent of discount was given?

7. Two successive discounts of 10% and 20% equal one discount of how much?

8. A merchant sold an overcoat for \$35 at a margin of 25% of the selling price. What was his margin?

9. An article that sells for $2\frac{1}{2}$ times the cost gives a margin of what per cent of the cost?

10. If the margin equals the cost, what part of the selling price does it equal?

Test B

1. A man's salary increased 20%. If he was earning \$25 a week, what was his increased salary?

2. John is earning \$10 a week delivering papers. That is 25% more than he received last year. How much did he receive last year?

3. What is the interest on \$150 for 6 mo. at 6%?

4. A man received a commission of 3% for selling a piece of property for \$3200. What was his commission?

5. A man sold brushes on a 40% commission. What would his sales total if his commission amounted to \$60 a week?

6. 75 is what per cent of 125?

7. Harry sold his bicycle for \$20 at a loss of 20% of the cost. What did it cost him?

8. Mr. Jones' interest at the bank for 6 mo. at 4% amounted to \$150. How much money did he have in the bank?

9. A square 4 inches on a side is what per cent as large as one 6 inches on a side?

10. A square 6 inches on a side is what per cent as large as one 4 inches on a side?

Test C

1. Express .0075 as a per cent.

2. What is the per cent of saving by buying old potatoes at $1 a bushel instead of new ones at $3 a bushel?

3. A radio tube that regularly sold for $1.50 was sold for $1. What was the rate of the discount?

4. A merchant sold a sweater costing $6 at a margin of 75% of the cost. What was the selling price?

5. A bill of goods listed at $200 was discounted 10% and 10%. What was the actual cost?

6. A man sold a horse for $200 at a loss of 20% of the cost. What was the cost of the horse?

7. The proceeds from a sale amounting to $200 were $194. What was the rate of commission?

8. What is the interest on $250 for 1 year at 4%?

9. A field 9 rd. by 10 rd. is what per cent as large as one 10 rd. square?

10. A house that cost $7200 was sold for $6600. That was a loss of what per cent of the cost?

Test D

1. What per cent of $5 is 5¢?

2. A certain kind of cloth shrinks 3% of its length. How long was a piece of cloth if it had shrunk 3 inches?

3. The area of a circle 4 in. in diameter is what per cent of the area of a circle 2 in. in diameter?

4. A man sold a lot for $600 at a gain of 20% of what it cost him. What did he pay for it?

5. Jane missed 3 examples on a test and her paper was scored 85%. How many examples were on the test?

6. An automobile tire was listed at $15. If a 15% discount was given, what did it cost?

7. A margin of 25% of the selling price is what per cent of the cost?

8. A man has a salary of $5000 a year. If his rent is 15% of his salary, what does he pay each month for rent?

9. A man wished to save 12% of his salary. If he saved $360 one year, what was his yearly income?

10. A company spent 5% of its income on advertising. If the advertising for one year cost $50,000, what was the yearly income of the company?

REVIEW OF PERCENTAGE

1. A man has a yearly income of $3500. He spent 65% of it and invested the remainder in real estate. At that rate of payment, how long will it take him to pay for a lot that cost $2500?

2. Mrs. Cole bought a piano on tne installment plan for $495. If she could have paid cash it would have cost her but $445. The cash price was what per cent of the price she paid for it?

3. A merchant sold a table for $75 that cost him $45. If the overheads amounted to 10% of the selling price, his profit was what per cent of the cost?

4. A retailer received a consignment of goods billed for $285 less discounts of 10% and 25%. If he sold the goods at the list price, his margin was what per cent of the cost?

5. A salesman received a monthly salary of $150, and also a commission of 2% of his sales. If his yearly earnings amounted to $3000, what was the value of his yearly sales?

6. Mr. Black sold a corner lot in a city for $55,000. What was his commission if he received $2\frac{1}{2}$% of the selling price?

7. A clothier had a suit marked for $45, but he sold it for $37.50. What per cent discount did he give?

8. What is the difference in the amount of the discount caused by giving a 25% discount or two successive discounts of 10% and 15% on a bill of goods listed for $350?

9. A pair of shoes cost a merchant $4.50. He wished to

sell them at a margin of 40% of the selling price. What was the selling price?

10. A dealer bought a piano at the factory for \$150. He wished to double his money but he wanted to mark it so he could allow a 25% discount. What was the marked price?

11. Betty had an allowance of \$18 each month. She put aside $33\frac{1}{3}$% of each of her 4 allowances prior to December in order to do her Christmas shopping. In December she spent \$6.40 for a tablecloth and deposited 20% of the remainder in the bank. How much did she then have left to spend for Christmas?

12. A merchant bought an article for \$18. He marked it 50% above cost, but he sold it at a discount of $12\frac{1}{2}$%. If he gave his salesman a commission of 2% of the selling price of the article, what was the merchant's margin?

13. Mary wished to buy $\frac{7}{8}$ of a yard of crepe de chine listed at \$3.75 a yard, on which she received a 30% discount. What did the material cost her?

14. A man bought a house for \$24,500 and sold it for \$32,000, but he gave his agent $2\frac{1}{2}$% commission for selling it. What per cent of the cost did he gain on his investment?

15. John, Tom, and Henry are saving money to buy a car that costs \$600. John has \$75, Tom has \$120, and Henry has \$90. If each boy is to contribute an equal amount, what per cent of the amount needed has each one?

16. A Persian rug was marked at \$820, but a 20% discount was given. If the margin then was 25% of the selling price, what was the cost?

17. If the total land area of the world is 56,000,000 square miles and the area of North America is 8,500,000 square miles, what per cent of the total land area does North America occupy?

18. What per cent may be saved by buying cigars at 3 for 25¢ instead of at 10¢ each?

19. Mrs. Brown noticed that there was a loss of about $33\frac{1}{3}\%$ in canning berries. If she wished to have 15 quarts of canned berries, how many quarts of the raw berries would she need to buy?

20. A merchant bought strawberries in 32-quart crates at \$4.80 per crate. To make 25% of the cost and allow a loss of 2 quarts in each crate, he must sell them at what price per quart?

21. The iceman pays \$8 a ton for ice and sells it at \$1.25 per 100 lb. If the freight charge is \$4 a ton and 20% is lost by melting, his margin is what per cent of the cost?

22. A dealer bought 1600 barrels of apples at \$2.25 per barrel. He sold $\frac{3}{4}$ of them at \$3.25 per barrel and the remainder at cost. His margin was what per cent of the cost?

23. A merchant bought an article for \$24.50 and marked it so that he would make 20% of the cost, after giving a discount of \$5.60. The discount was what per cent of the marked price?

24. What is the bank discount on a note of \$750 at 6%, if it was made on Mar. 20th, and expired in 4 months?

25. Find the proceeds on a note of \$1250 at 6% given on July 10th and to expire in 3 months.

26. Green coffee in roasting loses 20%. If the green coffee costs 24¢ per pound, what is the value of a pound of roasted coffee?

27. Ham loses 40% of its weight in boiling and slicing. What must be the cost of the raw ham if the sliced ham sells for 60¢ per pound?

28. Jane looked at a "weight and age table" and noticed that she was 3 lb. underweight. She should have weighed 84 lb. She was what per cent underweight?

29. Tom weighed 127 lb. and he found that he was 5 lb. overweight. He was what per cent overweight?

30. A man driving his car at 20 mi. per hour increased his

speed 25%, and this enabled him to reach his destination 2 hours sooner than if he had driven at his first speed. What was the length of the trip?

31. A pipe 2 in. in diameter will flow what per cent as much water as one 3 in. in diameter, if the water in each is flowing at full capacity?

32. Milk is bought by a factory at $2.50 per 100 lb. where it is pasteurized and put into quart bottles and then sold at 15¢ per quart. Allowing 8.2 lb. of milk per gallon, what is the per cent of margin on the cost made by the factory?

33. What is the amount of margin when an article costing $36 is marked at $33\frac{1}{3}$% above cost and then discounted 25% of the marked price?

34. If an article is marked 20% above cost and then it is discounted 20%, the new price is what per cent of the original price?

35. A man had a wage increase of 20% and then he had a decrease of 10%. His final wage was what per cent of his original wage?

36. After discounting his goods 20%, a merchant still made 20% of the selling price. They were marked to make a margin of what per cent of the cost?

37. After discounting his goods 20%, a merchant still made 20% of the marked price. They were marked what per cent above cost?

38. Goods were sold at a margin of 30% of the cost, but the cost of selling was 10% of the selling price. The profit was what per cent of the cost?

39. An article was sold at a margin equal to the cost, but the cost of selling it was equal to $\frac{1}{4}$ of the margin. The profit was what per cent of the cost?

40. In 1880, the life span of an individual was 40 years. In 1925, it had increased to $55\frac{1}{2}$ years. That was an increase of what per cent?

41. Because of better health conditions in New York City, the death rate has been reduced. In 1875, the death rate was 28.3 per 1000 of population; in 1925, it was 11.5 per 1000. That was a reduction of what per cent?

42. Statistics show that 2.14% of the population 21 years of age and over are college graduates; 25.08% of the same group have a full or part high school education; and 61.13% of the same group have part or full elementary education. Assuming the population of those who are 21 years of age and over to be 60,000,000, find the number in each group.

43. From a group of 24,278 people listed in "Who's Who in America," it was found that 14,055 are college graduates; 2756 are high school graduates; and 1880 have just an elementary education. Find the per cent who are in each group.

44. In Problem 42 you found the number of people in each group, and in Problem 43 you found the number from each group who are listed in "Who's Who." There is approximately one individual in each group listed in "Who's Who" from a population group of how many?

45. One of the leading universities in the country gave an expense account of what it costs the average student at this institution. The list does not include clothing and amusements. The rates allowed are as follows:

	Low	Average	Liberal
Books	$ 25	$ 30	$ 50
Room in Dormitory	135	185	215
Board (37 wk.)	335	380	535
Laundry	60	75	115
Incidentals	75	100	125

Find the cost of a 4-year course at this college.

46. In order for a boy to have a sufficient income from money invested at 5% to meet the average yearly cost at

this school, how much money would he need to have invested?

47. A father gave each of his two sons $1200 a year for 4 years to defray their college expenses. One boy kept his expenses below the "average cost," while the other boy's expenses equalled the "liberal allowance." How much more money did the first boy have saved to start in business when they graduated at the end of 4 years?

Problems Without Numbers

1. What is the meaning of per cent?

2. How do you find what per cent one number is of another? How do you tell which number is the divisor?

3. How do you find a per cent of a number?

4. When is the work simplified by changing the per cent to a common fraction?

5. Give the per cents that should be changed to common fractions before multiplying and show why it saves work.

6. When is it better to change the per cents to decimals before multiplying?

7. When you know the cost and the rate of gain on the cost, how do you find the selling price?

8. When you know the cost, how can you find a selling price that will give a rate of gain on itself?

9. If you knew the selling price of an article and the rate of gain on the cost, how could you find the cost?

10. How do you find the rate of discount when you know the former price and the reduced price?

11. How is the net price found when successive discounts are given?

12. When successive discounts are given, show that it does not matter which is used first.

13. What is simple interest? Bank discount?

14. When you know the date a note was given and the date of settlement, how do you find the time the note has run?

15. How do you find the interest on a loan for less than one year?

16. What is meant by commission? How is it found when the rate is known?

A Completion Test

Fill in the blank space the term which will complete the meaning:

1. Per cent means ————, or by the ————.

2. Discount is a ———— ———— reduction from the ———— ————.

3. A shipment of goods is called a ————. The party who sends the goods is called the ———— and the party to whom it is sent the ————.

4. In business, the margin is the difference between the ———— and the ————.

5. If the overheads are greater than the ————, there is a ————; if they are less than the ————, there is a ————.

6. A single discount equal to two successive discounts is their ———— less their ————.

7. The margin may be reckoned on the ———— or on the ———— ————.

8. When money is borrowed, a ———— ———— is usually given as an evidence of the loan.

9. The rate of per cent usually charged when money is borrowed is ————.

10. When a note is given at a bank, it is usually necessary to give some ————.

11. If the interval of time for a note to run is less than 1 yr. the interest is reckoned on the————number of———— the note runs.

12. The proceeds are found by ———— the bank discount from the ———— of the loan.

13. In order to change any decimal to a per cent, move the point———— ———— to the ————.

14. In order to change any per cent to a decimal, place the point ———— ———— to the ————

15. If the rate and the amount of the commission are given, the value of the sales can be determined by ———— the ———— by the ————.

True and False Test

If a statement is true, mark it +; if false, mark it −:

1. Any number may be expressed as a per cent of another.

2. To find what per cent one number is of another, you divide the smaller by the larger.

3. Expressing any number by per cent is the same as multiplying the number by 100.

4. The law pertaining to "$\frac{1}{2}$ of one per cent of alcohol," means that 1 to 20 must be the ratio of the alcoholic to the non-alcoholic contents.

5. A school tax of 12 mills on the dollar is .0012% of a dollar.

6. $\frac{3}{4}$%, .0075, $\frac{75}{10000}$ and .00$\frac{3}{4}$ all express the same value.

7. To find a per cent of a number, you change the per cent to a fraction or decimal and multiply the number by it.

8. If you know that a given number is a certain per cent of another number, you can find the second number by dividing the given number by the given per cent expressed as a decimal or a fraction.

9. Any number may be expressed as per cent by first changing it to a fraction form with 100 as the denominator and using the per cent sign instead of the denominator.

10. Any decimal may be expressed as a per cent by dropping the decimal point and annexing the per cent sign.

11. If two successive discounts are given, it does not matter which is deducted first.

12. Discounts of 10% and 10% have the same value as a 20% discount.

13. If a dealer gives a discount on an article which totals as much as the margin, there is no profit on the article.

14. If the margin computed on the cost is the same per cent as a discount reckoned on the selling price, there will be a loss on the article sold.

15. Most merchants do not have any overheads.

16. There can never be a sale in which the margin and the profit are the same.

17. If a salesman's commission is 5% of his sales and his salary from it amounts to $5000 per year, then his sales must have totaled $100,000.

18. Bank discount is simple interest paid in advance when a loan is made.

19. The proceeds and the amount are the same.

20. If a note dated Mar. 1st has 3 months to mature, the interest is computed for $\frac{1}{4}$ of a year.

CHAPTER VII

THRIFT AND INVESTMENTS

If you want to enjoy comfort and security throughout your life, it is necessary to save money and make wise investments. If you do not provide for the future by saving money, at some time business reverses or failing health will show you the danger of your position. The far-sighted man keeps **prepared.** He knows that his personal earning power must diminish sooner or later; but, in the meantime, he tries to accumulate good investments, sufficient to maintain the income which he requires.

The necessity for saving is not so much to provide for a temporary emergency, as it is to assure one of an adequate income through the years of declining earning power. In his prime, the average successful man accustoms himself to a scale of living which cannot be sustained indefinitely unless there is a well invested surplus which will yield sufficient income to maintain that standard of living.

The idea that thrift is a national asset is recognized by the leaders of our country. For this reason a week in each year has been set aside as **National Thrift Week.** In order to encourage thrift among school students, school banks have been established in many schools. In the year 1929 there were over 4,000,000 depositors in these banks, with a total of over $20,000,000 in deposits.

Do you have such a bank in your school?

Below there is given a *Thrift Code* which is worthy of your consideration.

THRIFT CODE

1. *Work and earn.*
2. *Make a budget and have your spending correspond to it.*
3. *Keep an accurate record of your expenditures.*
4. *Have a bank account.*
5. *Have a life insurance policy.*
6. *Own your home.*
7. *Make a will.*
8. *Invest in safe securities.*
9. *Pay your bills promptly.*
10. *Share your money with others.*

THE DIFFERENCE BETWEEN SPENDING AND INVESTING

Few people realize the difference between spending and investing. Money is spent whenever a purchase is made for something which decreases in value as soon as the purchase is made. Money is spent for an automobile, or a radio, or a suit of clothes. Money is invested when something is purchased which not only keeps its value, but also produces a regular fixed income. Wise investment consists in purchasing securities which are safe and which guarantee a good yield. This chapter is going to show you how to invest wisely and the necessity of not spending all of your earnings

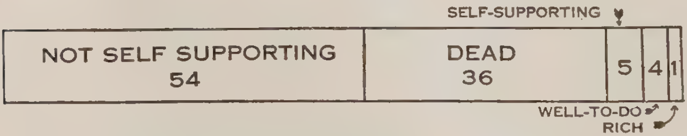
NECESSITY OF SAVING WHILE EARNING

One of the largest insurance companies in this country issued a statement called, "The Signals of Life." These

signals were **54—36—5—4—1.** This company classified a large number of people and found that, of each 100 who had insured their lives at the age of 25, the following was true 40 years later at the age of 65:

54 were not self supporting.
36 were dead.
5 were self-supporting.
4 were well-to-do.
1 was rich.

This may be illustrated by the following graph:



IN WHICH GROUP WILL YOU BE FOUND?

Statistics also show that 98% of American men at the age of 65 are financially incompetent. "Financially incompetent" means that a man has been unable throughout his lifetime to save enough to yield an income of $50 a month. To have an income of $50 a month, or $600 a year, a man must have saved, and have invested at 5%, the amount of $12,000.

A MAN'S FINANCIAL AGE

The American Provident Society has determined the amount of money in cash or securities a man should have at a given age. This amount represents the individual's so-called financial age.

The amounts to be saved at different ages are divided into three schedules. First, the **minimum** which represents enough

to assure the necessaries of life. Second, the **middling** which represents enough to assure the necessaries and comforts. Third, the **master** which represents enough to assure the necessaries, comforts, and some luxuries.

The table which follows shows the amounts needed for each schedule and is taken from the *New York Times*, January 26, 1930.

"FINANCIAL AGE" OF MAN

AGE	MINIMUM		MIDDLING		MASTER	
	Monthly Savings	Total Reserves	Monthly Savings	Total Reserves	Monthly Savings	Total Reserves
20	$ 7.25	$ 185.57	$12.00	$ 328.34	$22.00	$ 607.85
25	5.50	826.01	9.50	1,400.60	16.50	2,607.74
30	10.00	1,460.41	16.50	2,482.07	26.75	4,525.83
35	13.50	2,557.06	21.50	4,271.36	39.00	7,632.84
40	14.75	4,042.65	24.00	6,714.09	49.00	12,281.00
45	17.50	5,958.23	26.50	9,948.54	56.50	18,820.00
50	18.75	8,580.13	29.00	14,184.00	63.00	27,712.00
55	18.25	11,824.00	28.00	19,672.00	67.50	39,582.00
60	17.50	15,966.00	23.50	26,610.00	68.00	55,318.00
65	....	21,002.00	....	35,001.00	....	75,000.00

Exercises

1. Find the yearly incomes if the amounts accumulated at age 65 in each of the three groups are invested at 5%.

2. If not over 40% of a man's income should be spent for rent, what is the greatest amount per month that may be thus spent in each of the three groups? Use the incomes found in Ex. 1.

3. Some authorities on budget-making state that a man should save 15% of his salary. If a man in the middling group is 40 years of age and is saving according to the schedule given, what must be his yearly salary if he saves 15% of it?

4. If a man becomes disabled at the age of 55, what will be the annual income from his savings if they are invested at 5%? Consider each of the three groups.

5. At what age (approximately) will a man in the master group have accumulated as much as one in the minimum group has accumulated at the age of 65?

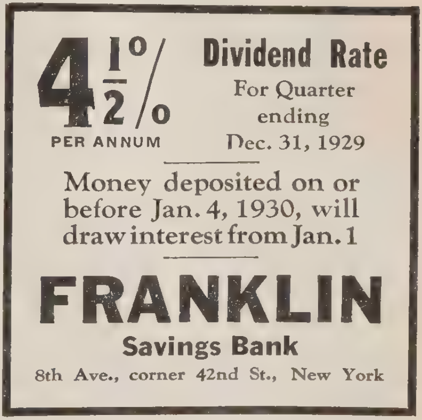
THE SAVINGS BANK

One of the most familiar ways of saving and accumulating money is by depositing it in a **savings bank.** Interest is allowed at the rate of 3% or 4% a year and thus the amount deposited grows. When the interest is added to the principal at the end of a year and the new interest is figured on this new principal, the interest is said to be **compounded annually.**

Thus, if you deposit $100 in a bank which pays 4% interest, compounded annually, at the end of a year the amount due you will be $104. The interest for the next year will be $.04 \times \$104 = \$4.16$, thus making the amount due at the end of two years $108.16.

Sometimes interest is compounded **semi-annually** or **quarterly.** Thus, if the rate is 4% compounded quarterly and $100 is deposited January 1, the amount due on April 1 will be $1.01 \times \$100 = \$101$; the amount due July 1 will be $1.01 \times \$101 = \$102.01$; the amount due October 1 will be $103.03; and the amount due at the end of the year $104.06. Thus the amount at the end of a year is somewhat greater than it would be if it were compounded once a year.

Exercises



1. In the advertisement shown in the margin on the right, what is meant by a "4½% per annum dividend rate"?

2. What is meant by the statement, "Money deposited on or before January 4, 1930, will draw interest from January 1"?

3. How is it possible for a bank to pay interest to its depositors?

4. Is the interest that is due a depositor sent to him, or is it credited to his account?

5. How often is the dividend paid in the Franklin Savings Bank?

6. The quarterly rate paid by the Franklin Savings Bank will be at what per cent?

7. Savings banks usually pay from 3% to 4½% annually. Visit some savings bank in your neighborhood and find the rate paid and how often the interest is paid. In most savings banks interest is paid semi-annually. What does the word "semi-annually" mean?

A COMPOUND INTEREST TABLE

By the use of a compound interest table it is very easy to find the amount of any given principal for a certain number of years.

COMPOUND INTEREST TABLE

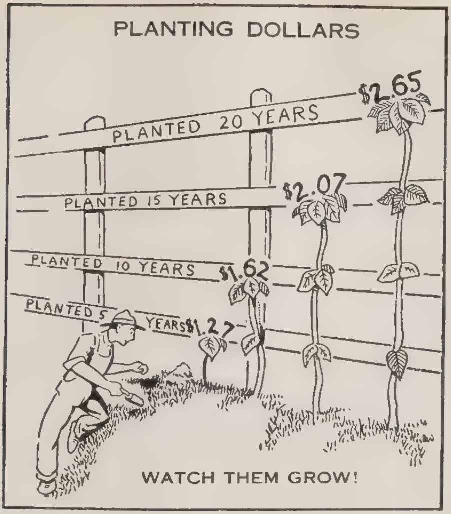
(*The amount of one dollar principal*)

Years	2%	2½%	3%	3½%	4%	5%	6%	Years
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0500	1.0600	1
2	1.0404	1.0506	1.0609	1.0712	1.0816	1.1025	1.1236	2
3	1.0612	1.0769	1.0927	1.1087	1.1248	1.1576	1.1910	3
4	1.0824	1.1038	1.1255	1.1475	1.1699	1.2155	1.2625	4
5	1.1041	1.1314	1.1593	1.1877	1.2167	1.2763	1.3382	5
6	1.1262	1.1597	1.1941	1.2293	1.2653	1.3401	1.4185	6
7	1.1487	1.1887	1.2299	1.2723	1.3159	1.4071	1.5036	7
8	1.1717	1.2184	1.2668	1.3168	1.3686	1.4775	1.5938	8
9	1.1951	1.2489	1.3048	1.3629	1.4233	1.5513	1.6895	9
10	1.2190	1.2801	1.3439	1.4106	1.4802	1.6289	1.7908	10
11	1.2434	1.3121	1.3842	1.4600	1.5395	1.7103	1.8983	11
12	1.2682	1.3449	1.4258	1.5111	1.6010	1.7969	2.0122	12
13	1.2936	1.3785	1.4685	1.5639	1.6651	1.8857	2.1329	13
14	1.3195	1.4130	1.5126	1.6187	1.7319	1.9800	2.2609	14
15	1.3459	1.4483	1.5580	1.6754	1.8009	2.0789	2.3966	15
16	1.3727	1.4845	1.6047	1.7340	1.8729	2.1829	2.5404	16
17	1.4002	1.5216	1.6529	1.7949	1.9479	2.2920	2.6928	17
18	1.4283	1.5597	1.7024	1.8575	2.0258	2.4066	2.8543	18
19	1.4568	1.5987	1.7535	1.9225	2.1069	2.5269	3.0256	19
20	1.4860	1.6386	1.8061	1.9898	2.1911	2.6533	3.2071	20
21	1.5156	1.6796	1.8603	2.0594	2.2788	2.7860	3.3996	21
22	1.5461	1.7216	1.9161	2.1315	2.3700	2.9253	3.6035	22
23	1.5770	1.7646	1.9736	2.2055	2.4647	3.0715	3.8198	23
24	1.6076	1.8087	2.0328	2.2835	2.5633	3.2251	4.0489	24
25	1.6405	1.8539	2.0938	2.3628	2.6658	3.3864	4.2919	25

Exercises

1. Find the amount of $500 at 5% for 8 years.

From the table it is seen that the amount of $1 *at* 5% *for* 8 *years is* $1.4775. *Hence, for* $500 *it will be* 500 *times the amount of* $1, *or* 500 × $1.4775 = $738.75.



2. Find the amount of $500 for 8 years at 5%, compounded semi-annually.

> *The amount of* $500 *for* 8 *years at* 5% *compounded semi-annually is the same as the amount of* $500 *for* 16 *years at* $2\frac{1}{2}$%, *compounded annually. Thus the answer is* 500 × $1.4845 = $742.25.

3. Find the amount of $600 for 20 years at 4% compounded annually and also semi-annually.

NOTE—Since the table does not give 40 periods, find 1.486×1.486× $600. Why?

By using the table find the amount of:

4. $500 at $3\frac{1}{2}$% for 20 yr.
5. $800 at 4% for 15 yr.
6. $300 at $3\frac{1}{2}$% for 12 yr.
7. $250 at 4% for 13 yr.
8. $375 at 4% for 9 yr.
9. $1200 at 3% for 5 yr.
10. $1500 at $3\frac{1}{2}$% for 18 yr.
11. $250 at 3% for 24 yr.
12. $750 at 4% for 11 yr.
13. $900 at 3% for 19 yr.

14. What rate of interest is necessary to have one dollar accumulate to the amounts shown in the picture on page 231?

15. Using the figures shown in the picture, how much would a man need to invest in order to have $10,000 at the end of 10 years? At the end of 20 years?

16. If interest were paid at the rate of 4%, how much would he need to invest to have it amount to $10,000 at the end of 10 years? At the end of 20 years?

17. Referring to the table on page 230, it is seen that in 10 years at 2% $1000 will amount to $1219. In how many years would this sum have been reached if the rate had been 3%? 4%? 5%? 6%?

18. About how many years will it take for money to double itself at 3%? At 4%? At 5%? At 6%?

LOOKING AHEAD FINANCIALLY

The average business life of a man is divided into four normal periods about as follows:

1. *Period of getting started*............age 25 to 35
2. *Period of surplus earnings*..........age 35 to 50
3. *Period of declining earnings*.........age 50 to 65
4. *Period of retirement*..............age 65 onward

Until he is 35, the average energetic young man is establishing himself in a business way and acquiring family responsibilities. His expenses are probably growing at a rate that is somewhat near his income. But, even if his income is growing, he cannot go on expanding his standard of living indefinitely. He must know where to stabilize it; that is, at a point where he can be reasonably sure of maintaining it.

Here the individual is in much the same position as a business concern which, during the period of its establish-

ment, has had to put its earnings back into the business to pay for plant and equipment. Business judgment tells the management when expansion has gone far enough in relation to resources. Unless the business is stabilized it may easily acquire an overhead which it cannot carry. Good management provides a margin of safety. Further expansion is delayed until the margin of safety justifies it. The same principles apply in the personal finances of the individual. Before a man reaches 35 years of age, he should begin seriously to work toward a financial objective. He should have a financial policy which will stabilize his expenditures and increase his surplus with all reasonable progress. **What the financial objective should be depends upon circumstances, but in a general way it should be an amount of income-earning assets sufficient to enable the individual to maintain permanently a satisfying standard of living.**

The First Step—Set Your Objective

The first step in planning to build up a surplus fund to provide future income, is to determine the amount of principal that will be necessary to furnish this desired income.

After you have decided upon an amount which you wish to attain, then it is necessary to do systematic saving throughout a period of years during which you are earning.

HOW MONEY ACCUMULATES FROM REGULAR DEPOSITS

The table on the next page shows how one dollar deposited yearly grows over a period of 40 years.

While savings may be made weekly or monthly, this table will be found sufficient to solve most problems in which you are interested.

TABLE SHOWING AMOUNT ACCUMULATED AT END OF A PERIOD OF YEARS BY PAYING $1 AT BEGINNING OF EACH YEAR IN THE PERIOD.

Year	2 Per Cent	2½ Per Cent	3 Per Cent	3½ Per Cent	4 Per Cent	5 Per Cent	6 Per Cent	Year
1	1.020	1.025	1.030	1.035	1.040	1.050	1.060	1
2	2.060	2.076	2.091	2.106	2.122	2.152	2.184	2
3	3.122	3.153	3.184	3.215	3.246	3.310	3.375	3
4	4.204	4.256	4.309	4.362	4.416	4.526	4.736	4
5	5.308	5.388	5.468	5.550	5.633	5.802	5.975	5
6	6.434	6.547	6.662	6.779	6.898	7.142	7.394	6
7	7.583	7.736	7.892	8.052	8.214	8.549	8.897	7
8	8.755	8.955	9.159	9.368	9.583	10.027	10.491	8
9	9.950	10.203	10.464	10.731	11.006	11.578	12.181	9
10	11.169	11.483	11.808	12.142	12.486	13.207	13.972	10
11	12.412	12.796	13.192	13.602	14.026	14.917	15.870	11
12	13.680	14.140	14.618	15.113	15.627	16.713	17.882	12
13	14.974	15.519	16.086	16.677	17.292	18.599	20.015	13
14	16.293	16.932	17.599	18.296	19.024	20.579	22.276	14
15	17.639	18.380	19.157	19.971	20.825	22.657	24.673	15
16	19.012	19.865	20.762	21.705	22.698	24.840	27.213	16
17	20.412	21.386	22.414	23.500	24.645	27.132	29.906	17
18	21.841	22.946	24.117	25.357	26.671	29.539	32.760	18
19	23.297	24.545	25.870	27.280	28.778	32.066	35.786	19
20	24.783	26.183	27.676	29.269	30.969	34.719	38.993	20
21	26.299	27.863	29.537	31.329	33.248	37.505	42.392	21
22	27.845	29.584	31.453	33.460	35.618	40.430	45.996	22
23	29.422	31.349	33.426	35.667	38.083	43.502	49.816	23
24	31.030	33.158	35.459	37.950	40.646	46.727	53.865	24
25	32.671	35.012	37.553	40.313	43.312	50.113	58.156	25
26	34.344	36.912	39.710	42.759	46.084	53.669	62.706	26
27	36.051	38.860	41.931	45.291	48.968	57.403	67.528	27
28	37.792	40.856	44.219	47.911	51.966	61.323	72.640	28
29	39.568	42.903	46.575	50.623	55.085	65.439	78.058	29
30	41.379	45.000	49.003	53.429	58.328	69.761	83.802	30
31	43.227	47.150	51.503	56.334	61.701	74.299	89.890	31
32	45.112	49.354	54.078	59.341	65.210	79.064	96.343	32
33	47.034	51.613	56.730	62.453	68.858	84.067	103.184	33
34	48.994	53.928	59.462	65.674	72.652	89.320	110.435	34
35	50.994	56.301	62.276	69.008	76.598	94.836	118.121	35
36	53.034	58.734	65.174	72.458	80.702	100.628	126.268	36
37	55.115	61.227	68.159	76.029	84.970	106.710	134.904	37
38	57.237	63.783	71.234	79.725	89.409	113.095	144.058	38
39	59.402	66.403	74.401	83.550	94.026	119.800	153.762	39
40	61.610	69.088	77.663	87.510	98.827	126.840	164.048	40

Exercises

1. If a man saves \$500 each year for 12 years at 5%, how much will he have?

> *From the table it is seen that* \$1 *saved yearly at* 5% *interest for* 12 *years amounts to* \$16.713. *Hence* \$500 will *amount to* 500 × \$16.713, *or* \$8356.50.

2. If you deposit \$250 yearly in a savings bank paying 4%, how much will you have in 10 years? In 20 years?

3. If a man saves \$300 a year from the time he is 25 until he is 60, how much will he have accumulated if he gets $3\frac{1}{2}$% interest?

4. At 4%, could a greater amount be realized by depositing \$250 a year for 20 years, or by depositing \$500 a year for 10 years? How great is the difference?

5. Using the data in Ex. 4, what would be the difference at $3\frac{1}{2}$%?

6. If a parent invests \$100 each year at 5% for his son who is 5 years old, how much money will be accumulated in 15 years to help pay for his college education?

HOW MONTHLY PAYMENTS GROW

Very often it is not convenient to deposit a yearly sum, but a certain monthly saving is made and deposited for future earnings. This is especially true of people who are working on a salary basis. If the habit of thrift has been formed, a regular amount each month is set aside to "prepare for a rainy day." It is true that savings banks seldom pay more than 4%, but after one's savings have grown to a few hundred dollars, it is easy to invest in stocks or bonds which earn 5% or $5\frac{1}{2}$%. Sometimes it is possible to earn 6% or more and still have the money safely invested. The tables given are based upon the assumption that all interest is reinvested at the given rate as it falls due.

RESULTS OF SYSTEMATIC INVESTING

This table shows the accumulation of principal and interest at compound interest, if money is invested monthly at the rates given below.

Rate	Years	$10 A Month	$15 A Month	$20 A Month	$25 A Month	% of Fund: Interest	% of Fund: Principal
4%	5	665	997	1,329	1,662	10	90
	10	1,475	2,212	2,950	3,687	19	81
	15	2,462	3,694	4,925	6,156	27	73
	20	3,666	5,500	7,333	9,166	35	65
	25	5,134	7,701	10,268	12,835	42	58
	30	6,923	10,384	13,846	17,307	48	52
	35	9,104	13,655	18,207	22,759	54	46
	40	11,372	17,058	22,743	28,429	58	42
5%	5	682	1,023	1,364	1,705	12	88
	10	1,555	2,333	3,110	3,888	23	77
	15	2,673	4,009	5,345	6,681	33	67
	20	4,103	6,155	8,206	10,258	42	58
	25	5,934	8,902	11,869	14,836	49	51
	30	8,278	12,418	16,557	20,696	57	43
	35	11,279	16,919	22,558	28,198	63	37
	40	15,120	22,680	30,241	37,801	68	32
$5\frac{1}{2}$%	5	691	1,036	1,382	1,727	13	87
	10	1,597	2,396	3,194	3,993	25	75
	15	2,786	4,178	5,571	6,964	35	65
	20	4,345	6,517	8,689	10,862	45	55
	25	6,390	9,584	12,779	15,974	53	47
	30	9,072	13,608	18,144	22,679	60	40
	35	12,590	18,885	25,180	31,475	67	33
	40	17,204	25,807	34,409	43,011	72	28
6%	5	700	1,050	1,400	1,750	14	86
	10	1,640	2,461	3,281	4,101	27	73
	15	2,904	4,357	5,809	7,261	38	62
	20	4,603	6,905	9,206	11,508	48	52
	25	6,886	10,329	13,773	17,216	56	44
	30	9,954	14,932	19,909	24,886	64	36
	35	14,078	21,117	28,156	35,194	70	30
	40	19,619	29,429	39,238	49,048	76	24

Exercises

From the table find the amount of:

1. \$15 a month for 15 years at 5%.
2. \$10 a month for 10 years at $5\frac{1}{2}$%.
3. \$20 a month for 20 years at $5\frac{1}{2}$%.
4. \$5 a month for 20 years at 6%.

5. Of the amount accumulated at 5% for 5 years, what per cent is principal? What per cent is interest? Of the amount at the end of 10 years?

6. Answer the questions in Ex. 5 if the money is accumulated for 15 years. For 20 years.

In the following exercises, the rate at which the money is accumulated is 4%.

7. If a man is 30 years old, how much must he save each month to have \$40,000 at the age of 65?

NOTE.—If \$25 is saved each month at 4%, in 35 years it will amount to \$22,759. $\$40{,}000 \div \$22{,}759 = 1.758$. Then $1.758 \times \$25$ will give the required monthly payment.

8. Mr. Jones begins at age 20 to save enough to have \$50,000 when he is 60 years old. Mr. Black begins at age 40 to save enough to amount to the same sum at age 60. How much more money does Mr. Black need to deposit each month than does Mr. Jones?

9. In Ex. 8, why is the monthly amount Mr. Black deposits almost three times the amount deposited by Mr. Jones?

10. Two men, whose ages are 30 and 40, respectively, begin to save enough to amount to \$25,000 at the age of 65. How much more actual cash is deposited by the older than by the younger man?

11. Mr. Brown at the age of 30 decided that he wished to have a yearly income of $2000 from his savings in the bank when he reached the age of 65. What will he need to save each month in order that he may accumulate enough to earn $2000 per year at 5% interest?

NOTE.—The yearly interest from his savings must amount to $2,000. Then $.05 \times ? = \$2{,}000$.

12. Make a vertical bar graph to show the actual amounts of money that must be deposited to accumulate to $40,000 at the age of 65, by individuals who are 20, 30, 40, and 50 years of age, respectively.

13. Does the graph in Ex. 12 prove to you the value of beginning to save at the earliest age possible?

14. If the monthly payments were changed so that the individual paid in one sum at the end of the year the total of the 12 payments, would the accumulation be the same as that given in the table?

BUILDING AND LOAN ASSOCIATIONS

One of the best forms of monthly saving is in a **building and loan association.** Building and loan associations are subject to state supervision just as are banks. The rate of interest varies for different organizations. For people who are working on a salary, the building and loan association represents a good type of investment.

8% COMPOUNDED Semi-Annually
on investments; in monthly payments or lump sums; Safety; Real Estate Security; Tax Exempt; State Supervision.
Send for Details
Okmulgee Building & Loan Ass'n
Okmulgee Oklahoma

Have you ever seen in a newspaper or a magazine an advertisement like the one that appears on the left? See if you can interpret it.

HOW BUILDING AND LOAN ASSOCIATIONS OPERATE

Building and loan associations issue stock which has a value of $100 or $200 a share when it matures. This stock is different from a share of stock in a business. In case of a business stock, the share is paid for in advance and the earning is paid in the form of a dividend on the par value of the stock. In a share of building and loan stock, the earnings help to pay for, or mature, the share.

Generally an individual subscribes for shares in a building and loan association on what is known as the **serial** plan. Instead of selling and issuing all stock at the same time, it is divided into series, the first being sold at the date the series begins. Subscribers who come in after its first organization are required to pay back dues. After this series has run for a certain length of time, it is closed and a new one is opened.

The method of payment varies with different organizations. An individual may subscribe for a share of stock and pay $1 a month for a period of about 11 years, when the share will be worth $200. Companies have made provision for people who do not wish to wait so long for their stock to mature, so that an individual may pay $2 per month for each share for a period of about 6½ years, when the value of the share will be $200.

The table (page 240) shows the actual earnings of a building and loan association in New Jersey where the dues are $1 a month. The table shows the dues paid, the book value, and the withdrawal value of one share at periods up to the end of 11 years. In this association it takes 11 yr. 4 mo. for one share to mature, or to be worth $200.

The withdrawal value is determined as follows: After the first year it is the amount of dues actually paid in, plus the following percentage of profits declared by the last preceding

REPORT OF A BUILDING AND LOAN ASSOCIATION

At the End of	Dues Paid	Earnings	Book Value	Withdrawal Value
3 mo.	$ 3.00	$ 0.02	$ 3.02	$ 3.00
6 mo.	6.00	.09	6.09	6.00
9 mo.	9.00	.21	9.21	9.00
12 mo.	12.00	.37	12.37	12.17
15 mo.	15.00	.59	15.59	15.27
18 mo.	18.00	.84	18.84	18.38
21 mo.	21.00	1.17	22.17	21.53
24 mo.	24.00	1.54	25.54	24.77
27 mo.	27.00	1.98	28.98	27.99
30 mo.	30.00	2.46	32.46	31.23
36 mo.	36.00	3.61	39.61	37.99
42 mo.	42.00	5.00	47.00	44.75
48 mo.	48.00	6.61	54.61	51.97
54 mo.	54.00	8.47	62.47	59.08
60 mo.	60.00	10.60	70.60	66.89
66 mo.	66.00	13.00	79.00	74.45
72 mo.	72.00	15.66	87.66	82.96
78 mo.	78.00	18.60	96.60	91.02
84 mo.	84.00	21.84	105.84	100.38
90 mo.	90.00	25.39	115.39	109.04
96 mo.	96.00	28.54	124.54	118.83
102 mo.	102.00	33.34	135.34	128.67
108 mo.	108.00	37.87	145.87	140.19
114 mo.	114.00	42.65	156.65	150.25
120 mo.	120.00	47.80	167.80	160.63
126 mo.	126.00	53.32	179.32	171.32
132 mo.	132.00	59.22	191.22	182.34
136 mo.	136.00	64.00	200.00	200.00

annual report; during the second year, 45 per cent; the third year, 50 per cent; the fourth year, 55 per cent; the fifth year, 60 per cent; the sixth year, 65 per cent; the seventh year, 70 per cent; the eighth year, 75 per cent; the ninth year, 80 per cent; the tenth year and thereafter until the maturity of the series, 85 per cent.

Exercises

1. What is the difference between *book value* and *withdrawal value?*

2. Why is it the two are not the same?

3. If a man subscribed for 20 shares of stock in the association described, what would be the book value of his stock at the end of 6 years? The withdrawal value?

4. How much greater is the book value of 10 shares at the end of ten years than the amount which could be saved in that time at 6% interest by depositing $10 a month? (See table on page 236.)

5. In Ex. 4, compare the withdrawal value of the ten shares with the amount that could be saved at 6%.

HOW BUILDING AND LOAN ASSOCIATIONS EARN MONEY

You have noticed from the table that building and loan associations frequently earn more than 6%. If an individual wishes to borrow money from the association, he is charged interest at the rate of 6%. How is it possible for an organization to pay its depositors more than 6% and yet charge its debtors the same rate?

The associations have other earnings besides the interest on the loans. Some of these are:

1. Often it is necessary for an individual to discontinue his monthly payments. He may then withdraw the full amount of money he paid in, but the withdrawal value will not be the same as the book value. The individual who withdraws before maturity receives only a certain per cent of the earnings on what he has paid in.

2. A fine is imposed on all who are not regular in their payments. This fee varies with different associations, but it is very common to charge 5¢ for each share subscribed.

3. Often a bonus is paid by the person who borrows from the association. The amount of the bonus varies with the value of money on the market. If money is easily secured at 6%, there may not be a bonus charged, but if money is scarce, a bonus may be demanded.

4. Interest is charged on the full amount of the loan from the date of the application for it. The association pays out to the contractor, who is building the house, only that portion which represents the value of the house as it is being constructed. In that way it is possible for an organization to have a 6% rate of interest on a loan for a period of several months and to have paid out but a small part of it.

HOW A LOAN IS PAID BACK

The length of time that is required to pay up a loan is dependent upon the following things:

1. **The amount of regular weekly or monthly payments.**
2. **The earnings of the association.**

Generally the individual who wishes to make a loan from a building and loan association is required to subscribe for enough stock to equal the loan. For example, suppose Mr. Jones wishes to borrow $6000 to build a house. Since each share has a maturity value of $200, it will be necessary for him to subscribe for 30 shares. If the association is earning about 6%, it will require a little over 11 years in which to pay off the loan. The interest charge on $6000 at 6% is $360, or $30 a month. This makes Mr. Jones' monthly payment $60.

Exercises

1. If a man borrows $4000 from a building and loan association to build a house, to how many $200 shares will

he have to subscribe? What will be his monthly dues at $1 per share?

2. What will be the monthly interest at 6% on the loan of Ex. 1? What will the total monthly payment be?

3. If the earnings amount to 6%, find by the table, page 236, about how long it will take to pay for the home.

4. If the earnings amount to 5%, find by the table, page 236, about how long it will take to pay for the loan.

5. In a certain building and loan association, the dues are 50¢ a month for each $100 share and the stock matures in about 11 years. At this rate, how much could you accumulate in 11 years by paying $16 per month? By paying $15 per month?

6. The interest on $5000 at 6% is how much per year? The interest will carry how many $200-shares at $1 per share per month?

7. If $2 a month is paid for each share, about how long will it take for a $200 share to mature?

8. A man subscribed to 10 shares of stock at $1 per share and paid up his dues for 6 years. He then withdrew. From the tables, how much is the company making as the result of his withdrawal?

9. A father wishes his 6-year old son to have $3000 when the boy is ready to enter college at 18 years of age. How much money will the father need to invest in building and loan to accumulate that amount?

10. In Ex. 9, should the father have taken the same amount as he invested in the building and loan and put it in a savings bank at 4%, what would have been the amount of the accumulation?

11. If you have a building and loan association in your city, find the amount of dues charged, the earnings, the rate of interest charged for loans, and the average time required for stock to mature.

LOANING MONEY ON MORTGAGE

A **mortgage** is a written promise, secured by property, to promptly repay a loan and to pay the interest when it is due. It gives the holder the right, under the direction of a court, to sell the property if the loan is not repaid. A **real estate mortgage** is secured by real estate.

The oldest form of investment is loaning money on a mortgage on real estate. The antiquity of real estate mortgages is one of their most impressive features. According to writings in cuneiform characters which have been unearthed in Mesopotamia, real estate mortgages existed 2100 B. C. Thus it is seen that this historic form of investment is 4000 years old. Land is, and always has been, recognized as the most stable commodity, and man, in the very nature of things, has always been dependent upon it for meeting his every need.

A real estate mortgage is not only the oldest, but also the soundest and most stable of all investments. The truth of this statement is shown by the provision of the law which permits savings banks in most states to invest a large part of their deposits in first mortgages, and by the fact that the laws of every civilized country give preference to real estate first mortgages as an investment for the most carefully guarded trust funds.

A **second mortgage** is a mortgage given on real estate that already has been mortgaged. The rates of interest are often higher on a second mortgage than on a first mortgage, for there may be a greater risk involved. If the property has to be sold to pay off the mortgages, the person who holds the first mortgage is paid first. After the first mortgage has been paid, then should there be any money remaining, the second mortgage is paid. For this reason, a person who takes a second mortgage assumes, in many cases, a risk too great for safe investment.

Exercises

1. It is not considered safe to loan more than 50% or 60% of the value of the property taken as security. See if you can give a reason for this.

2. If buildings are taken as a part of the value of the real estate, they should be insured in favor of the holder of the mortgage. Can you tell why?

3. If a man loans money on a real estate mortgage, he makes sure that all taxes have been paid and sees that they are kept paid. Give a reason for this.

4. If one loans money on real estate, he makes sure that the property is free from debt. That is, that some other party does not hold a mortgage on it. Why?

5. In order that one may find out whether there is a former mortgage against property, all mortgages must be properly registered by the register of deeds and mortgages of the county in which the property is located. This person has different titles in different places. Find how mortgages are registered in your county.

Carefully selected mortgages on real estate are one of the safest forms of investment and usually pay a little higher rate of interest than other forms of investment with the same degree of safety.

One reason for their higher yield of income is that they are not as **marketable** as some other forms of investment. That is, if one holds a mortgage and wishes money on it before the note or bond is due, he may not be able to find some one to buy it as quickly as if it were a "listed bond," to be discussed in the next class of investments.

Two things that affect the rate of income that an investment pays are:

1. **Its safety; and**
2. **Its marketability.**

Exercises

1. What is meant by saying that, "Mr. Jones has a $6500 mortgage on his farm"?

2. If Mr. Smith holds a 6% bond and mortgage on Mr. Jones's farm for $6500, interest payable yearly, what income per year will he get until the bond is paid?

3. A man bought a farm for $18,000, paying half cash and giving a 5½% mortgage on the farm for the rest, interest payable annually. How much interest must he pay each year?

4. Mr. Taylor bought a house of Mr. Barnes for $12,500, paying $6500 cash and giving him a 6% mortgage on the property for the rest, interest payable yearly.

(*a*) What is the face of the note?

(*b*) Who gives the note and who holds it?

(*c*) Who pays the interest? How much and when?

(*d*) What security has Mr. Barnes that he will get the interest when due, and the face of the note when due?

(*e*) Why would Mr. Barnes refuse to take a note for the whole value of the property secured by a mortgage on this property alone?

5. It is estimated that there are $12,000,000,000 invested in mortgages in the United States. At 5½%, how much is the yearly interest?

6. A Country Club bought a club house and a golf course for $100,000. A first mortgage of $50,000 at 6% was given, and a second mortgage of $25,000 at 8%. What was the yearly interest that the club had to pay?

7. If this club had a membership of 250 and the yearly dues were $200, what per cent of each member's dues went to pay the interest on the mortgages?

BONDS AS A FORM OF INVESTMENT

Bonds represent the most popular type of investment. If you turn to the financial section of a daily paper, you will see the large number of bonds that are bought and sold each day. These represent but a small part of the total number issued. The bonds that are found quoted are called **listed bonds.** This means that they are the ones bought and sold by brokers who belong to some **exchange.**

A bond is simply a promise to pay, signed and executed under legal safeguards. Generally, payment of interest and principal is assured by some kind of security. Thus a bond is a form of mortgage.

Most of the business of the country is carried on by corporations. When they need money to carry it on, they borrow money just as an individual does. They do this by issuing bonds of $100, $500, $1000, or larger sums, which they sell to investors. Such bonds are usually secured by a mortgage on the property of the corporation.

City, county, and state governments, and our National Government often borrow money to meet certain expenditures by issuing bonds.

Interest on bonds is usually paid semi-annually. It is paid in two ways. Some bonds are called **registered bonds.** That means that the corporation has the name of the holder of each bond and sends him a check for the interest when due. Other bonds are called **coupon bonds,** which means that they have small coupons attached. These coupons are certificates representing the interest due each interest period. When due, they are cut off and deposited with a bank for collection.

CLASSIFICATION OF BONDS

There are several ways by which bonds may be classified. One of the most common ways is to name them according to the kind of company or organization that issues the bonds.

1. **Government bonds** are those issued by the federal government as a means of raising money to defray certain expenses. You are all familiar with the Liberty Bonds which were issued during the World War. The money from the sale of these bonds was used to meet the expenses of conducting the World War.

2. **Municipal bonds** are those issued by cities, counties, and other political units, and are paid by special taxation. The money used for the building of school houses is raised, in most cases, by money from the sale of municipal bonds.

3. **Railroad bonds** are those issued by railroads. The security back of them is a mortgage on the company's property. The safety of the security lies in the value of the property and the company's earning capacity.

4. **Public utility bonds** are those issued by electric light, gas, power, street railway, and similar companies.

5. **Industrial bonds** are those issued by manufacturing concerns, oil, coal, steel, automobile companies, and others of a similar nature. Industrial and public utility bonds are usually secured by mortgages on the property, just as in the case of the railroads. As these depend upon trade conditions, their market value varies with the general state of the industry.

THE SAFETY OF BOND INVESTMENTS

Just because bonds are "promises to pay," you must not feel that all bonds are equally safe investments. The safety depends upon the ability of the corporation to pay the interest and the face of the bonds when due. Before buying a bond, one not familiar with investments should consult his banker or some person well informed on the subject. One can get a good idea of what investors think of the safety of an investment from the rate of interest the bond pays. In general,

A low rate of income means great safety; and a high rate means greater risk.

The safest investments usually yield from 4% to $5\frac{1}{2}$%. Good investments yield from $5\frac{1}{2}$% to $6\frac{1}{2}$% or 7%. Investments yielding more than this usually involve greater risk.

When one buys a bond yielding above 7% or 8%, it is considered a *speculation* instead of an *investment*, for in order to get a large return it is necessary to take a great risk of not getting it back.

NOTE.—Before investing in mortgage bonds, ask the following questions:

a. Is the corporation issuing the bonds one that is likely to be a permanent institution of sound financial structure?

b. Is its management recognized by authorities as being efficient and economical?

c. Does the corporation unconditionally guarantee its mortgages?

d. Does the value of the property, judged by its earnings, justify the amount of the bond issue?

1. What part of the advertisement on page 250 gives the data that will help you to form the best answer to *a?*

2. Does any part of the advertisement give you the necessary information to help you answer *b?*

3. What part of the advertisement gives the data that will help you to make the best answer to *c?*

4. What part will help you to answer *d?*

5. What is meant by a **first mortgage 6% gold bond?**

6. The operating expenses represent what per cent of the gross earnings?

7. The net earnings represent what per cent of the gross earnings?

8. The annual interest requirement is what per cent of the net earnings?

9. Do you think the security is sufficient to amply cover the bonds?

$5,000,000

The Cincinnati Street Railway Company

First Mortgage Gold Bonds, Series B 6%

To mature April 1, 1955

Guaranty Trust Company of New York, Trustee

The following is contained in a letter written to us by Mr. Walter A. Draper, President of the Company:

Business

The Cincinnati Street Railway Company (formed by consolidation in 1880) owns and operates the entire street railway system in Cincinnati, together with an extensive supplementary bus system.

Fares

The present railway fare is 8⅓¢ for tickets or 10¢ cash, a reduction from a straight 10¢ fare having been made on November 1, 1925. The bus fares are 10¢ and 15¢ cash.

Capitalization

Outstanding after giving effect to this issue

First Mortgage Gold Bonds:	
Series A 5½%	$ 6,784,500
Series B 6% (this issue)	5,000,000
Equipment Trust Certificates 6%	217,500
Total Funded Debt	$12,002,000
Capital Stock	23,761,950
Total Capitalization	$35,763,950

Security

The mortgage under which these Bonds are to be issued has a direct first lien on all fixed property and all equipment now owned by the Company, except about one-eighth of its railway cars, on which the mortgage has a lien subject to the $217,500 outstanding equipment trust certificates.

Retirement of Bonds

The Company will be obligated to make sinking fund payments, beginning January 1, 1931, which are calculated to retire more than 75% of the Series B Bonds by maturity.

Earnings

Calendar Yrs.	Gross Revenues	Operating Expenses, Maintenance, Taxes and Depreciation	Net Earnings Available for Interest
1926	$8,102,518	$6,580,205	$1,522,313
1927	8,748,279	7,103,800	1,644,479
1928	8,846,665	7,073,508	1,773,157
1929	8,844,733	6,725,909	2,118,824

Net earnings for 1929, as shown above, amounted to more than 3 times annual interest requirements of total funded debt presently to be outstanding.

General

The Company has paid dividends without interruption since its formation nearly 50 years ago. At present prices the outstanding stock has an indicated aggregate market value of more than $20,000,000.

All of the Company's directors and over 80% of its stockholders of record, of whom there are approximately 6,475, are residents of the Cincinnati area.

99½ and accrued interest, to yield over 6%

10. Explain what is meant by the statement, "To mature April 1, 1955."

11. Would you consider this bond an investment or a speculation? On what do you base your answer?

THE PRICE OF A BOND

In the advertisement you saw this statement, "Price $99\frac{1}{2}$ and accrued interest, to yield over 6%." By the price is meant the market value of the bond. If it sells for more than its face value, it is selling **above par**; if it is sold below the face value, it is selling **below par.**

Bonds are not listed in terms of their money value, but in terms of per cent of their par value.

A bond quotation of "Penn. R. R. 5s, 1968, 109," means that the 5% bonds of the Pennsylvania Railroad, due in 1968, are selling for 109% of their face value. That is, a $1000 bond is selling for $1090.

Exercises

1. Tell what the following mean: (Taken from the New York Stock Exchange report of Jan. 21, 1930.)

Armour & Co. $4\frac{1}{2}$s, 1939, $89\frac{1}{4}$.
Beth. Steel $5\frac{1}{2}$s, 1953, 107.
Am. Tel. & Tel. 5s, 1943, $105\frac{5}{8}$.
Penn. R. R. $4\frac{1}{2}$s, 1965, $97\frac{7}{8}$.
Chile Copper 5s, 1947, $95\frac{1}{2}$.
Southern Railway $6\frac{1}{2}$s, 1956, $122\frac{3}{4}$.

2. Find the market value and the yearly interest on a $1000 bond in each of the above companies.

3. Which do you consider the safest? Why?

4. Which do you consider the least safe? Why?

5. On Jan. 21, 1930, Italian Government 7s were selling at 96, and Dominion of Canada 5s were selling at 103. Find

the yearly interest on a \$1000 bond of each. Which one would you rather own? How do you account for the fact that one was selling below par and the other above par?

6. If an individual bought one of each of the bonds listed on page 251, what would be the total cost, not counting brokerage? What would be the amount of the semi-annual interest derived from the six bonds?

THE YIELD ON A BOND

Aside from the safety and the marketability of a bond, an investor is interested in the **yield** on a bond. The **yield** means the average yearly rate of income received on the investment if kept to maturity, or until the bond is due.

A 5% bond maturing in 1960 is advertised in 1930 as, "Price 96 and interest, yielding over 5.25%." What is meant by the statement?

A \$1000 bond will cost \$960. But if the bond matures in 1960, \$1000 will be paid to the owner by the company in 1960. Thus there is a gain at maturity of \$40; that is, at the end of 30 years.

The yearly interest at 5% on a \$1000 bond is \$50.

If you will turn to the table on page 234, you will see that \$1.00 per year for 30 years at 5% amounts to \$69.76. Thus, at 5%, it will take about 56 cents a year for 30 years to amount to \$40. Hence the gain at maturity is about 56¢ per year, giving a yearly income of \$50.56.

$$\$50.56 \div \$960 = .0526 = 5.26\%$$

Usually the small investor is interested in an "approximate yield," hence he does not bother to use a set of tables to distribute the gain or loss at maturity over each year, but merely divides the gain or loss by the number of years the bond has to run. When the time is short, as 5 to 10 years, this gives a close approximation to the yield. But in the

solution above, $40 ÷ 30 would have given $1.33 instead of $0.56 to add to the $50. How would this have affected the answer?

1. A man buys a $1000 bond paying 6% interest, due in 5 years. If the purchase price is 102, find the approximate yield.

```
          .0549 = 5.49%
1020)56.0000
     51 00
      5 000
      4 080
        9200
        9180
          20
```

Think, "He pays $1020 for it and gets back $1000 at the end of 5 years. This is an average loss of $4 per year." Then think, "He gets $60 per year interest. Deducting the loss of $4 per year, he has a net income of $56 per year from an investment of $1020."

Then find what per cent $56 is of $1020.

2. A man buys a $1000 bond paying 4% interest, due in 4 years. If he pays 92, find the approximate yield.

```
         .0652 = 6.52%
920)60.0000
    55 20
     4 800
     4 600
      2000
```

Think, "He pays $920 for it and gets back $1000 in 4 years. This is an average gain of $20 per year." Then think, "He gets $40 per year interest. Adding the gain, he has a yearly income of $60 from an investment of $920." Then find what per cent $60 is of $920.

$30,000,000
Associated Gas and Electric Company

Convertible 5% Sinking Fund Gold Debentures, due 1950

Dated February 1, 1930 Due February 1, 1950

Interest payable February 1 and August 1. Redeemable as a whole or in part at any time on not less than 30 days' published notice at 103 on or before January 31, 1940; thereafter at 102 on or before January 31, 1945; thereafter at 101 on or before January 31, 1949; thereafter at 100 to maturity; in each case with accrued interest. Coupon Debentures in denominations of $1,000. The Public National Bank and Trust Company of New York, Trustee.

Capitalization

Consolidated capitalization of Associated Gas and Electric Company and its subsidiary companies as of December 30, 1929, assuming the sale of all of this issue of Debentures and after giving effect to recent financing and to the acquisition or retirement of securities and to calls for redemption since that date, is as follows:

Associated Gas and Electric Company:	Outstanding
Class A, B and Common Stocks (no par value)	7,768,320 shares
Preferred Stocks (all of equal rank) Liquidation Value	$ 24,983,700
Debenture Obligations Convertible now or later into Stocks at Company's option	86,832,000
Convertible 5% Gold Debentures, due 1950 (this issue)	30,000,000
Other Funded Debt of Company	208,433,232*
Subsidiary Companies' Funded Debt and Preferred Stocks:	
Associated Electric Company 4½s, due 1953	17,851,000
Other Subsidiary Companies' Funded Debt	143,705,200
Subsidiary Companies' Preferred Stocks—Liquidation Value	53,431,750
Minority Common Stocks and Surplus applicable thereto	2,274,472

*Includes 5½% Investment Certificates and $8 Interest Bearing Allotment Certificates.

Earnings

The consolidated earnings of the Company and subsidiary companies, irrespective of dates of acquisition, for the twelve months ended November 30, 1929, and annual charges on securities outstanding at that date, assuming the sale of all of this issue of Debentures and after giving effect to recent financing and to the acquisition or retirement of securities and to calls for redemption since that date, were as follows:

Gross Operating Revenues and Other Income		$89,177,899
Operating Expenses, Maintenance and Taxes (except Federal Income Taxes) and amounts applicable to minority common stocks		40,756,446
Consolidated Net Earnings before Interest, Depreciation, Dividends, etc		$48,421,453
Annual Interest and Dividends on Funded Debt and Preferred Stocks of Subsidiary Companies (less $775,576 credit for interest during construction) and Annual Interest* on entire funded and unfunded debt of Company to be outstanding		21,284,644*
Depreciation	$4,238,210	

Consolidated net earnings as above were, before depreciation, over 2.27 times and after depreciation, over twice the above annual interest and dividend charges.

*Includes interest on 5½% Investment Certificates and on $8 Interest Bearing Allotment Certificates, but excludes interest on obligations now convertible at Company's option into stocks.

Over 92% of the gross operating revenues was derived from electric and gas operations.

We Recommend These Debentures for Investment

Price 90 and interest, yielding 5.85%

Find the approximate yield at maturity on each of the following purchased in 1930:

Bond	Maturity	Purchase Price	Yield to Maturity %
3. Kansas City Southern, 3s..	1950	$73\frac{3}{4}$	?
4. Georgia Midland, 3s......	1946	$73\frac{1}{4}$	?
5. Wabash R.R., Omaha Div., $3\frac{1}{2}$s..................	1941	84	?
6. N. Y., N. H., & H. R.R., $3\frac{1}{2}$s..................	1947	72	?
7. Chateaugay Ore & Iron, Ref., 4s...............	1942	89	?
8. Cincinnati, Lebanon & Northern, 4s...........	1942	$90\frac{3}{4}$	?
9. Lehigh & New York, 4s...	1945	90	?
10. Providence Terminal, 4s...	1956	84	?
11. Louisville & Jeffersonville Bridge, 4s.............	1945	$90\frac{1}{4}$	?
12. Southern Ry., 4s.........	1956	$86\frac{3}{4}$	?

From the advertisement on the preceding page you find there are introduced some terms that you may not have seen before. The expression, "Convertible 5% sinking fund gold debentures," contains several new terms.

Convertible means that any time before the date of maturity, the value of the bond may be converted into **common stock.** The rates are given in the first paragraph of the bond.

A sinking fund is an accumulation of money which is set aside each year to be used for the redemption of the bonds when they mature. Usually this money is taken from the yearly earnings of the company.

A gold debenture bond is a mere promise to pay, in gold, without offering any security to back it up. From the bond

listed on page 249, you noticed that it was a first mortgage bond. A debenture has for its security the name and the reputation of the company offering the bond. If the company which issues the bonds does not pay the interest nor redeem the bonds, legal suit may be brought against the company.

Before buying a debenture, be sure that the organization issuing the bonds is sound financially. That is, that the company has enough unmortgaged property to secure the bonds and that the earnings are much more than the interest charges.

BUYING A BOND

Bonds are bought and sold through agents called **brokers.** Or, your banker will buy or sell a bond for you.

When buying a bond the cost, or what the buyer actually pays, includes the *market value* of the bond, the *broker's fee*, and the *accrued interest.*

The **accrued interest** is the interest the bond has earned since the last interest was paid. If one buys a 6% bond, "interest payable Jan. 1 and July 1," on March 10, it has been 68 days since the last interest was collected. So he pays the interest at 6% for 68 days, and on July 1 he will get the interest earned since Jan. 1.

1. Find the accrued interest on Aug. 15 on a \$1000, 6% bond whose last interest was paid on June 1.

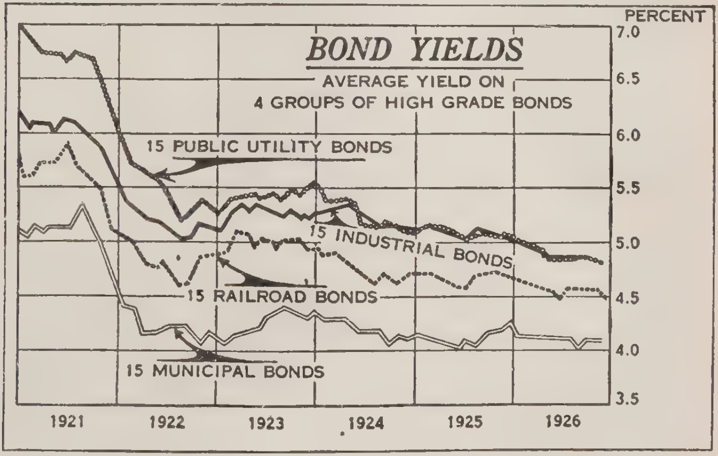
2. A man bought five \$1000 Kingdom of Belgium $7\frac{1}{2}$s at $109\frac{1}{2}$ on Oct. 16. The last interest was paid on Aug. 1. Here is the bill he got from his broker. Check it.

5 M K'n'g Bel. 7½% at 109½	\$5,475	00
Brokerage at \$2.00	10	00
Accrued interest at 7½% for 76 da.	79	17
Total	\$5,564	17

3. Find the total cost of three \$1000 bonds, interest 5%,

bought at $96\frac{1}{2}$ on Aug. 20, last interest paid June 1, brokerage $2.00 each.

WHY MARKET PRICES CHANGE



If you will select several bonds and watch the market quotations for a few weeks, you may find very little change in price. But bonds are usually issued to run from 20 years to 50 years or even longer. During that time money rates may change and this will cause a change in the price of the bond. Thus, if money on "bond and mortgage" is worth 6%, one would not pay par value for a bond paying but 4%. On the other hand, if money is worth but 4%, a bond paying 6% would be worth more than par.

Another element that enters into the price of a bond is the security back of it. During such a long period the value of the property may change and thus affect the price of the bond. In general, the four leading factors that regulate the price of bonds are:

1. The security back of the bonds.

2. The rate of interest the bond is paying compared with the general interest rates of money.

3. The length of time the bond has to run.

4. The confidence of the buying public in the stability and general earning power of the corporation issuing the bonds.

Exercises

1. Which type of bond gives the greatest yield as determined by the graph on page 257?

2. What may have been the cause of the gradual decline in the yield since 1921?

3. Why is it that municipal bonds give such a small yield?

4. Has the yield from municipal bonds changed much within the period from 1922 to 1926?

5. When general interest rates are 5%, would you expect a 6% bond issued by a prosperous corporation to sell for more or less than par?

6. When general interest rates are but 5%, could you afford to pay 102 for a 6% bond which will mature in 10 years?

7. From the graph given on page 257, you see that the yield from bonds has gradually declined since 1921. What was the reason for the high yield during 1921?

8. Mr. Brown said, "If a bond is selling above par, it is a safe investment; and if it is selling below par, it is unsafe." Was he right? Defend your answer.

CORPORATIONS AND STOCKS

You have noticed from advertisements that most of the things you use are produced by some **company** or **corporation.** Thus, you see the advertisements of Swift & Co.; The Quaker Oats Co.; The National Biscuit Co.; Colgate & Co.; General Electric Co.; etc. These companies consist of a number of

individuals united by the consent of the state, and empowered by the state to transact a certain form of business. The list of powers, rights, and duties are stated in writing in an instrument called their **charter.**

Stock is a name given the **capital** with which they do business. This capital stock is divided into **shares,** usually $100 each, called the **par value,** but they may be of any size. Thus, if a company has a **capital** of $1,500,000 divided into $100 shares, there will be 15,000 of them. Any one may become a part owner of the company by buying one or more of these shares. If one owned 15 of these 15,000 shares, he would own one one-thousandth of the business and be entitled to that part of its earnings.

The owner of one or more shares of stock in a company is called a **stockholder** in the company. As evidence of ownership, each stockholder receives a **stock certificate** showing the number of shares he owns and the par value of each.

NOTE.—It is becoming quite common to issue shares with no par value. The stock certificate shows the number of shares issued and the number represented by that certificate.

The earnings of a corporation that are divided among its stockholders are called the **dividends.** They are distributed as a per cent of the par value of the stock. Thus, a 12% dividend gives the stockholder $12 for each $100 share that he owns.

When no par value is given the dividend is stated in dollars and is found by dividing the total dividend by the number of shares outstanding.

EXERCISES

1. If the capital of a corporation is $500,000 and it is divided into $100 shares, how many shares will there be? The holder of 100 of these shares owns what part of the business?

2. If a man owns 100 of the 1000 shares in a company,

he owns what part of the business? If $8000 in earnings (dividends) are distributed, how much will he get?

3. If a company with a $100,000 capital divides it into $50 shares, how many shares will there be? For each share held, one will own what part of the business?

4. If a company with a capital of $500,000 distributes $75,000 in dividends, what per cent of the capital is this?

5. If one owns twenty $100 shares in a company and an 8% dividend is declared, how much will he get?

6. If one owns twenty $50 shares and a 10% dividend is declared, how much will he get?

7. When a company with a capital of $1,000,000 declares a 6% dividend, how much will the whole dividend be? How much will a man get who owns fifteen $100 shares?

8. When a man gets $180 in dividends from fifteen $100 shares of stock, what rate of dividend has been declared?

THE MARKET VALUE OF STOCK

The **market value** of stock is the price at which it can be bought or sold in open market. A number of factors affect the market price of stock, chief among which are:

1. **The real or prospective earning power of the corporation; and**

2. **The confidence of the buying public, or the lack of it, in the general stability of the enterprise.**

When the real or prospective rate of dividend is small, the price is low; when large, the price is high.

As these two factors change with cost of labor and material, and the public demands for the company's products, and for numerous other reasons, the price of stock varies greatly. For that reason there is much speculation in stocks. By **speculation** is meant buying in expectation of a rise in price, or selling in expectation of lower prices, with the intention of buying back. In other words, speculation is dealing in uncertainties.

A stock investment is more speculative than a bond investment, owing to the fluctuation of the market value.

Owing to the fluctuation in stocks, many people buy them as a speculation instead of an investment. That is, they buy them hoping to sell soon at a higher price. Here are the prices of certain stocks on January 29, 1930, listed on the New York Stock Exchange.

Sales	Stock	Open	High	Low	Close	Net Change
67,300	Anaconda Copper......	$73\frac{3}{4}$	$76\frac{1}{8}$	$73\frac{1}{4}$	$74\frac{3}{4}$	+1
900	Eastman Kodak.......	$194\frac{1}{2}$	$194\frac{1}{2}$	$192\frac{1}{2}$	$193\frac{1}{2}$	$-1\frac{1}{4}$
4,900	General Electric.......	264	$264\frac{3}{4}$	261	261	−3
6,300	Standard Oil of N. J...	64	$64\frac{1}{4}$	$63\frac{1}{8}$	$63\frac{1}{4}$	$-1\frac{1}{2}$
800	Western Union Tel.....	206	206	203	203	−3

The first line is read as follows: "On January 29, 1930, 67,300 shares of common stock of Anaconda Copper were sold. The opening price was $73.75 per share. The highest price paid during the day was $76.125. The lowest was $73.25. The closing price was $74.75. There was a net gain on each share of $1. The "Net Change" shows the change from the closing price of the preceding day.

1. Read each of the other quotations given.

2. Get a daily newspaper at the time you study this and see how the prices of these stocks have changed.

STOCKS AND BONDS COMPARED AS INVESTMENTS

Stocks	Bonds
1. The dividends depend upon the earning power of the corporation.	1. The interest is a fixed rate.
2. The dividends are not due until they have been declared by the board of directors.	2. The interest is paid at regular fixed periods.
3. Subject to sudden fluctuations in value.	3. Only slight fluctuations in value.

THE NEW YORK STOCK EXCHANGE

The **New York Stock Exchange** is a market place for securities. It is a voluntary association and its rules and regulations are drawn up by its own members. There are at present 1375 members, but not all of them have their places of business in the New York Exchange. More than 100 members reside in other cities and handle business for their customers in those places.

HOW STOCKS ARE LISTED ON THE EXCHANGE

In order to protect the public from buying fraudulent stock, the Stock Exchange requires each company that wishes to offer its stock on the Exchange to submit certified statements concerning its business. If these statements show that the company is in good standing, the Exchange may list the stock of that company.

BUYING ON MARGIN

When stock is bought on **margin,** the purchaser pays only a certain per cent of the value of the stock in cash, and the rest of the payment is made by the broker who executes the transaction. If 100 shares of Eastman Kodak are bought on margin, the purchaser may pay $5,000 in cash and the broker will supply the amount necessary to pay the cash value of the stock. The broker will keep the stock as collateral and charge the purchaser a certain rate of interest for the loan. Buying stock on margin is dangerous. The person who buys stock on margin usually intends to keep the stock a short period of time, hoping to sell whenever the stock increases in value a few points. If the value of the stock decreases, the broker will call upon the purchaser to make another payment to cover the decline. When this amount is more than the purchaser is able to meet, the broker may be compelled to sell

the stock in order to protect the loan which he made to the purchaser. If there is a very sudden drop in the value of stocks, such as occurred in November, 1929, it is possible for one who has bought on margin to lose in a few days all the money which he invested. For that reason, buying stock on margin always involves a high degree of risk.

COMMISSION CHARGED BY BROKERS ON THE NEW YORK STOCK EXCHANGE

When a broker buys or sells securities he charges a **fee** or **commission** for his services at a fixed rate, depending on the market value of the securities. The following rate is charged:

Stocks selling at $	1 and under $	10, $	7.50 per 100 shares.
Stocks selling at	10 and under	25,	12.50 per 100 shares.
Stocks selling at	25 and under	50,	15.00 per 100 shares.
Stocks selling at	50 and under	75,	17.50 per 100 shares.
Stocks selling at	75 and under	100,	20.00 per 100 shares.
Stocks selling at	100 and under	200,	25.00 per 100 shares.

Stocks selling at $200 per share and over, not less than 30¢ per share, plus 5¢ per share for each $50 or fraction thereof beginning at $250.

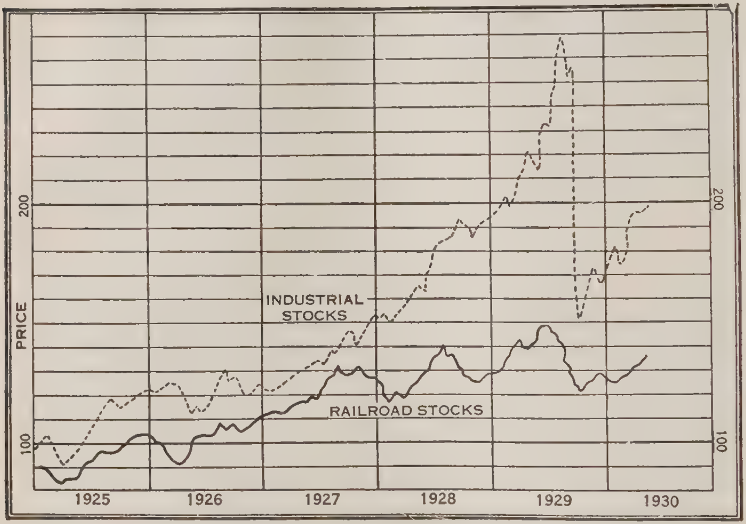
The commission rates apply proportionately to transactions in odd lots (less than 100 shares), with a minimum charge on any transaction of $5.00.

Bonds or notes.............. $2.00 per $1000 par value.

Exercises

1. From the graph on the following page, can you tell what has been the trend of prices on the New York Stock Exchange from 1925 to 1928?

PRICE TREND ON THE NEW YORK STOCK EXCHANGE



2. When were industrial and railroad stocks on about the same level?

3. Can you explain why industrial stocks should sell for more than railroad stocks?

4. From the graph given, in what year did industrial stocks have the greatest value? The least value?

5. Of the two types of stocks given, which sells at the higher price?

6. How would you compare industrial stocks with railroad stocks for the year 1925?

7. When did railroad stocks have the least value?

8. What date represents the time it would have been the least advisable for the investor to buy industrial stock?

THE RATE OF INCOME ON STOCKS

Since many stocks do not have a par value, the dividend on stocks is usually expressed in dollars. The yield is found by comparing the dividend with the market value.

Exercises

Find the yield on the following stocks:

1. General Motors, selling at 39; dividend, $3.
2. American Telephone and Telegraph, selling at 220; dividend, $9.
3. United States Steel, selling at 175; dividend, $7.
4. Anaconda Copper, selling at 75; dividend, $7.
5. Westinghouse, selling at 145; dividend, $5.

INVESTING IN PREFERRED STOCK

The stocks already discussed, in which the stockholder becomes a part owner of the corporation by his investment, has a vote in the control of the business, and shares its profits, are called **common stocks.**

There is a growing tendency among industrial corporations to obtain capital requirements through the issue of a type of stock called **preferred stock.** This differs from the common stock in that the holder has no vote in the control of the corporation, and does not share in the earnings except to the extent of the dividends guaranteed in the certificate. Preferred stock usually guarantees, in the certificate, a dividend of 7%. It is sometimes but 6%. Since preferred stock is not in the form of a note or bond against the company, secured by a mortgage on the property of the corporation, and having a date at which it matures, it may be less safe as an investment and hence often has to pay a higher rate in order to find buyers. In certain well-established corporations, however, preferred stock is as safe as their bonds.

$1,000,000

H. C. Bohack Co., Inc.

CHAIN STORES, GROCERIES, AND MEATS

Seven Per Cent Cumulative First Preferred Stock

Preferred as to assets and dividends. Dividends are cumulative and payable quarterly, February, May, August and November 1st when declared by Board of Directors. Redeemable in whole or in part on ninety days' notice at the option of the Company, at $115 per share and accrued dividends. Par value $100 per share.

Capitalization

The Company has no funded debt.

After giving effect to the sale of $1,000,000 First Preferred Stock, the Company will have the following capitalization:—

	Authorized	Presently Outstanding
First Preferred Cumulative 7% Stock, par value $100	$3,000,000	$3,000,000
Second Preferred Cumulative 6% Stock, par value $100	150,000	150,000
Common Stock, par value $100	1,850,000	1,850,000

Business

Organized in 1903, the Company has grown to proportions of great size as shown below. It is operated on the "Cash" basis. Stores throughout Brooklyn, Queens and Long Island.

Dividend Record

Average earnings for past years have covered preferred dividend requirements several times over.

Regular dividends have been paid on the Seven Per Cent First Preferred and on the Six Per Cent Second Preferred Stock since date of issuance.

Ten Per Cent has been paid on the Common Stock for the first seven years.

Purpose of Issue

The proceeds of the Preferred Stock presently to be issued are to reimburse the Company for its expenditures in building its new Pork Packing Plant now in successful operation, and for other additions to plant and equipment.

From the advertisement on page 266, you noticed the expression, "Seven per cent **cumulative** first preferred stock." You have already found out what is meant by preferred stock, but you may not have heard of cumulative stock.

In **cumulative** stock if, for any reason, the company is unable to pay dividends, the dividends accumulate until such time as the company makes enough money to pay them. Thus, in the stock advertised, if dividends are not paid for 3 years, there will be due to the stockholders 21% in back dividends. These must be paid before any dividends are paid on the common stock.

Exercises

1. What is the yearly income from 40 shares at $100 each of the 7% preferred stock?

2. Find the difference in income between $10,000 invested in 7% preferred stock at par, and the same amount invested in $4\frac{1}{2}$% bonds at par.

3. When "General Motors pf." is quoted at $119\frac{1}{4}$, what per cent of return is the investor getting if the dividend is 7%?

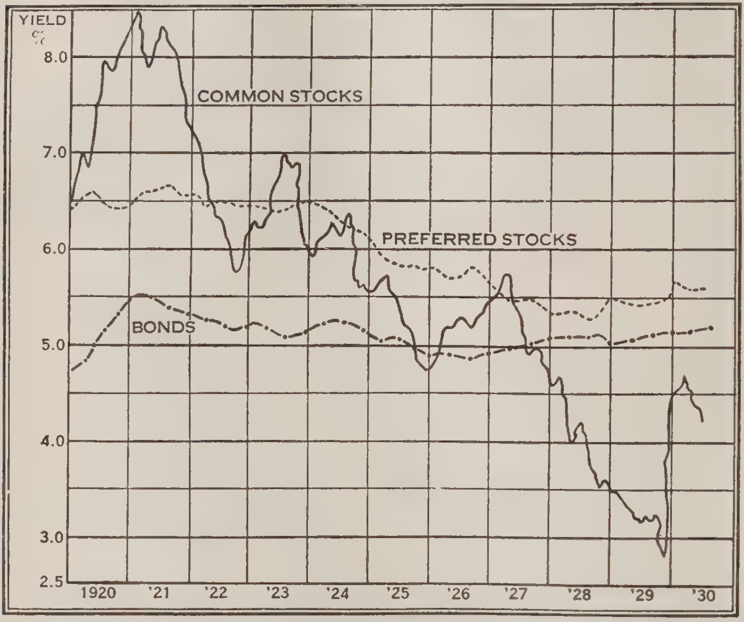
4. "Public Service pf." is quoted at 110 and pays a dividend of 6%, and "United States Steel 7% pf." is quoted at 140. Considering the safety of the two to be the same, which represents the better investment?

5. A man owns 25 shares of the stock listed in the advertisement on page 266. If the company has been able to pay a dividend of only 5% for each of two years, what will be the amount of dividend payment owed to him the next year?

YIELD ON STOCKS AND BONDS COMPARED

There is no definite guide for knowing which of the two forms of investment, stocks or bonds, will give the greater yield. Ordinarily bonds fluctuate less than stocks in both

value and yield. The graph given below shows the average yield for each.



1. When would it have been most profitable to own stocks, if one considered only the yield?

2. Has the decline in the yield in bonds been as great as that for stocks?

WISE PURCHASING AS A FORM OF THRIFT

"One of the best means of saving is by careful spending."
THEODORE ROOSEVELT.

One of the most popular forms of purchasing at the present time is buying on the **installment plan.** By this plan a deposit is made at the time of the purchase, and then a regular weekly or monthly payment is made until the total payment is made. Because only a small amount of money is needed at the time

of purchase and because the terms seem very easy, the purchaser often never considers the high rate of interest that necessarily must be charged for this type of buying. A few of the reasons a high rate of interest is charged may be given as follows:

1. *There is a very much increased cost to the company for added bookkeeping.*

2. *There is a risk involved in that the purchaser may not make the total payments, thus necessitating that the company reclaim the article.*

3. *Interest must be paid on the unpaid balance.*

The opinions of leading financiers and business men differ as to the advisability of encouraging installment buying. In the year 1926, the value of the articles purchased on the installment plan ranged from $7,500,000,000 to $10,000,000,000. This amount represents about one-fifth of the total retail sales in the country for this same year. From these facts, it is readily seen that installment buying is one of the most popular forms there is.

The following extract is part of an editorial from the New York *Herald-Tribune* of December 6, 1927:

> A banker in a Middle Western manufacturing town was approached recently by a man who wished to negotiate a loan of $250. The would-be borrower explained to the banker that he worked in a local factory and that his weekly pay envelope averaged about $50.
>
> The banker naturally inquired about collateral. Did the man own his home? Did he own it clear and free, or was there a mortgage on it? The house was mortgaged and would not be entirely his until he had completed his installments, which had 12 years to run. Well, let's see what else did he own. He had a car which would be paid for in another year. He also boasted of a number of other possessions, including a piano, a radio, a vacuum cleaner, and an electric washing machine—all of them in various stages of installment purchase.
>
> The banker jotted down the different items to figure the cost each month and said, "It is costing you all but $40 a month of what you earn. You don't want to borrow money. You want to stop buying things you can't afford."

The other side of installment buying may be seen by studying a report of Professor Seligman of Columbia University.

Professor Seligman is one of America's leading economists and he has made an intensive study of installment buying. His findings lead him to conclude that while this method of purchasing may be more expensive, on the whole, it has worked for the betterment of industrial conditions. He maintained that it stabilized industry because there was more purchasing done than if cash payments were always demanded.

To appreciate the cost of buying on this plan, it is necessary to take a problem and work it to see the actual rate charged.

1. A radio sold for \$130 cash, or on the installment plan, \$30 cash and 8 monthly payments of \$15 each. What rate of interest is charged on the loan?

It is seen that the difference between the cash price and the installment price is \$20. *This may be considered as the interest charged by the company for granting the privilege of partial payments. Since the installment price is* \$150, *of which* \$30 *is paid at the time of purchase,* \$120 *may be considered as a loan made to the purchaser by the company. The monthly payments may be considered as a means of reducing this loan until it is completely paid back. Suppose the purchaser buys the radio March* 1, *at which time* \$30 *in cash is paid. At intervals of* 1 *month,* \$15 *will be paid for* 8 *consecutive months.*

Installment of Apr. 1 *repays a loan of* \$15 *for* 1 *mo.*
Installment of May 1 *repays a loan of* \$15 *for* 2 *mo.*
Installment of June 1 *repays a loan of* \$15 *for* 3 *mo.*
Installment of July 1 *repays a loan of* \$15 *for* 4 *mo.*
Installment of Aug. 1 *repays a loan of* \$15 *for* 5 *mo.*
Installment of Sept. 1 *repays a loan of* \$15 *for* 6 *mo.*
Installment of Oct. 1 *repays a loan of* \$15 *for* 7 *mo.*
Installment of Nov. 1 *repays a loan of* \$15 *for* 8 *mo.*
36 *mo.*

(*Explanation continued on opposite page.*)

> *From this it is seen that the different installments have been equivalent to a loan of* \$15 *for* 36 *months. This period of time,* 36 *months, is equal to* 3 *years. The interest charged was* \$20 *for* 3 *years, or* \$6.67 *for* 1 *year. Hence for a loan of* \$15 *an interest charge of* \$6.67 *was made. Then the rate may be found by comparing* \$6.67 *with* \$15, *or* 44.46%.

2. A piano sold for \$1200 cash, or \$200 cash and \$100 a month for 12 months. Find the rate of interest charged for the accommodation.

> *The installments were* \$100 *each; one a loan for one month; the next a loan for* 2 *months; and so on for* 12 *months.* $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 = 78$. *Hence the accommodation was equivalent to a loan of* \$100 *for* 78 *months, or* $6\frac{1}{2}$ *years. The interest charged was* \$200, *the difference between the cash price and the installment price. Hence the interest charged per year was* $\$200 \div 6\frac{1}{2}$, *or* \$30.77. *This was the interest charged on a loan of* \$100. *Then* $\$30.77 \div \$100 = .3077 = 30.77\%$, *the rate of interest that the accommodation cost.*

3. Find the rate of interest charged for purchasing a saxophone on the installment plan. The cash price is \$75, or \$5 down and \$5 weekly for 18 weeks.

4. A gas range sells for \$163 cash, or on the installment plan for \$15 cash and 9 monthly payments of \$20 each. What rate of interest is charged for installment buying?

5. A radio sells for \$120 cash, or for \$135 on the installment plan. On the installment basis \$30 is paid when the purchase is made and 7 monthly payments of \$15 each. What rate of interest is charged?

6. A set of books sells for \$7.85 cash, or \$1 down and \$2 a month for 4 months. What rate of interest is charged?

7. A washing machine sells for \$155 cash, or \$25 in cash and 14 monthly payments of \$10 each. What rate of interest is charged?

8. A popular car has a cash selling price of \$540. If bought on the installment plan, $\frac{1}{3}$ of the cash price must be paid at the time of purchase. The remaining payment of \$360, plus an extra charge of \$48, is to be made in 12 monthly payments. What is the rate of the interest charged for the privilege of buying on this plan?

NOTE.—The financing company requires the buyer to keep the car insured against fire and theft, but this is not part of the cost of the car.

9. A 6-cylinder sedan has a cash price of \$1485. If bought on the 10 payment plan, it has a selling price of \$1614, $\frac{1}{3}$ of which must be paid at the time of purchasing. Of the extra \$129 in the cost price, \$14 is for fire and theft insurance and the remaining \$115 is for financing charge. Find the rate of interest charged by the financing company.

10. A furniture store advertised a table for \$16.50 cash, or for \$18.50 on the easy payment plan. On this plan the purchaser paid \$2.50 down and \$4 a month until the price of the table was paid. Find the rate of interest charged.

11. A large mail order house advertised a table for \$39.85 cash, or for \$43.50 on the easy payment plan. On this plan the purchaser paid \$3.50 down and \$5 a month until the price of the table was paid. Find the rate of interest charged.

12. This same mail order house advertised a radio for \$50 cash, or \$55 on the easy payment plan. On this plan the purchaser paid \$7 down and \$8 a month until the full payment was made. Find the rate of interest charged.

13. An automobile dealer sold a car for \$1225 cash. The partial payment plan was 40% cash and \$68 plus the balance

in two equal payments, one in 4 months and the other in 8 months. Find the rate of interest charged.

14. Assuming that not over $10,000,000,000 was spent for the purchase of articles on the installment plan during the year 1929, and that $\frac{1}{4}$ of this amount was paid in cash and the remaining amount was paid in monthly installments, what would be the interest charge on this amount at a 20% rate for a period of 6 months?

15. How much greater would the interest charge be in Ex. 14 than if the money had been borrowed from savings banks at 6%?

POINTERS ON THRIFT AND INVESTMENTS

1. *Form the habit of saving a part of your earnings when you are young.*

2. *Remember the code of thrift that was drawn up by the organization which provides for National Thrift Week.*

3. *The ideal of investment is the greatest rate of interest where the safety of the principal is assured.*

4. *If abnormally high dividends or extremely large profits are promised, watch out.*

5. *If bond or stock salesmen are going to "let you in on the ground floor," be very careful before you do any buying.*

6. *For a wage earner working on a salary basis, the building and loan associations offer a very splendid means of saving and accumulating money at a good rate of interest.*

7. *First mortgages are good investments. They pay a good rate of interest and have but a small degree of risk.*

8. *Buy bonds rather than stocks for an investment, as they are safer and the income is regular.*

9. *United States Government, state, and municipal bonds are among the safest types of investment.*

10. *If you buy stocks or bonds, it is advisable not to invest all your money in any one type. Get a list from your banker of several bonds or stocks that are reliable and buy a few of each.*

11. *Remember it is always cheaper to pay cash for an article, rather than to buy it on the installment basis.*

12. *If you have a sum of money and have not decided what form of investment to make, remember that a savings bank represents a standard form of investment.*

The summary of pointers given below in regard to investment was taken from the October, 1927, issue of the *American Magazine.*

Here Are Some Rules That May Save Your Money

IF YOU want to get rich surely, do not try to get rich quickly. An unusually high rate of return is a warning of danger.

Be sure that what you *want to believe* coincides with the facts.

Buy through an absolutely responsible, honest, and competent business house.

Never buy from a friend *merely because he is a friend.*

Remember that the best invention ever patented never built a successful business all by itself.

Do not pick a security merely because *one* factor looks favorable.

Good investments will await investigation. Don't go too fast.

Tips, rumors, and "hunches" dig the graves of millions of easy-go dollars every year.

Do not sell a good security the moment the price drops a little; it may go up again.

CHAPTER VIII

THE MEANING AND NEED OF INSURANCE

You have heard people speak of **carrying insurance** on their property or on their health or life. **Insurance** is an agreement by an **insurance company,** for a consideration called a **premium,** to compensate the insured party for actual losses or damages arising from certain causes. The agreement or contract is called the **policy.** The sum of money specified in the policy to be paid in case of loss is called the **face of the policy.**

The three most common forms of property insurance are **fire, theft,** and **liability.**

A **fire insurance** policy is an agreement to compensate or indemnify the insured against **actual** losses arising from **accidental** fires. The loss by fire includes any damage resulting from chemicals or water used in extinguishing the fire. Fire caused by lightning is usually included under accidental fires.

Theft insurance is that type of insurance in which the company will compensate the insured for a loss of property due to theft. Many people have their automobiles, jewelry, and other valuables insured against theft.

Liability insurance is that type of insurance in which the company will assume the responsibility up to a fixed sum for the damage done by the insured to other persons or to their property. Many people who drive cars carry this form of insurance in order to protect themselves in case the automobile they are driving injures some person or property.

WHY INSURANCE IS NECESSARY

All of us in our daily lives take a great many **chances.** If we own an automobile, there is the chance that it will be stolen or that it will injure someone. If we own a house, there is a chance that it may be destroyed by fire. Each of us runs the chance of death, or of loss of earnings because of sickness. And in each of these cases there is the chance that we may have large financial loss in case any of these disasters come to us.

Insurance averages these losses up among a great many different persons so that each one pays a small part of the loss in his premium. Hence the premium that one pays for any kind of insurance depends on the **chance** that a particular loss will occur.

In many parts of the country, particularly in farming districts, coöperative fire insurance companies are formed. Then the premium each member pays is based on the actual fire losses which occur. Thus, if the members insure property worth \$200,000, and if fire losses during the year have amounted to \$2,000, each one is assessed \$10 on each \$1000 worth of property he has insured as his premium for the year.

Thus, insurance of every kind is based on the following principles:

1. The fact that there exist forms of known danger to which all of a group of persons are equally exposed.

2. That loss from this danger is not likely to fall on all exposed to it.

3. That when the loss does occur, it is better for each of a large group to pay a small part of it, rather than have the loss fall on a single individual.

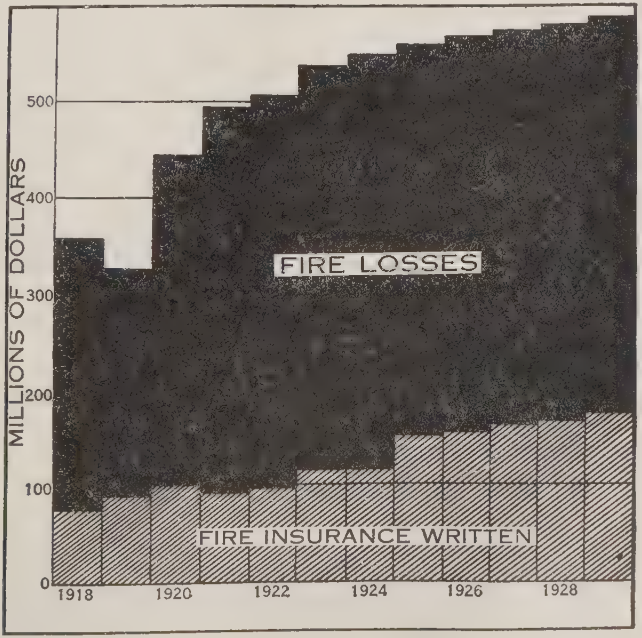
In order to determine how much the premium must be, one must have a fairly accurate knowledge of the amount of loss that may be expected in a given year. This knowledge is the result of statistics in the past and it enables companies to calculate the cost with a reasonable degree of certainty.

THE NEED OF FIRE PREVENTION

Although fire insurance is taken by an individual to help protect him from loss due to a fire, it is necessary to take every precaution that fires do not occur. Thus, one should observe the suggestions made by fire prevention agencies:

1. *Do not allow any fire hazard about the home.*
2. *Do not allow waste and rubbish to accumulate.*
3. *Be careful how you dispose of lighted matches.*
4. *Remember that the cheapest form of protection is prevention.*

ANNUAL FIRE LOSSES IN THE UNITED STATES



The above graph shows the property loss. The greatest

loss, however, is in human lives. Each year about 15,000 people lose their lives by fire.

Exercises

1. From the graph, has the amount of insurance that is written increased as rapidly as has the loss from fire?

2. From 1919 to 1929, what year showed the greatest increase in the loss due to fire?

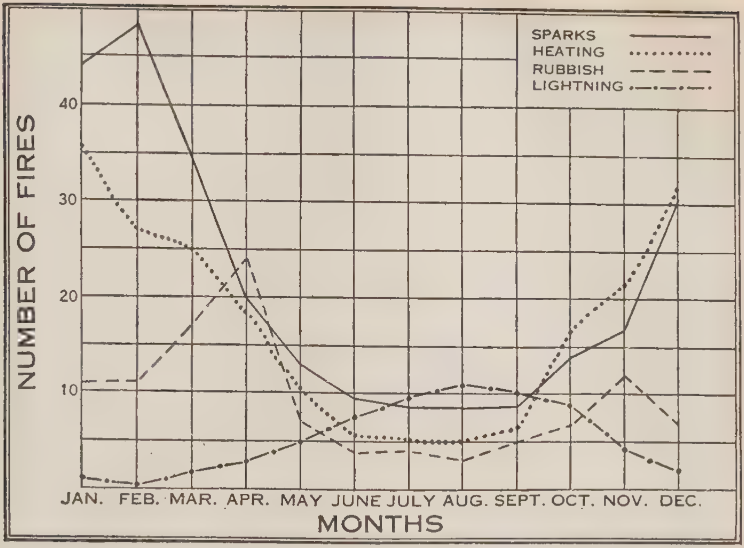
3. About how much more insurance was written in 1929 than in 1918?

4. About how much greater was the loss in 1929 than in 1918?

The **rate of premium** for fire insurance varies with conditions and is usually stated as a specified sum on each $100 of the face of the policy. Fire insurance is usually issued for periods of from one to three years. The premium paid is determined by the actual losses incurred over a period of years. In order to get cheaper rates, the hazards and losses must be reduced. Mr. W. E. Mallalieu, General Manager of the National Board of Fire Underwriters, in an address before a Convention of the National Editorial Association, June 7, 1918, in speaking of the causes of fires stated: "There is one cause of fire which concerns almost every individual in the United States. To state it in a sentence, dozens of recognized fire causes are practically reducible to one great fire cause, *American Carelessness*." Then the cost of fire insurance can be reduced only as the losses become less. The best way to do this is to avoid undue carelessness.

FIRE HAZARD AT PEAK IN WINTER

The greatest number of fires occur in the winter months. The following graph shows the causes and the number of fires which occur throughout the year.



1. What is the cause of the greatest number of fires during the summer months? During the winter months?

2. What is the cause of the greatest number of fires throughout the entire year?

Exercises

1. For which do you think an insurance company would charge the larger premium: a \$5000 policy on a house in a town with no fire protection, or the same amount on a house in a city having good protection?

2. For which would a company charge more in the same city: a policy on a modern fireproof building, or one of the same size on a wooden building with a shingle roof?

3. Which do you think should pay the higher rate of premium on his policy: the owner of a frame house, or the owner of a frame factory?

4. Which would cost more, to insure a building in a neighborhood of brick or stone buildings, or the same building if it were surrounded by frame buildings?

5. Here is a table of rates per $100 for a certain city. See if you can give a reason for the different rates.

Construction	1 Yr.	3 Yr.	Construction	1 Yr.	3 Yr.
Brick House	$0.18	$0.45	Frame Flat	$0.75	$1.875
Brick Flat	.234	.585	Brick Garage	.27	.675
Frame House	.50	1.25	Frame Garage	.75	1.875

Using the table in Problem 5, answer:

6. Why is the rate higher on a brick flat than on a brick house?

7. Why is it higher on a frame house than on a brick house?

8. Why is it higher on a garage than on a house?

9. Find the cost to insure a brick house for $8000 for 3 years.

10. Find the cost to insure a frame house for the same time and amount.

11. Find the cost to insure a frame flat for $12,000 for 1 year.

12. Find the cost to insure a frame garage for $4000 for 3 years.

The premium rates upon a building depend upon:

1. **The location;**
2. **The nature of the construction;**
3. **The use made of the building; and**
4. **The construction and the use of adjoining buildings.**

13. Some towns are divided into zones and the rates vary in each zone according to their distance from a fire department station. If you can, visit some local agent and get from him the rates where you live.

14. Rates are usually quoted as so many cents, or dollars and cents per \$100. 24¢ per \$100 is what rate per cent?

15. If the rate on a 3-year policy in a certain city is \$1.62 per \$100, what is the rate per cent?

16. Show two ways of finding the premium on a 3-year policy of \$6000 when the rate is 96¢ per \$100.

17. For 15 years a man has kept his house insured at \$5000 by taking out 1-year policies at 48¢ per \$100. He could have taken out 3-year policies for $2\frac{1}{2}$ times the rate on a 1-year policy. How much could he have saved in 15 years by taking out the longer policies?

18. Mr. Reed pays \$1.70 per \$100 on a \$5000 policy on a summer cottage in the country, and but 36¢ per \$100 on a \$5000 policy on his home in town. How much does he pay on each, and how do you account for the great difference in the rate?

19. A man has his property insured for \$2000 in one company and \$3000 in another. In case of a \$4000 loss, how much will he collect from each company?

20. On account of the fire protection in most cities, a total loss of property by fire is unusual. For this reason a man often carries insurance to protect but partially the full value. Find the premium at 24¢ per \$100 on a policy covering but 80% of property valued at \$12,000. In case of a loss of \$12,000 how much would the policyholder receive? How much in case of a loss of \$6000? A loss of \$500?

21. In some states, if a man agrees, by accepting a certain clause in the policy, to carry a certain amount of insurance and fails to do so, he can collect (in case of loss) but such a part of it as the face of the policy bears to the amount agreed upon. Under such a contract, if a man agrees to carry \$5000 and carries but \$3000, what part of any loss up to \$3000 can he collect?

22. Some policies contain an 80% co-insurance clause,

which is an agreement to carry 80% of the value of the property. How much insurance would a man have to carry under such a contract if his property is worth $12,000?

23. A man having property worth $10,000 insures it for $6000. If there is an 80% co-insurance clause in the contract, how much does he thus agree to carry? What part of any loss up to $6000 can he collect?

24. If a man accepts an 80% co-insurance clause in the contract, should the rate be higher or lower than in an ordinary policy? Give reasons for your answer.

25. The cost of an ordinary policy on a certain building is 45¢ per $100 for three years, or 41¢ per $100 for three years on an 80% co-insurance clause. Find the difference in the amount paid in premiums in 15 years in the two plans if the value of the property is $12,000 and it is insured at 80% of its full value.

26. If the house in Ex. 25 is insured for $8000 and at the end of 15 years there is a fire that causes damage of $10,000, which policy would have been the cheaper and by what amount?

27. Some business organizations take out a *blanket policy.* This means that there is a certain amount of insurance to cover the building and the contents. This saves the bookkeeping involved in the enumeration of all of the contents of the building. A school building and its equipment is worth $150,000 and it is covered by a blanket policy of 60% valuation. What will be the amount that can be collected, if a fire destroys property to the value of $25,000?

28. A man living in a small city paid $10 for a 3-yr. term policy on his furniture, valued at $1000. Express in mills the rate per dollar that was charged for each year.

29. A man had his house insured for $7500 at the rate of 60¢ per $100 for 3 years. Three years later he had a fire that

caused damage of $1500. What was the average yearly cost for each $100 of settlement?

AUTOMOBILE INSURANCE

Many people who drive and own cars have them insured against fire and theft. Insurance against damage done to the property or persons of other people is known as **liability** insurance. In the United States there is one car for every six people and the number is rapidly increasing. Thus, traffic is becoming more congested all the time, and the chances for an accident are greater than they were when there were not so many cars. For that reason it is better to take out liability insurance than to run unprotected the risk of an accident. The person who owns a car may be held responsible for accidents caused by his car and the injured person may sue the owner for damages which may total many thousands of dollars. If the car owner is insured, the company will pay damages up to the amount of the insurance.

Many of the standard liability insurance policies have limits of $5000 for one individual, and $10,000 for two or more people who may be injured. Then there is a property damage limit of $1000. The rates for insurance vary in different places. In the following two tables, *W* is for a low priced car, *X* is for a medium priced car, and *Y* is for a more expensive car.

RATES FOR NEW YORK CITY FOR ONE YEAR

Symbol	Public Liability	Property Damage
W	$ 87	$24
X	107	30
Y	136	38

RATES FOR PLACES OTHER THAN NEW YORK CITY

Symbol	Public Liability	Property Damage
W	$26	$11
X	32	13
Y	40	16

Fire and theft insurance varies with the kind of car and also with the age of the car. The rates per $100 for two popular cars are given below.

Fire Rates

		Age Groups			
A popular low priced car:		Up To 6 Mo.	6-18 Mo.	18-30 Mo.	Over 30 Mo.
The Oranges, N.J.	Per $100	$0.35	$0.45	$0.50	$0.65
New York City	Per 100	.60	.80	1.00	1.20
A popular moderate priced car:					
The Oranges, N.J.	Per $100	$0.15	$0.20	$0.25	$0.35
New York City	Per 100	.30	.40	.50	.60

Theft Rates

A popular low priced car:		
The Oranges, N.J.	Per $100	$0.80 any age
Newark, Jersey City	Per 100	1.55 any age
New York City	Per 100	3.40 any age
A popular moderate priced car:		
The Oranges, N.J.	Per $100	$0.55 any age
Newark, Jersey City	Per 100	1.05 any age
New York City	Per 100	2.35 any age

In figuring the value of a car, a general rule is to allow 85% of the original price the first year, 60% of it the second year, 40% of it the third year, and 25% of it the fourth year. This is not always true, as there are many factors that enter into the value of a car which might change this percentage.

Exercises

1. Why do you think that liability insurance in New York City costs more than in a smaller city?

2. The premiums given in the table are for "$5,000-$10,000" liability. The premiums for liability policies double this amount will be 20% more than the rate given in the table. Find the premium for double liability in New York City and elsewhere for a car in each classification.

3. Mr. Jones owns a car that is in the *Y* group. How much more would it cost for 3 years insurance on a $5,000-$10,000 liability policy in New York City than for the same policy outside of that city?

4. Mr. Brown, who lives in New York, has a car that is in the *X* group. He paid his insurance premiums for 3 years on a $5,000-$10,000 liability policy and then had an accident which cost the company $2,500. The cost of the accident is how much more than the premiums paid?

5. Using the data in Ex. 4, how much would Mr. Brown have to deposit at 4% compounded annually to amount to $2,500 in three years? (Use the table on page 234.)

6. Mr. Smith, who lived in Newark, N. J., had a car classified in the *W* group. After he had paid premiums for property damage for 6 years, he had an accident which caused damage of $250. Were the premiums paid more or less than the damage caused?

7. Using the data in Ex. 6, if he had carried a $5,000-$10,000 liability policy in connection with the property damage policy, would he still be saving money by carrying the policies?

8. Even though premiums are paid for many years and no accident occurs, what benefits has the insured man paid for and received?

9. Why do you think that the rates for fire and theft on a low priced car should be greater than those for a moderate priced car?

10. Mr. White owns a car two years old in the low priced class. If the car cost $650 when new, find the cost of fire and theft insurance necessary to cover it for 60% of the purchase price in each of the localities given.

11. Mr. Brown's car is a moderate priced car. If it cost $1,750 new, find the cost of fire and theft insurance in each locality for the first year. Assume that it will be assessed at 85% of the purchase price.

12. A man's insurance premium on his car for fire amounted to $36. If he had a short circuit that caused a fire loss of $500, the loss would represent the interest on how much money invested at 6%?

13. A car equipped with 4-wheel brakes has a cheaper liability rate than a car with 2-wheel brakes. Do you know of any reason why this should be true?

14. A man bought a 6-cylinder car for $1,800 and kept it for 4 years. He had it insured in New York City against theft for the amounts and rates given in the table. How much did it cost him each year for insurance?

15. If the same man had the car insured against fire for the amounts and rates as given in the table, how much did it cost him each year for insurance?

16. Why is it that the rate per $100 for fire insurance increases each year?

17. Why should theft insurance cost so much more than fire insurance?

18. During the year 1930 the insurance companies found that it was necessary to increase the cost of liability insurance. What may have been some of the reasons for this increase in rate?

LIFE INSURANCE

A life insurance policy provides, in return for the payment of a fixed premium by a man during his life, that at his death his dependents will be paid a certain sum of money. Thus, in

a certain kind of life insurance, if a man pays $27.32 a year throughout his life, the insurance company will pay his dependents $1,000 when he dies. So you see that life insurance provides a certain estate for a man even though he dies before he can accumulate one.

A class of high school seniors was asked to write out the answer to this question:

"A company insured a married man's life for $10,000 and 2 months after the first premium of $273.20 was paid the man died. How could the company afford to pay the widow $10,000?"

One student gave this answer: "The company could not afford to pay the $10,000, but out of sympathy for the widow it paid the amount."

Do you think that this answer is correct?

THE NET COST OF INSURANCE

If it is known that, of a group of 1000 men aged 25, 8 will die during the next year, then it is possible to find how much each of the 1000 men must pay at the beginning of the year to provide, at the end of the year, $1000 for each of the 8 who die. $8000 will be necessary to meet the payments at the end of the year. From interest tables it is found that, at 4% interest, $.9615 will amount to $1 in a year. Hence the sum necessary is 8000 × $.9615, or $7692. Hence each of the men must pay $7692 ÷ 1000 = $7.69. This amount is called the **net premium.** To get the actual premium charged by the companies, there must be added to this a share of the expenses of the company.

Thus you see that, in finding the cost of an insurance, it is necessary to know how many, out of a large group of men, will die during each year. These figures are obtained from a table called a **mortality table.** The mortality table shown on the next page was made through the experience

AMERICAN EXPERIENCE TABLE OF MORTALITY

Completed Age	Number Surviving at Each Year	Deaths in Each Year	Death Rate per 1000	Average Future Lifetime* Years	Completed Age	Number Surviving at Each Year	Deaths in Each Year	Death Rate per 1000	Average Future Lifetime Years
10	100,000	749	7.490	48.7	**53**	66,797	1,091	16.333	18.8
11	99,251	746	7.516	48.1	**54**	65,706	1,143	17.396	18.1
12	98,505	743	7.543	47.5	**55**	64,563	1,199	18.571	17.4
13	97,762	740	7.569	46.8	**56**	63,364	1,260	19.885	16.7
14	97,022	737	7.596	46.2	**57**	62,104	1,325	21.335	16.1
15	96,285	735	7.634	45.5	**58**	60,779	1,394	22.936	15.4
16	95,550	732	7.661	44.9	**59**	59.385	1,468	24.720	14.7
17	94,818	729	7.688	44.2	**60**	57,917	1,546	26.693	14.1
18	94,089	727	7.727	43.5	**61**	56,371	1,628	28.880	13.5
19	93,362	725	7.765	42.9	**62**	54,743	1,713	31.292	12.9
20	92,637	723	7.805	42.2	**63**	53,030	1,800	33.943	12.3
21	91,914	722	7.855	41.5	**64**	51,230	1,889	36.873	11.7
22	91,192	721	7.906	40.9	**65**	49,341	1,980	40.129	11.1
23	90,471	720	7.958	40.2	**66**	47,361	2,070	43.707	10.5
24	89,751	719	8.011	39.5	**67**	45,291	2,158	47.647	10.0
25	89,032	718	8.065	38.8	**68**	43,133	2,243	52.002	9.5
26	88,314	718	8.130	38.1	**69**	40,890	2,321	56.762	9.0
27	87,596	718	8.197	37.4	**70**	38,569	2,391	61.993	8.5
28	86,878	718	8.264	36.7	**71**	36,178	2,448	67.665	8.0
29	86,160	719	8.345	36.0	**72**	33,730	2,487	73.733	7.6
30	85,411	720	8.427	35.3	**73**	31,243	2,505	80.178	7.1
31	84,721	721	8.510	34.6	**74**	28,738	2,501	87.028	6.7
32	84,000	723	8.607	33.9	**75**	26,237	2,476	94.371	6.3
33	83,277	726	8.718	33.2	**76**	23,761	2,431	102.311	5.9
34	82,551	729	8.831	32.5	**77**	21,330	2,369	111.064	5.5
35	81,822	732	8.946	31.8	**78**	18,961	2,291	120.817	5.1
36	81,090	737	9.089	31.1	**79**	16,670	2,196	131.734	4.7
37	80,353	742	9.234	30.4	**80**	14,474	2,091	144.466	4.4
38	79,611	749	9.408	29.6	**81**	12,383	1,964	158.605	4.1
39	78,862	756	9.586	28.9	**82**	10,419	1,816	174.297	3.7
40	78,106	765	9.794	28.2	**83**	8,603	1,648	191.561	3.4
41	77,341	774	10.008	27.5	**84**	6,955	1,470	211.359	3.1
42	76,567	785	10.252	26.7	**85**	5,485	1,292	235.552	2.8
43	75,782	797	10.517	26.0	**86**	4,193	1,114	265.681	2.5
44	74,985	812	10.829	25.3	**87**	3,079	933	303.020	2.2
45	74,173	828	11.163	24.5	**88**	2,146	744	346.692	1.9
46	73,345	848	11.562	23.8	**89**	1,402	555	395.863	1.7
47	72,497	870	12.000	23.1	**90**	847	385	454.545	1.4
48	71,627	896	12.509	22.4	**91**	462	246	532.466	1.2
49	70,731	927	13.106	21.6	**92**	216	137	634.259	1.0
50	69,804	962	13.781	20.9	**93**	79	58	734.177	.8
51	68,842	1,001	14.541	20.2	**94**	21	18	857.143	.6
52	67,841	1,044	15.389	19.5	**95**	3	3	1000.000	.5

*Average Future Lifetime is sometimes called "Expectation of Life."

of many American companies in writing life insurance. Hence it is called the **American Experience Table of Mortality.**

Exercises

1. If an insurance company has 12,500 policies in which the age of the insured is 35, how many of these policy holders may be expected to die within 1 year?

Note.—From the table it is seen that the death rate is 8.95 per 1,000 population. Then for 12,500, it will be 12.5×8.95. The answer should be rounded off to the nearest whole number.

2. If an insurance company has 8,640 policies in which the age of the insured is 25, how many of the insured may be expected to die within 1 year?

3. What per cent of the people who are 20 years of age may be expected to reach the age of 60?

4. What per cent of the people who are 50 years of age may be expected to reach the age of 70?

5. What per cent of the people who are 25 years of age reach the age of 80?

6. Do you now understand how it is possible for an insurance company to predict the number of mortalities that may be expected in any given year?

7. If a man began saving at 30 years of age, how much would he need to save at 4% compounded annually until 55 to provide his family an income of $1,800 per year from the interest at 6% on the savings?

8. A man, age 30, has a wife and three children. He is able to save yearly about $450. This would carry about $20,000 worth of life insurance. In case he should die, he would leave no estate for his family. If he carried the insurance, what would be the yearly income from the amount of the insurance if it were invested at 5%?

9. If this man instead of carrying insurance should invest $450 yearly in a savings bank paying 4%, how much would he have at the end of 10 years? At the end of 20 years?

In finding the net cost of insurance it is necessary also to use a table showing how much money must be deposited today to amount to a given sum in a certain number of years.

PRINCIPAL WHICH WILL AMOUNT TO $1
AT 4% IN A CERTAIN NUMBER OF YEARS

Years	Principal	Years	Principal	Years	Principal	Years	Principal
1	$0.9615	11	$0.6496	21	$0.4388	31	$0.2965
2	.9246	12	.6246	22	.4220	32	.2851
3	.8890	13	.6006	23	.4057	33	.2741
4	.8548	14	.5775	24	.3901	34	.2636
5	.8219	15	.5553	25	.3751	35	.2534
6	.7903	16	.5339	26	.3607	36	.2437
7	.7599	17	.5134	27	.3468	37	.2343
8	.7307	18	.4936	28	.3335	38	.2253
9	.7026	19	.4746	29	.3207	39	.2166
10	.6756	20	.4564	30	.3083	40	.2083

Exercises

1. Using the table above, how much must be deposited now to amount to $10,000 in 38 years?

$$\begin{array}{r} \$0.2253 \\ 10{,}000 \\ \hline \$\ \ 2253 \end{array}$$

From the table we see that $.2253 *will amount to* $1 *in* 38 *years. Hence* 10,000 × $.2253 = $2253. *This principal at* 4% *compounded annually will amount to* $10,000.

Compound interest at 4% *is assumed in these problems.*

2. A man wishes to deposit an amount which will provide $10,000 for his son's education in 18 years. How much must he deposit?

3. A man's will provides: "Out of my estate I direct that a sufficient sum be set aside to provide my son with $45,000 when he reaches the age of 35." If the son is 18 when his father dies, how much must be set aside to provide for the bequest?

4. How much money must be deposited to provide for $1,000 to each of 7,915 men in 35 years?

5. How much must be deposited now to pay three debts, each of $1,000, due at the end of 25, 26, and 27 years, respectively?

6. Find the net cost of an insurance of $1,000 for 1 year at age 40.

$0.9615 765000 $735,547	*By referring to Table I you see that of* 78,106 *men living at age* 40, 765 *will die within a year. Hence* $1,000 *must be provided for each of these* 765 *men, or* $765,000 *in all.*
78,106)$735,547 $9.42	*By referring to Table II it is seen that* $.9615 *will amount to* $1 *in a year. Hence the sum to be deposited now to provide* $765,000 *a year from now will be* 765,000 × $0.9615 *or* $735,547. *Then each man must pay* $735,547 ÷ 78,106 *or* $9.42.

Find the net cost of an insurance of $1,000 *for* 1 *year at the following ages:*

7. 20	9. 30	11. 40	13. 50	15. 85
8. 25	10. 35	12. 45	14. 60	16. 95

17. Find the net cost of an insurance of $1,000 for 2 years at age 28.

$.9615 $\times$ 718,000 $690,357 $.9246 $\times$ 719,000 $664,787	*From Table I you see that of* 86,878 *men living at age* 28,718 *will die the first year and* 719 *the second year.* *Hence* $718,000 *will be due in* 1 *year and* $719,000 *in two years. The sum which must be deposited to produce these amounts is (from Table II)* 718,000 $\times$ $.9615 *and* 719,000 $\times$ $.9246, *or a total of* $1,355,144.
$690,357 664,787 86,878 | $1,355,144 $15.60	*Since this must be paid by* 86,878 *men, each pays* $1,355,144 $\div$ 86,878 *or* $15.60.

Hence $15.60, *answer.*

Find the net cost of an insurance of $1,000, *using the following data:*

18. Age 20, for 2 years.

19. Age 30, for 2 years.

20. Age 25, for 3 years.

21. Age 35, for 5 years.

THE KINDS OF POLICIES

There are four general forms of life insurance policies: (1) *ordinary life;* (2) *limited life;* (3) *endowment;* and (4) *term insurance.*

In the **ordinary life policy,** the premiums are paid, usually annually or semi-annually, during the life of the insured, and the insurance company agrees to pay a fixed sum to the heirs of the insured, or to some other party designated in the policy, at his death.

The person named to receive this sum is called the **beneficiary.**

In the **limited life policy,** the premiums are paid for a fixed number of years, after which the policy is called **paid up;** but the face of the policy is not paid the beneficiary until the death of the insured.

In the **endowment policy,** the premiums are paid for a fixed number of years, as ten, fifteen, or twenty, and the face of the policy is paid the insured at the end of the period. The insurance is then no longer in force.

NOTE.—In both the limited life and the endowment, the face of the policy is paid the beneficiary in case of death before the end of the period during which premiums are payable.

In **term insurance,** the premiums are paid for a fixed period and the face of the policy is paid the beneficiary in case of death during this period. At the end of the period the contract ceases.

All of the exercises on the cost of insurance that you have solved have been term insurance: after the number of years the policy ran, the contract expired. Thus the man had insurance for a limited time only. This is the cheapest form of insurance for the younger ages, but is not the best form of insurance for two reasons: (1) the cost of insurance increases each year and finally becomes prohibitive; (2) in order to continue this form of insurance a new physical examination is required at the expiration of each contract. Then if a man's health fails he may find that he cannot get more insurance.

EXERCISES

1. Would you expect an ordinary life policy or a twenty-payment life to cost more? Give a reason.

2. Would you expect a twenty-year endowment or a twenty-payment life policy to cost more? Give a reason.

3. Arrange the four kinds of policies in order of what you consider the rate of premium.

THE TABLE SHOWS THE PREMIUM CHARGED BY A LEADING LIFE INSURANCE COMPANY FOR A $1,000 POLICY. THE PROBLEMS THAT FOLLOW ARE BASED UPON THESE RATES

Age of Insured	Ordinary Life	20-Payment Life	20-Yr. Endowment
20	$18.01	$27.82	$47.67
25	20.14	30.12	48.15
30	22.85	32.87	48.83
35	26.35	36.22	49.85
40	30.94	40.38	51.48
45	37.08	45.73	54.22
50	45.45	52.87	58.81

4. How much per year will a $5,000 ordinary life policy cost a man who insures at the age of 25? How much a year will it cost him if he insures at the age of 40?

5. Find how much a $5,000 policy of each of the three types will cost a man taking insurance at the age of 35.

6. Suppose that a man 30 years of age, taking out a $15,000 policy, dies after making the 15th payment. His beneficiary will get $15,000 under any of the three policies named above. Show how much he would have paid out in each.

7. If a man of 30 takes out a 20-year endowment policy of $10,000 and lives 20 years, he will receive the face of the policy. How much less is this than the amount of the premiums if placed in a savings bank paying 4%?

8. If a man of 30 takes out an ordinary life policy of $10,000 and dies in 20 years (after making 20 payments), would his beneficiary get more or less than the amount of the premiums if placed in a savings bank paying 4%?

9. Make the same kind of comparison as in Problem 8, supposing that he died in 10 years.

10. Suppose a man of 50 should take a 20-payment life policy of $20,000 and die at the end of 20 years. Compare what the beneficiary would receive with the amount of the premiums placed in a savings bank paying 4%.

11. Make the same kind of comparison as in Problem 10, supposing the man to have been but 20 years of age when taking out the insurance and dying in 20 years.

12. At the beginning of 1927, the total value of the written life insurance in the United States was $80,000,000,000 from 108,429,000 policies. How much was that for each policy?

13. In 1906 the total amount of insurance for all companies in this country was $13,700,000,000. What was the per cent of increase from 1906 to 1927?

14. How do you account for the fact that there are almost as many policies in force as there are people in the country, when a large percentage of the people do not carry life insurance?

15. Of the $40,000,000,000 of government life insurance written during the World War, only $2,774,936,000 was still in force at a recent date. What per cent of the total amount that was written is still in force?

16. A man took out a life insurance policy at age 20 and paid premiums for 10 years. Then he wrote and asked the company to return the amount he had paid, since he enjoyed perfect health and the company had been to no expense through him. Discuss this.

17. Insurance is bought to protect a man's dependents against loss because of his death. Give two reasons why a man should not delay taking out life insurance.

18. A group of 100 young men determine to form their own life insurance company and to pay no premiums except on the death of a member, when each survivor would pay his share of the $1,000 insurance.

a. If all the members are 20 years of age, will the cost be very large at first?

b. Will men who cannot get into regular companies because of ill health try to join?

c. As the members become older, would the assessments increase much?

d. When the assessments became very large, would it be difficult to get healthy young men to join?

e. What would happen when the group consisted of relatively old men, many in poor health and without ability to earn money?

THE THREE ELEMENTS THAT MAKE UP THE PREMIUM

The **annual premium** paid by the insured is made up of three items: (1) **mortality cost**; (2) **reserve**; and (3) **expense loading.**

The **mortality cost** is the amount reckoned as necessary to collect each year to pay the death claims of that year. This is determined by mortality tables compiled from long experience, showing the deaths expected each year out of a certain number at any age.

The **expense loading** is the amount estimated as necessary to meet the expenses of the management of the company. It is usually about one-fifth or one-sixth of the total premium.

In the exercises on page 291, you have seen that as a man's age increases, the mortality cost of term insurance for a single year increases. The following table shows how rapidly this occurs:

COST OF $1,000 INSURANCE FOR ONE YEAR

Age	Cost	Age	Cost	Age	Cost	Age	Cost	Age	Cost
25	$7.75	30	$8.10	35	$8.60	60	$25.67	85	$226.49
26	7.82	31	8.18	40	9.42	65	38.59	90	437.06
27	7.88	32	8.27	45	10.73	70	59.61	95	961.54
28	7.95	33	8.38	50	13.25	75	90.74		
29	8.02	34	8.49	55	17.86	80	138.91		

The costs shown in this table are the actual **mortality costs** of the insurance. Generally, one pays for his insurance in equal annual installments, each of which is larger than the mortality cost of insurance at early ages and smaller than the mortality cost of insurance at later ages. The difference between the premium paid and the sum of the mortality and expense loading cost of insurance is called the **reserve.** It is a sort of savings bank account of the insured with the company, bearing interest at from 3% to 4%. It may be withdrawn (except for a small *surrender charge*) at any time by surrendering the policy and thus terminating the contract, and is thus called the **cash surrender value** of the policy.

RESERVE AND CASH SURRENDER VALUE OF AN ORDINARY LIFE POLICY

Age 25. Annual premium for $1,000 insurance is $20.14.

End of Year	Reserve	Cash Surrender Value	End of Year	Reserve	Cash Surrender Value
1	$ 8.20	....	9	$ 84.98	$ 78.98
2	16.41	$ 6.41	10	95.44	90.44
3	25.55	15.55	11	106.62	102.62
4	34.86	24.86	12	118.57	115.57
5	44.15	34.15	13	130.75	128.75
6	54.17	45.17	14	143.11	142.11
7	64.32	56.32	15	155.69	155.69
8	74.60	67.60			

Exercises

All rates refer to the table given on page 294.

1. From the table, what is the cash surrender value of a $5,000 policy at the end of 5 years?

2. What would have been the amount of this accumulation if the yearly premium had been deposited in a savings bank paying 4% interest compounded annually?

3. A man 40 years of age taking out a $10,000 ordinary life policy at the rates given on page 294 may surrender it at any time after 2 years and get the reserve or cash surrender value. At the end of 20 years, this cash value is $3,834.70. This is how much less than he has paid out? Why should it be less?

4. If a man of 30 takes out an ordinary life policy of $1,000, he may surrender it in 15 years and receive $276.02. This is how much more or less than he has paid out?

5. A man insuring for $1,000 at age 25 on the 20-payment plan may surrender it for $504.58 after having made the last payment. Compare this with the amount paid out. Explain the difference.

6. A man insuring for $10,000 at age 30 on the 20-year endowment plan may surrender his policy in 15 years and receive $6,748.50. Compare this with what he has paid out.

7. The cash surrender value, at the end of 10 years, of a $5,000 20-payment policy taken by a man of age 25 is $1,044.75. Compare this with the amount of the premiums at 4% compound interest.

8. A man of 40 took out a 20-payment life policy for $10,000 and at the end of 15 years he died. Compare the amount his beneficiary will receive with the amount the premium would have accumulated in the bank at 4%.

DIFFERENCE BETWEEN INSURANCE AND INVESTMENT

You have seen from the exercises that you have solved that insurance is not a form of investment. *Life insurance is a commodity purchased for the sake of the protection it gives.* Its investment merit is secondary and yet policies are sometimes taken out because of the opportunity they offer for systematic saving. Although it is not advisable to be without insurance, one should carry it to protect dependents, and not with the idea of making a good investment. Many people put all of

their savings into life insurance. It hardly seems wise to carry more insurance than is needed for protection, as the investment of the premiums in some other way such as you have studied would accumulate much more rapidly, except in case of premature death. *One should carry enough life insurance to protect his dependents in case of premature death.* Should an individual be able to save more than the yearly premium to carry this amount, it should be invested in some other form.

WHAT TYPE OF POLICY SHOULD BE TAKEN

There is no fixed rule by which one can decide for you the type of policy you should take. There is one outstanding thing which should be uppermost in the mind of the individual when he takes out an insurance policy in order to protect those depending upon him for support. That thing is, *the maximum amount of protection should be obtained for the money invested.* On that basis an ordinary life policy seems to be the best type.

You have learned how to determine some of the factors in the cost of insurance of the three common types: whole-life; 20-payment life; and 20-year endowment. Let us consider these three types for a young man about to take out $1,000 life insurance at age 25.

The average married young man has a small income and his dependent children need protection against his death more than they will at a later period. So he wants the most protection he can receive at the cheapest possible cost.

You have seen that while term insurance is cheap at first, there are arguments against its purchase. In an ordinary life policy the insured pays for the insurance throughout his life; the table on page 288 shows that a man aged 25 years may expect to live, on the average, about 39 years. Hence the cost of the life insurance is spread over 39 equal annual payments.

In a 20-payment life policy, the cost of the insurance is paid in 20 equal annual installments.

In a 20-year endowment policy, the man pays for two things: (1) he must pay the cost of insurance for 20 years; (2) he must pay enough to accumulate $1,000 at 3% or 3½% interest.

Hence it is seen that the cheapest form is the ordinary life policy. By taking that form of insurance, the man can get the greatest amount of protection while he needs it most, and at the least cost when he has least money. Later on, when his children have grown up and he has saved enough to provide an income for himself and wife, he may decide he no longer needs insurance. He may then surrender his policy for its cash surrender value.

GROUP LIFE INSURANCE

Group life insurance is offered to a group of 50 or more people who are working at a common trade or profession. It is a form of term insurance and the premium for each individual depends upon the age of the insured at the time he enters the group.

A factor that may cause the premiums to vary is the type of work that the insured is doing. In extra hazardous work, such as steeple painting, the rates would be higher than for a group of clerks in a large department store. The table below gives the cost of a policy for a group of teachers.

In some states employers of labor are required by law to provide insurance of a certain amount for their employees. Group insurance is a satisfactory way in which to meet this obligation. The amount of the premium may be deducted from the yearly earnings of the worker, or it may be considered part of the overhead expense, and thus paid by the employer. In most cases, however, the employees carry the insurance independently of the employer.

GROUP LIFE INSURANCE

ONE YEAR RENEWABLE TERM

Attained Age of Insured Nearest Birthday	Premium per $1,000	Attained Age of Insured Nearest Birthday	Premium per $1,000	Attained Age of Insured Nearest Birthday	Premium per $1,000
15	$5.39	35	$ 6.76	55	$19.87
16	5.47	36	6.92	56	21.47
17	5.57	37	7.11	57	23.20
18	5.65	38	7.32	58	25.08
19	5.76	39	7.56	59	27.12
20	5.87	40	7.85	60	29.39
21	5.97	41	8.18	61	31.82
22	6.08	42	8.58	62	34.45
23	6.14	43	8.99	63	37.33
24	6.21	44	9.49	64	40.44
25	6.27	45	10.02	65	43.83
26	6.31	46	10.62	66	47.47
27	6.35	47	11.30	67	51.45
28	6.38	48	12.04	68	55.72
29	6.40	49	12.88	69	60.35
30	6.43	50	13.78	70	65.34
31	6.45	51	14.78		
32	6.48	52	15.89		
33	6.56	53	17.09		
34	6.65	54	18.43		

Exercises

1. What will it cost a man 25 years of age to carry a $5,000 policy in group insurance for 1 year?

2. Using the result in Ex. 1, how much cheaper is that policy than a $5,000 ordinary life?

3. If the insured were 50 years of age, what would be the difference in the premium for that year, using the two policies given in Ex. 2?

4. A manufacturer employed 650 men and insured each man for $2,500. He found that the average yearly premium

for each employee was the same as that for an individual 42 years of age. Find the total yearly cost to carry the insurance for the group.

5. How much more expensive per $1,000 is it for a man 40 years of age to carry a 20-payment life policy than to carry a group policy for 20 years?

TERM INSURANCE

Term insurance, as the name implies, means that one who holds a policy of this type is covered by insurance for a fixed term of years, usually from 1 to 5 years, and then the policy or contract terminates. The insured may renew it or transfer it to a policy of one of the types you have studied, but very probably the need of the insurance, which caused the individual to take out a term policy, is no longer felt. Some definite need of the insured has been met. He no longer considers the insurance necessary.

What was the need? He may have undertaken an enterprise hazardous either for himself or for his company. Suppose that he was the head of an engineering organization with contracts to erect a large bridge. This man's personal direction seemed necessary to assure success. But he might die within the year from natural causes or might lose his life in the construction work. Any one of a dozen accidents would imperil either himself or his company. Thus the individual himself, or his company, takes out a term policy for the period of time required to do the construction.

PART PLAYED BY TERM INSURANCE

Term insurance is likely to have a permanent place in the big business man's insurance schedule. He can buy it much cheaper than other and more enduring forms of insurance. Sometimes he even buys it as part of his financing. If

$1,000,000 is needed in his business, he can buy a term policy for that amount and it will form a basis for the collateral with which to obtain a loan.

OTHER TYPES OF INSURANCE

The types of insurance you have just studied are the ones that are most familiar, because a large part of the insurance business is made up of policies of this description. But these do not constitute all of the field of insurance. One large insurance company advertises in this manner, "*We insure anything.*"

Managers of large athletic events, such as baseball games, prize fights, and track events have insurance to guarantee there will be no deficit in case the weather is too inclement for the event to take place.

Farmers find it very essential in some parts of the country to carry insurance on their crops. Often a hail-storm or cyclone visits certain sections, and should the farmer not have his crops insured, he might suffer a total loss for the year's labor and expense in planting the crops.

Marine insurance is concerned with the insurance of goods shipped on water. The rates vary with the kind of goods and the destination.

It is quite common for people to insure themselves against accident, sickness, or disease. If one who carries such a policy is taken sick and cannot work, the company will pay him a certain amount each week of the illness. Often people insure just part of their bodies against accident. One company advertised having insured for $1,000,000 the voice of a famous singer.

Another type of insurance is that against property damage from a passing airplane. Should a passing plane drop on a building that is insured against such an accident, the company will be liable for the damage incurred to the building, provided

it does not exceed the face of the policy. The companies charge 50¢ per $1000, provided the building is located at a distance of half a mile or more from the nearest flying field. If the building is within a radius of a half mile of the flying field, the rate is $2.50 per $1000.

From these facts it is readily seen that wherever there is a great risk involved, it is advisable to pay a small amount to get protection against that risk. Companies can afford to take that risk because of the large number of policies which they have. It is on that basis that the companies are perfectly safe in assuming the risk, but it is hardly an act of wisdom for the individual to assume the same risk.

CHAPTER IX

BANKS AND BUSINESS FORMS

Do you know what a man means when he says, "I will pay you by check"? Why is it that all people cannot give a check to pay a bill? What is the advantage of having money in a checking account, rather than carrying it in cash? Can you tell the difference between commercial banks, national banks, and savings banks?

These questions pertain to banks and banking, and will be answered in this chapter.

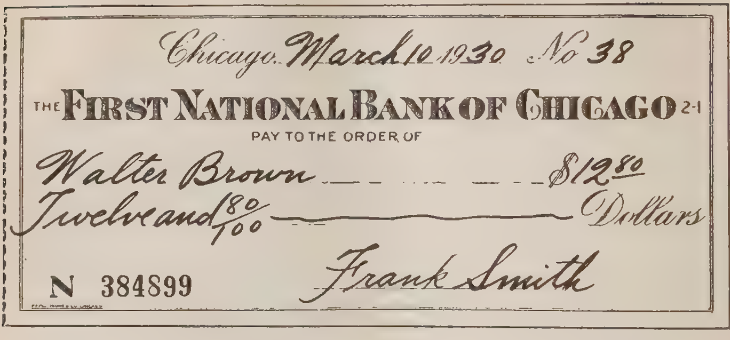
There are two kinds of banks: the **commercial bank** and the **savings bank.** Many banking institutions have both a commercial and a savings department.

The officials of some neighborhood bank of each kind may be glad to have your class visit their banks and learn of them some of the services they render the community.

THE WORKING OF A COMMERCIAL BANK

Commercial banks are institutions where money is deposited for safekeeping and is paid out on the order of the depositor. This order is called a **check. Drafts** may also be bought from banks to send in payment of a debt instead of sending the actual money. Banks also collect debts by means of drafts, and they loan money on security. The modern bank is essential to the commercial life of the country.

A COMMON FORM OF CHECK


Chicago March 10 1930 No 38

THE FIRST NATIONAL BANK OF CHICAGO 2-1

PAY TO THE ORDER OF

Walter Brown $12.80

Twelve and 80/100 Dollars

N 384899 Frank Smith

EXERCISES

1. Who is the maker of this order or check? Who, then, has money deposited in The First National Bank?

2. To whom is the money to be paid?

Before the bank will pay the money to Mr. Brown he must **indorse** it by writing his name, as it appears in the check, across the back of it. The words "to the order of" makes the check **negotiable.** That is, Mr. Brown may indorse it and get the money from his own bank or from any one who knows him and is willing to take it. The bank or person receiving it will then indorse it and collect the money.

When a bank cashes a check, it is marked "paid" and finally it is returned to the maker. He should keep it as a receipt to show that the money has been paid the person to whom it was given.

3. R. L. Brown has money deposited in The Merchants Bank of Chicago. He wishes to pay his gas bill of $9.60 to the Public Service Gas Co. Write the proper form of check and show the indorsement necessary before cashing the check.

Each check book has a **stub** or form for entering the frequent balances, new deposits, amount of the checks given, to whom

and for what given, and the balance still in the bank. These should be filled out carefully when a check is drawn so that the drawee may have an accurate record of all money paid out and the amount that he still has remaining in the bank.

This is a common form that is used in many check books.

No. 237 $14.75
May 3 1930
To Chas. E. Walls
For lawnmower

	DOLLARS	CENTS
BAL. BRO'T FORWARD	312	46
AM'T. DEPOSITED		
TOTAL	312	46
AM'T THIS CHECK	14	75
BAL. CAR'D. FORWARD	297	71

DEPOSITED FOR
Henry C Jones
NAME
331 No. Walnut St.
ADDRESS
IN THE
ESSEX COUNTY TRUST COMPANY
OF EAST ORANGE, N. J.
July 17 1930
PLEASE LIST EACH CHECK SEPARATELY

	DOLLARS	CENTS
BILLS	128	
SPECIE	3	42
CHECKS (When in City, state name of Bank; out of City, name of place where payable.)		
	103	
	24	38
	17	83
TOTAL $	276	63

SEE THAT ALL CHECKS AND DRAFTS ARE ENDORSED

Before it is possible to write out a valid check on a bank, there must have been deposited in that bank an amount equal to or greater than the amount of the check. Whenever you deposit any money in the bank, you should make out a deposit slip like the one given above, at the right.

Find the credit balance of the following:

1. Balance brought forward, $241.36; deposits, $137.50, $95.80, $150. Checks drawn, $12.80, $125, $83.90, $8.75.

2. Balance brought forward, \$216.80; deposits, \$197.60, \$75, \$135.50. Checks drawn, \$150, \$24.80, \$1.75, \$50.

3. Balance brought forward, \$297.80; deposits, \$200, \$125, \$93.75. Checks drawn, \$12.80, \$24.60, \$150, \$19.38.

4. Balance brought forward, \$93.84; deposits, \$175, \$100, \$65.70. Checks drawn, \$86.30, \$54.85, \$7.32, \$10.50.

5. Balance brought forward, \$398.46; deposits, \$150, \$110.50. Checks drawn, \$75, \$115.45, \$72.60, \$210.25.

FACTS ABOUT CHECKS

1. *Always fill out a check with pen and ink. When pencil is used it is easy for some one to change the amount of the check.*

2. *Never alter or erase an error made in filling out a check. Destroy it and make out a new one. If you do not, very probably the bank will reject the check when it comes in for payment.*

3. *The person to whom a check is made out must indorse the check before the bank will pay it. He may indorse it by signing his name across the back.*

4. *A person should not indorse a check until he is ready to cash it, for if it should be lost, any one finding it can cash it by adding his indorsement.*

5. *Payment of a check may be stopped by notifying the bank upon which it is drawn that you do not wish the check honored. This is known as protesting the check.*

6. *If your name is not spelled correctly or if a wrong initial is used on a check, first indorse it as it appears on the check and then rewrite your name correctly.*

7. *It is unlawful to write a check upon a bank in which you have no money.*

HOW BANKS EARN MONEY: BANK DISCOUNT

A bank does not charge a depositor for taking care of his money and paying it out on his order, unless his average

monthly balance falls below a fixed sum (usually from \$50 to \$200). But, for its services, it loans a large part of the deposits and has the interest on these loans.

Banks usually loan their money for short periods, ranging from a few days up to 3 or 6 months, and collect their interest in advance.

Interest collected in advance is called **bank discount.**

Banks demand some kind of assurance that the money loaned will be repaid when due. Sometimes the borrower submits a form showing his financial standing and is allowed to borrow up to a certain amount, depending upon his financial standing; and sometimes he secures the bank by turning over certain **collateral,** as stocks or bonds, which the bank may sell to repay itself if the drawer of the note does not pay it when due. When the note is paid, the collateral is returned.

On page 310 is a statement made by The Chase National Bank. The questions below are based on it.

Exercises

1. What is meant by *resources?* By *liabilities?*

2. What is meant by *capital?*

3. If one share of stock has a par value of \$100, how many shares of stock has the bank issued?

4. What is meant by *undivided profits?*

5. Since the surplus is equal to the value of the capital stock, would you think the stock would sell above or below par?

6. If the bank gets 6% on "Loans and Discounts," what is the daily income from this source?

7. If the U. S. Government Securities pay 4½%, what is the daily income from this source?

8. The deposits constitute what per cent of the total liabilities of this bank?

THE CHASE NATIONAL BANK

of the City of New York

PINE STREET CORNER OF NASSAU

STATEMENT of CONDITION

AT CLOSE OF BUSINESS MARCH 27, 1930

RESOURCES

CASH AND DUE FROM BANKS		$360,558,483.41
LOANS AND DISCOUNTS		800,609,535.09
U. S. GOVERNMENT SECURITIES		175,530,971.98
OTHER SECURITIES		45,614,888.56
REAL ESTATE		21,513,053.17
REDEMPTION FUND—U. S. TREASURER		386,825.00
CUSTOMERS' ACCEPTANCE LIABILITY	$86,600,626.51	
LESS AMOUNT IN PORTFOLIO	6,250,648.01	80,349,978.50
OTHER ASSETS		419,584.57
		$1,484,983,320.28

LIABILITIES

CAPITAL	$105,000,000.00	
SURPLUS	105,000,000.00	
UNDIVIDED PROFITS	33,589,271.25	$243,589,271.25
RESERVED FOR TAXES, INTEREST, ETC.		2,220,637.80
DIVIDEND PAYABLE APRIL 1, 1930		3,937,500.00
DEPOSITS		1,106,677,736.52
CIRCULATING NOTES		7,710,000.00
ACCEPTANCES	$87,697,972.67	
LESS AMOUNT IN PORTFOLIO	6,250,648.01	81,447,324.66
ACCEPTANCES, BILLS, ETC., SOLD WITH ENDORSEMENT		39,370,484.16
OTHER LIABILITIES		30,365.89
		$1,484,983,320.28

9. If the dividends are payable quarterly, what is the rate paid on the stock?

10. Mr. Jones borrowed $2500 at the bank for 90 days at 6%. Find the interest and proceeds, if the interest is paid in advance.

$$\frac{\overset{3}{\cancel{90}}}{\underset{\underset{2}{\cancel{60}}}{\cancel{360}}} \times \frac{\cancel{6}}{100} \times \$2500 = \$37.50$$

$$\$2500 - \$37.50 = \$2462.50.$$

The interest is $37.50, called ***bank discount.*** *Since it is paid in advance, the borrower has left $2462.50, called the* ***proceeds.***

Find the bank discount and proceeds of:

11. $3600 for 120 da. at 6%.
12. $4500 for 80 da. at $5\frac{1}{2}$%.
13. $1850 for 25 da. at 6%.
14. $750 for 36 da. at 6%.
15. $6000 for 42 da. at 5%.
16. $8500 for 18 da. at 6%.
17. $1725 for 30 da. at 6%.
18. $2000 for 15 da. at $5\frac{1}{2}$%.
19. $5000 for 10 da. at 6%.
20. $8550 for 28 da. at 5%.
21. $1650 for 112 da. at 6%.

Bank discount is only interest paid in advance.

The **proceeds** is the difference between the face of the note and the bank discount.

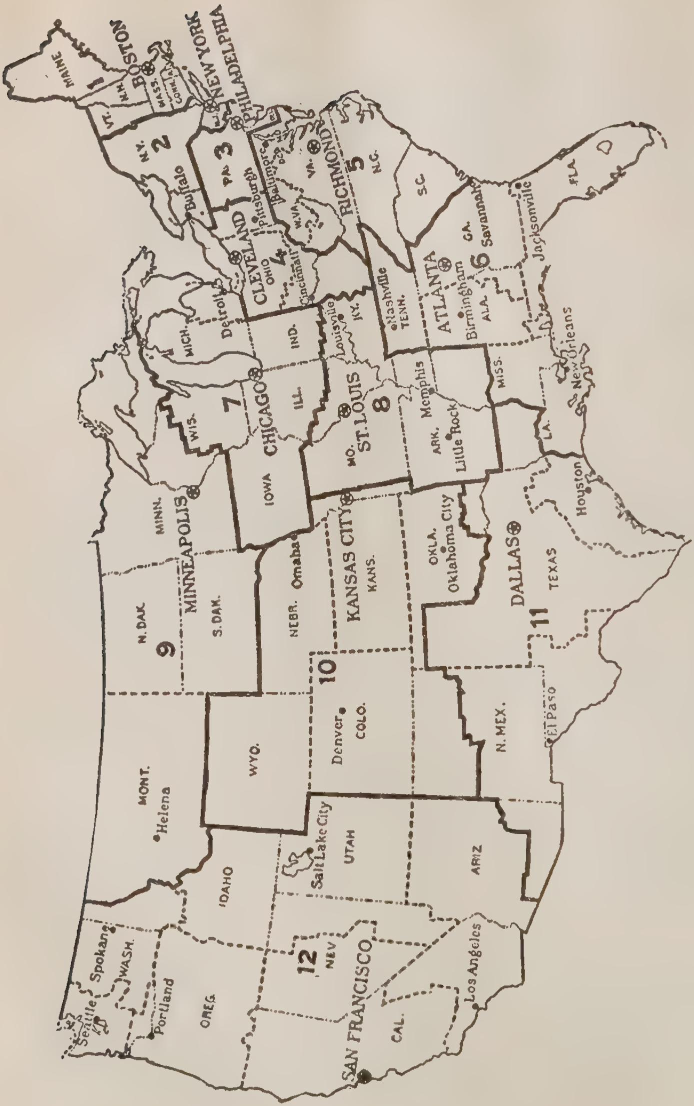
When the note is due, the maker of the note pays the face of the note only, for the interest has been paid.

The note given is a "promise to pay" the principal only, and no rate of interest is stated in the note.

Exercises

Find the bank discount at 6% and the proceeds of:

1. $500 for 30 da.
2. $750 for 60 da.
3. $980 for 30 da.
4. $765 for 90 da.
5. $1250 for 30 da.
6. $1575 for 60 da.
7. $1500 for 90 da.
8. $1650 for 45 da.
9. $1860 for 30 da.
10. $1780 for 90 da.
11. $1560 for 70 da.
12. $1350 for 20 da.



Location of Federal Reserve Banks
(The cities having the banks are starred)

For short periods (less than one year) 30 days are considered an interest month or $\frac{1}{12}$ of a year. Hence, at 6%, the interest is 1% for each 60 days. Thus, the interest of \$1350 at 6% for 60 days can be seen at sight to be \$13.50. For 30 days it would be half as much, or \$6.75.

At sight give the discount at 6% of:

13. \$1200 for 60 da.	**17.** \$1800 for 30 da.	**21.** \$1200 for 90 da.
14. \$1950 for 60 da.	**18.** \$2400 for 30 da.	**22.** \$1600 for 90 da.
15. \$2480 for 60 da.	**19.** \$1600 for 30 da.	**23.** \$2400 for 90 da.
16. \$1375 for 60 da.	**20.** \$1450 for 30 da.	**24.** \$3600 for 90 da.

THE FEDERAL RESERVE SYSTEM OF BANKS

There are twelve banks distributed in various parts of the United States which constitute the **Federal Reserve Banks.** These banks are for the convenience of the member banks located in their respective districts. No individual may make a deposit in one of these twelve banks. The depositors are the banks which are members of the Federal Reserve System. The purpose of this system of banks is to help the banks which are members of the Reserve System. If a member bank is in need of money, the federal bank will take notes issued by the member bank in order to tide over the financial difficulty. The Federal Reserve System was established to eliminate the possibility of a money panic such as was experienced in 1907.

There are about 30,000 banks in the United States. Of this number, about 8000 are national banks and the others are state or private banks. A **national bank** is a bank which is a member of the Federal Reserve System. These banks are given a charter to do business by the Secretary of the Treasury of the United States. Each national bank must keep 12% of its deposits in cash to meet the withdrawals

of its depositors. All national banks must become members of the Federal Reserve System, and state banks and trust companies may join. There are five distinct functions which a federal reserve bank performs. They are:

1. *To hold the member banks' reserve.*
2. *To lend to member banks in time of need.*
3. *To issue currency.*
4. *To collect checks.*
5. *To act as the government's agents.*

Any national bank may issue paper currency, provided that for each dollar of currency issued there is set aside a reserve in gold or acceptable security of a like amount. A gold reserve of not less than 40% of its outstanding currency must be provided.

An important function of the Federal Reserve banks is that they are check collection agencies for their member banks. Before the Federal Reserve System was begun in 1913, banks charged for collection of checks. The twelve Federal Reserve district banks have organized a most effective system of handling the checks of their members without cost. In 1925 these twelve banks handled a total of 779,000,000 checks with a total value of more than $225,000,000,000.

The Federal Reserve banks also act as agents for the government. The subtreasuries which once existed are closed. The Reserve banks supply currency and coin, and cash government checks. During the period of the World War, these banks handled the sale of all Liberty Bonds.

The map given on page 312 was taken from the *New York Times*, September 25, 1927. This map shows the cities in which the twelve Federal Reserve banks are located.

BILLS AND STATEMENTS

When goods are bought on a charge account at a retail store, the customer is sent a monthly bill by the store. This bill

shows the date and the amount of the purchases, the credits and the amount due.

ARNOLD, CONSTABLE & CO.
Fifth Avenue at Fortieth Street, New York

SOLD TO Mrs. J. J. Harris
67 North Mountain Ave
Montclair. N.J.

June 2, 1930

PURCHASES FOR THE MONTH OF MAY 1930

1930	MAY	ITEMS		CHARGES	CREDITS	BALANCE
	4	3 yds. silk $1.98	5.94			
		2 pr. hose $1.35	2.70	8.64		
	12	1 skirt $4.80	4.80			
		4 yd. gingham $.60	2.40	7.20		
	15	1 skirt ret'd			4.80	
		PAID		15.84		11.04
		June 10, 1930 Arnold, Constable & Co. R. F. T.				

EXERCISES

1. Check this bill to see if there are any errors in the computation.

2. Why is it advisable to keep a receipted bill?

3. What does the amount $11.04 in the balance column represent?

Using local names and recent dates, rule paper and make out bills for the following:

4. 4 pr. hose at 59¢; $3\frac{1}{2}$ yd. gingham at 72¢; $2\frac{3}{4}$ yd. lining at 60¢; 2 dresses at $13.50; one dress returned, $13.50.

5. 2 skirts at \$8.75; 3 waists at \$4.95; 4 yd. velvet at \$2.95; 3 pr. hose at 95¢; 2 waists returned at \$4.95.

6. 3 shirts at \$2.95; 2 ties at \$1.25; 4 pr. socks at 75¢; 1 pr. shoes at \$7.50; 1 hat at \$4.50; 1 suit at \$48; 2 shirts returned at \$2.95.

7. 6 table napkins at 75¢; 8 yd. of sheeting at 35¢; 1 dress at \$24.50; 2 hats at \$7.50; 5 radio tubes at \$1.75; 1 hat returned at \$7.50; 2 radio tubes returned at \$1.75.

HOW TO KEEP A RECORD OF EARNINGS AND EXPENDITURES

You can cultivate habits of thrift by being careful of the way you spend your money. One of the best ways to form good habits of spending is to keep an accurate cash account of all the money you receive and of all that you spend.

Exercises

1. A common form of cash account is shown below. The receipts are entered on the left-hand side and the expenditures

1930						
May	2	Cash on hand	18	12		
	2	Deposited in Savings Bank			10	00
	4	Bought knife				75
	5	" tie				80
	7	Rec'd for errand		75		
	7	Bought Book			1	25
	7	Contributed to athletic team			1	00
					13	80
					5	07
	9	Balance	18	87	18	87
May	9	Cash on hand	5	07		
	10	Earned	1	25		
	12	Bought gloves			1	75
	14	Hair cut				50
	14	Earned	1	50		
		Balance			?	??

on the right-hand side. Check the account to see if it is correct.

The *balance* is the excess of the amount received over the amount paid out.

Make out the cash accounts and balance them:

2. Receipts: May 1, cash on hand, \$2.10; allowance, \$1.50; May 8, errands, \$1.40; May 12, wages, \$3.25. Payments: May 2, baseball glove, \$2.25; May 8, tennis balls, \$1.20; May 15, movies, 30¢.

3. Receipts: June 15, cash on hand, \$3.40; June 17, wages for clerking in a store, \$3.25; June 20, mowing lawn, \$1.25; June 24, deliveries, \$1.15. Payments: June 16, club dues, 50¢; June 20, baseball, \$1.25; June 25, bathing suit, \$3.50.

4. Receipts: Sept. 4, cash on hand, 56¢; Sept. 8, mowing lawn, \$2.25; Sept. 12, sweeping school room, 75¢; Sept. 15, clerking in store, \$3.25. Payments: Sept. 6, battery recharged, \$1; Sept. 8, football, \$4.25; Sept. 15, class dues, 25¢.

5. Rule a form like that of Ex. 1 and keep your own personal account. Balance the account each week.

A CUSTOMER'S ACCOUNT AS KEPT BY A MERCHANT

On the following page is shown the form of ledger account between L. Jones & Son (merchants) and D. W. Booth, a customer:

1. Check the account given above and see if the computation is correct.

2. What does the balance on May 2 show? On June 1? Had there been a balance on the "Cr." side, what would it have shown?

3. What is meant by "Dr." and "Cr."?

4. Had Mr. Booth sold the merchants some produce, upon which side of the account would it have been placed?

D. W. Booth *Dr* *Cr.*

1930					1930				
Abr	3	2 aprons		98	Apr.	15	By check	75	00
		1 sweater	6	75					
	13	3 shirts	5	85					
		2 pr. hose		90					
		2 books	4	50					
	22	1 pr. shoes	9	45					
		1 suit	50						
					May	2	Balance	3	43
			78	43				78	43
May	2	Foward	3	43	May	11	By check	3	43
	8	2 pr. curtains	7	14					
		1 blouse	3	79					
	19	Camping outfit	19	39		23	By check	25	00
		Cot	3	95	June	1	Balance	9	27
			37	70				37	70

Rule forms for a merchant's personal account with his customers and balance the following:

5. Barr & Bros. in account with A. W. Smith. Sales: Sept. 5, furniture, \$285; Sept. 10, carpet, \$140; table, \$35; refrigerator, \$65; Sept. 15, gas range, \$56; electric washing machine, \$155. Credits: Sept. 5, cash, \$225; Sept. 10, cash, \$350.

6. Hoffman & Sons in account with M. C. Lewis. Sales: Dec. 2, groceries, \$5.57; Dec. 7, hardware, \$3.65; Dec. 10, toys, \$6.80; Dec. 12, groceries, \$8.22; Dec. 20, Christmas tree, \$2.50. Credits: Dec. 10, cash, \$7.50; Dec. 12, service rendered, \$6.25; Dec. 22, cash, \$15.

7. Birch Lumber Co. in account with C. A. Howard. Sales: June 3, lumber, \$12.80; June 5, paint, \$9.50; June 18, mason supplies, \$25.60. Credits: June 14, cash, \$25; June 20, paint returned, \$2.40; June 25, cement returned, \$4.15.

CHAPTER X

THE MEANING AND NEED OF TAXES

Do you know how the money is obtained to pay the expenses of the police and fire departments in your city and the expenses for the maintenance of your school?

Very likely you know that the money required to meet these expenses is raised by taxation. A **tax** is the money raised in some form to meet the expenses of the various units of government. The reason for all general taxation is that it is best to pool the effort and cost to provide and maintain those things which have come to be regarded as for the common welfare of all. When an individual is compelled to contribute his share towards the maintenance of these things, the amount that individual pays is called a tax.

Towns and **cities** must raise money to pay for fire and police protection, to build and maintain schools and other public buildings, to pay its officers, etc.

Townships and **counties** must meet the expenses of building roads and bridges, and of maintaining public institutions, courts, charities, etc.

The **state** has many salaried officials to pay, and helps build the roads of the state. It also keeps up certain state institutions, as prisons, schools, and asylums, all of which demand the expenditure of large sums of money.

The **United States Government** also requires large sums of money to meet its expenses. Among these are the salaries of the officials, the maintenance of the army and navy,

interest on the national debt, the pension of disabled soldiers, and many other items. The total government expense of 1916 was about $725,000,000; during our first year in the World War it rose to over $18,000,000,000. For the year 1930-31, the Budget Bureau of the federal government asked for $3,830,445,232.

HOW CITY, COUNTY, AND STATE EXPENSES ARE MET

Most of the expenses of towns, cities, counties, and states are met by a tax levied by the proper officers upon the **property** of the town, city, county, or state. The property is divided into two classes for taxation: (1) **real estate,** regarded as immovable property, as lands and buildings, mines, railroads, etc.; and (2) **personal property,** including all movable property, as money, stocks, bonds, furniture, live stock, etc.

Assessors, elected or appointed, estimate the value of the property to be taxed. This is called the **assessed valuation** of the property. From the total assessed valuation and the tax to be raised, the **tax rate** is determined.

In addition to a property tax, some states also have an **income tax** to meet the expenses of the state. This is a tax levied upon one's income.

A FORM OF TAX BILL

RATE $2.02

THIS BILL MUST BE RETURNED WHEN YOU PAY YOUR TAXES

Mr. *John Doe* — Page *123* Line *39*
No. *56 N. Walnut St.* — Map *3* Block *C* Lot No. *38*

Real Estate	Personal Property	Total Valuation	State, School, and County Tax	School Tax	Town Tax	Poll	Total Taxes
7200	900	8100	55 64	35 08	72 90		163 62

WAYS OF EXPRESSING THE TAX RATE

The tax rate of any district is usually expressed in one of three different ways: (1) as mills on each $1 of valuation, as 32.5 mills on $1; (2) as dollars and cents on each $100 of valuation, as $3.25 on $100; (3) as dollars and cents on each $1000 of valuation, as $32.50 on $1000. Since one mill is one-tenth of a cent, it is .1 × $.01, or $.001. Notice that mills are represented by three decimal places. The following will help to show the different ways of writing mills:

1 cent	= $.01	12 mills	= $.012
1 mill	= .001	25.5 mills	= .0255
5 mills	= .005	10 mills	= .01

Exercises

1. A tax rate of $2.96 per hundred is how many mills on the dollar?

2. A tax rate of $34.70 per $1000 is how many mills on the dollar?

3. A school tax of 12 mills on the dollar is how much per $1000?

4. A tax rate of $2.44 per $100 is how much per $1000?

5. A tax rate of $32.75 per $1000 is how many mills on the dollar?

The tax rate is found by dividing the tax levy, or the total amount of tax to be raised, by the total assessed value.

Exercises

1. A town with an assessed valuation of $6,500,000 needs $185,250 for the year's expenses. Find the tax rate.

The amount to be raised by taxation is \$185,250. *Think,* "? × \$6,500,000 = \$185,250."

To find this number, divide the amount to be raised by the assessed valuation.

\$185,250 ÷ \$6,500,000 = .0285.

Then the tax rate is $28\frac{1}{2}$ *mills on the dollar, or* \$2.85 *on the* \$100.

When the tax rate is known and the assessed value of the property is known, the amount of taxes an individual needs to pay is found by multiplying the assessed value of the property by the rate.

2. When the tax rate is $28\frac{1}{2}$ mills on the dollar, what will be the amount of the taxes paid by a man who owns a home assessed at \$8,500?

Think, "*The rate is* \$2.85 *per* \$100; *then for* \$8500, *it will be* 85 *times as much as for* \$100." 85 × \$2.85 = \$242.25, *the amount of taxes to be paid on the property.*

3. If the assessed valuation of the property of a village is \$6,500,000 and \$97,500 is to be raised, the tax rate is how many mills on the dollar? How many dollars per \$100? What per cent?

4. If the school tax of a town is 12 mills on the dollar, how much school tax must a man pay whose property is assessed at \$12,000?

5. When the town tax is $8\frac{1}{2}$ mills on the dollar, how much is the tax on property assessed at \$9500?

6. A man has property worth \$12,000, which is assessed at 75% of its value. He pays taxes at the rate of \$2.40 per \$100 of the assessed valuation. If the property were assessed

at full value, what would be the tax rate equivalent to his present rate?

7. A man's total tax at \$2.02 on \$100 was \$163.62. From this find at what value his property was assessed.

8. If taxes increase from \$1.65 to \$2.17 per \$100, how much will it increase one's tax whose property is valued at \$18,500?

9. Who is paying the highest rate, one who pays $10\frac{1}{2}$ mills on the dollar, \$1.01 per \$100, or 1.1%?

Give the rate per \$100:

	ASSESSED VALUATION	TAX TO BE RAISED		ASSESSED VALUATION	TAX TO BE RAISED
10.	\$ 4,800,000	\$ 36,000	**14.**	\$245,000,000	\$ 1,250,000
11.	16,500,000	288,750	**15.**	356,000,000	1,850,000
12.	51,000,000	750,000	**16.**	758,000,000	3,762,000
13.	89,000,000	763,000	**17.**	986,000,000	10,248,000

Give the tax on:

	ASSESSED VALUATION	TAX RATE		ASSESSED VALUATION	TAX RATE
18.	\$12,500	$4\frac{1}{2}$ mills on \$1	**22.**	\$ 6,780	\$12.25 per \$1000
19	9,750	$9\frac{1}{2}$ mills on \$1	**23.**	11,250	\$17.60 per \$1000
20.	10,500	\$1.65 per \$100	**24.**	17,750	$1\frac{1}{3}\%$
21.	13,750	\$1.08 per \$100	**25.**	16,350	\$2.02 per \$100

26. A town with an assessed valuation of \$80,000,000 spent \$20,000 one year to improve its parks. That one item had what effect upon the tax rate? (Answer in cents per \$100.)

27. In Problem 26, how much did a man whose property was assessed at \$4,000 have to pay toward the improvement of the parks?

28. A man has property worth \$12,000 which is assessed at 60% of its value. What will be his tax, if the rate is \$3.12 per \$100?

29. A man paid \$90 towards the support of schools in a town in which the school tax was 15 mills on the dollar. If his property was assessed at three-fourths of its value, what was the value of the property?

PERSONAL PROPERTY TAX

Personal property may be of two kinds: (1) **tangible property,** which includes furniture, farming implements, and other property of a like nature; and (2) **intangible property,** which includes stocks, bonds, and mortgages.

Most tax authorities are agreed that tangible property should be taxed at a much lower rate than real estate. Most tangible property depreciates very rapidly and does not have a permanent taxable value.

Most people agree that intangible property should not be taxed. There are two reasons for this belief. *First,* since stocks, bonds, and mortgages may be easily concealed, it is difficult for the taxing authorities to find and assess them. *Second,* a tax on intangible property generally means that such property is taxed twice. For example, a man may own property worth \$15,000 on which he has given a \$10,000 mortgage. He is, however, taxed on the full value of the property, although his *equity* in it is only \$5,000. If, in addition, the man who holds the \$10,000 mortgage is taxed on it, that part of the property pays a double tax.

Part of an editorial in the New York *Herald Tribune,* February 3, 1930, entitled "The Foolish Personalty Tax," which shows the view of the editorial staff of this newspaper, is given below:

> The personal income tax covers income from securities and loans as well as earnings. The direct tax on furniture, books, pictures, clothing, jewelry and household effects is a relic of the past, when such things bulked large in the possessions of a small well-to-do class. Now they have lost their old meaning as objects of luxury taxation. It is almost impossible to assess the sales value of such property and the assessors merely levy amounts at random on names taken out of the directories and then compel the victims to prove that the levies are unreasonable. This is a travesty on fair and intelligent taxation. Only the very conscientious or the very timid are caught in the dragnet. The others swear off or pay small fractions of the amounts originally charged against them.

Exercises

1. A man owns a $1,000 bond which yields $5\frac{1}{2}\%$ and is taxed at $3.25 per $100. What is the net yield?

2. A man owns $100,000 worth of common stock. What would be his yearly tax, if the rate were $3.12 per $100?

3. A farmer has live stock and farming implements valued at $3,000, assessed at half value. How much is his yearly tax on these things, if the rate is $2.86 per $100?

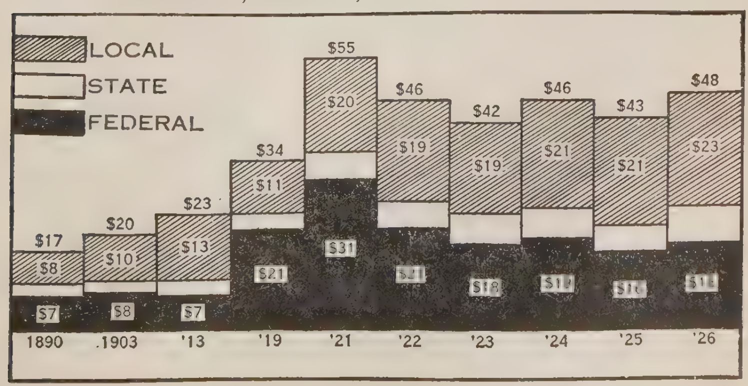
4. One state taxes intangibles at 7% of their value. What would have been the total tax in Ex. 2, if the stock had been taxed at this valuation?

5. Some tax authorities agree that intangibles should be taxed at a rate of 3 mills on a dollar of their full valuation. If a person owned a $1,000 bond paying $5\frac{1}{2}\%$, what would be the net return after paying the tax?

THE RISING COST OF GOVERNMENT

As the cost of government increases, so must the amount of taxes increase. The following graph taken from the *New York Times* for June 5, 1927, shows how local, state, and federal taxes have increased over a period of 36 years.

FEDERAL, STATE, AND LOCAL TAXES



Exercises

1. Why do you think the amounts are expressed in terms of 1913 dollars?

2. Find the per cent of increase in the amount of taxes paid per capita from 1913 to 1926 for local, state, and federal purposes.

3. In which one of the three departments of government has there been the greatest increase in the interval from 1913 to 1926?

4. From your study of the graph, would you say the burden is becoming lighter for the tax payer?

5. Make a graph to show the increased cost of government for each person gainfully employed. These amounts are not expressed in 1913 dollars.

1890	\$36.67	1923	\$242.15
1903	49.71	1924	257.53
1913	74.90	1925	259.24

6. It will be easier to understand why the cost of government has increased so rapidly if some of the expenditures for different purposes are considered over a period of years. These amounts are expressed in the nearest million dollars.

	1904	1915	1922
General government	29	65	114
Highways	37	66	111
Education	91	182	500

Show these relations graphically. In which of the three is there the greatest increase from 1904 to 1922?

7. What is the per cent of increase for each of the three items given?

HOW A CITY BUILDS A SCHOOL BUILDING

It would not be fair to tax people enough to pay for a school in a single year, for it will be used many years and those using it should help pay for it. The money which is needed to erect new school buildings is obtained by issuing bonds. These bonds usually mature in about 20 or 30 years. The bonds may be **sinking fund** or **serial bonds.** With **sinking fund bonds** enough money is placed in a fund each year so that, at the maturity of the bonds, there is enough money in the fund to pay or *retire* the bonds. **Serial bonds** are those which are issued in such a manner that a certain number will mature each year from the date of issue until at the final date of maturity they are all paid.

When a school bond, or any other municipal bond, is issued, the interest on the bond must be paid by part of the money collected from taxes. Not only must the interest on the bond be paid, but the bond itself must be paid when due. Since many school bonds are serial, it is necessary to pay the interest on the value of the outstanding bonds and also to pay part of the principal. This is known as amortization of the bond issue.

Exercises

(See advertisement on page 328)

1. Over how long a period does this issue run?

2. Is this a serial or a sinking fund issue?

3. How much must the City of Englewood pay in taxes each year to pay the interest on this issue?

4. What is meant by the expression, "Tax free in New Jersey"?

5. Do you think that these bonds represent a good investment?

Exempt from all Federal Income Taxes
Tax Free in the State of New Jersey

$1,016,000

City of Englewood, New Jersey

$4\frac{1}{2}\%$ Gold Bonds

Dated April 1, 1930 — Due serially April 1 as below

Principal and semi-annual interest (April 1 and October 1) payable in gold coin at the Chemical Bank and Trust Company, New York City. Coupon bonds in the denomination of $1,000, with the privilege of registration as to principal only or as to both principal and interest.

Legal Investment for Savings Banks and Trust Funds in New York and New Jersey

FINANCIAL STATEMENT
(as officially reported)

Assessed Valuation, 1929		$36,399,609
Total Debt (including this issue)		3,255,300
Less: Sinking Funds	$333,428	
Net Debt		2,921,872

Population, 1920 Census..........11,627
Population, 1930 Estimated.......18,000

These bonds issued for School and Improvement purposes, constitute direct and general obligations of the entire City of Englewood, payable from unlimited ad valorem taxes levied against all the taxable property therein.

AMOUNTS AND MATURITIES

$35,000	due	April	1,	1932 to 1948, inclusive
40,000	"	"	"	1949 " 1955, "
32,000	"	"	"	1956
20,000	"	"	"	1957 to 1961, inclusive
9,000	"	"	"	1962

PRICES

Maturities 1932-1935 to yield 4.15%

Maturities 1936-1962 to yield 4.20%

AMORTIZATION OF A BOND ISSUE

In order that the tax burden may be more uniform, it is necessary to pay off a certain amount of the principal each year besides paying the yearly interest. When bonds are issued, this must be taken into consideration.

If a sinking fund is created, enough money is paid into this fund each year to provide for the payment of the bonds at their maturity. Then the annual payment into the sinking fund is so calculated that, with the annual interest, the cost of the bonds is the same each year. Thus, if a school district has a $4\frac{1}{2}\%$ bond issue of \$100,000 for 20 years, \$5,000 a year for 20 years would provide the \$100,000 necessary to pay the bonds when they are due. However, if the school district gets $4\frac{1}{2}\%$ interest on the money deposited in its sinking fund, payments of only \$3,187.61 a year are necessary. Thus the total annual cost of the bond issue will be:

Interest at $4\frac{1}{2}\%$ on \$100,000	\$4,500.00
Sinking fund	3,187.61
	\$7,687.61

When serial bonds are issued, a certain number of the bonds are paid each year. Thus the interest charges get smaller year by year and with this saving more bonds can be retired.

Thus, in the bond issue above, if serial bonds are used, three bonds can be retired each of the first three years. Then sufficient interest will have been saved to make it possible to retire four bonds the next year. In 20 years, or at the date of the maturity of the bonds, there may be only seven \$1,000 bonds outstanding. That year the total payment will be \$7,000 to pay the bonds plus interest of \$315 on these bonds, a total of \$7,315.

A statement of this financing follows:

	BONDS OUTSTANDING	INTEREST	BONDS RETIRED	TOTAL PAYMENT
1st year	$100,000	$4,500	$3,000	$7,500
2nd year	97,000	4,365	3,000	7,365
3rd year	94,000	4,230	3,000	7,230
4th year	91,000	4,095	4,000	8,095
5th year	87,000	3,915	4,000	7,915
....	......	......	......	......
....	......	......	......	......
....	......	......	......	......
18th year	20,000	900	6,000	6,900
19th year	14,000	630	7,000	7,630
20th year	7,000	315	7,000	7,315

Notice that in the first year the interest was a very large part of the payment, while in the last year it is but a very small part.

INTERNAL REVENUE

Certain things that are made and sold in this country are taxed to help contribute to the operating expenses of the government. These taxes are called internal revenue taxes. The chief items in the list of taxable articles are cigars, cigarettes, and chewing and smoking tobacco.

In 1926 the internal revenue of our government amounted to over $850,000,000.

Exercises

1. During a recent year 6,953,552,000 cigars were taxed $4 per 1000. Find the amount of this tax.

2. In a recent year the tax paid on cigarettes was $3.50 per 1000. Find the tax on 84,957,206,000.

3. At 18 cents per pound, find the amount of tax paid on 376,176,881 lbs. of chewing and smoking tobacco.

4. The revenue from the sales of automobiles in 1926 was

$1,646,797. If the tax rate was 3%, find the value of the automobiles sold during the year.

INCOME TAX

The **income tax,** as the name suggests, is a tax upon incomes. This is a new form of raising money to support the government, having first been made a law in 1913. The rate has changed several times to meet new demands upon the government. The income tax is upon individuals and corporations. Since the beginning of this form of taxation the *personal* or *individual tax* has been divided into a **normal tax** and a **surtax.**

Under the income tax law of 1930, there is an exemption of $1500 for a single person and $3500 for a married person or the head of a family and $400 for each child. The **net income,** or the amount upon which one is taxed, is his total income less his exemption. The **normal rate** on the first $4000 on net incomes in excess of personal exemptions and other credits is $\frac{1}{2}$% of the amount; the second $4000 is 2%; the balance of net income is taxed 4%.

There is no surtax on incomes less than $10,000. From $10,000 to $14,000 it is 1%; from $14,000 to $16,000 it is 2%; and it increases 1% for each $2,000, up to $24,000. From $24,000 it increases 1% for each $4,000 of income until the income is $64,000. The maximum amount of surtax is 20% for incomes of $100,000 or over.

Exercises

1. Find the amount of income tax a man will pay, if he has a net income of $2,500 after personal exemptions and other credits.

2. Find the amount of income tax a married man without children will pay, if he has a gross income of $6,000.

3. How much more income tax will a single man pay than a married man without children on a gross income of $5,000?

4. How much income tax will a man pay who has a net income of $15,000?

5. Find the amount of income tax a man will pay who has a gross income of $25,000, if he is allowed $5,000 exemption.

The rates given above may be changed by the time you are studying this text. If you wish to find out the exact rates, go to your banker and he will advise you on the matter.

6. In the 1928 income tax reports, the following large incomes were shown:

ANNUAL INCOME	NUMBER OF PERSONS
$1,000,000—$1,500,000	241
1,500,000— 2,000,000	105
2,000,000— 3,000,000	89
3,000,000— 4,000,000	20
4,000,000— 5,000,000	17
Above 5,000,000	24

Make a distribution graph showing the number of persons having incomes as given in this table.

7. The average income for the group having an annual income of over $5,000,000, is $10,000,000. How much capital invested at 5% will give the annual income of $10,000,000 for this group?

TARIFFS, DUTIES, OR CUSTOMS

Some imported goods are not subject to duty. Such goods are said to be on the **free list.** The duties are of two kinds: (1) **ad valorem duty,** which is a per cent of the invoice price of goods at the place of purchase; and (2) **specific duty,** which is a certain amount per unit, as pound, ton, bushel, barrel, yard, etc. Some goods are subject to one duty and some to both.

The customs revenue is collected at **custom-houses** situated at the various ports of entry.

The tariff rates are frequently changed by Congress to meet new needs or to meet new theories of taxation. In 1909

we had rates established by a law known as the Payne-Aldrich Tariff Law; in 1913 the rates were greatly changed by the Underwood-Simmons Tariff Law; in 1922 we had a new law called the Fordney-McCumber Tariff Law; in 1930 the Smoot-Hawley Tariff Law was enacted.

Exercises

1. Under the act of 1922 the duty on woolen woven fabrics, as dress goods, was 45 cents per pound and 50% ad valorem on a certain grade. Find the duty on 10,000 lbs. valued at $18,500.

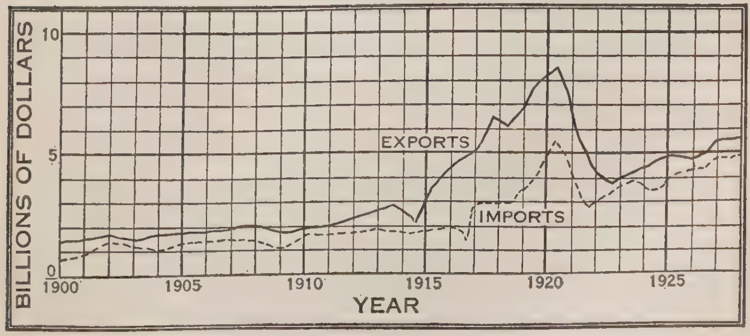
2. The duty on knit outerwear including sweaters was 45 cents a pound and 50%. Find the duty on a sweater weighing $1\frac{1}{4}$ lbs. valued at $3.80.

3. The duty on Axminster, Wilton, Brussels, and velvet rugs was 30%. A merchant imported $185,000 worth of rugs one year. How much duty did he have to pay?

4. Sewing silk, twist, and floss are subject to a duty of $1.50 per pound and 40%. Find the duty on 100 lbs. valued at $960.

5. The duty on china and porcelain, undecorated, is 60%; and, when decorated, 70%. Find the duty on an undecorated dinner set that cost the merchant $35. On a decorated dinner set costing the merchant $42.

OUR FOREIGN TRADE



6. The graph given on page 333 shows the value of our imports and exports. Approximately what was the value of our imports in the year 1925? Our exports?

7. In what years did our exports exceed our imports by the greatest amount? Do you see why this should be true for this particular period?

8. In a recent year the value of the "free list" in round numbers was $2,709,000,000. The value of the imports on the duty list was $1,467,000,000. The free list constituted what per cent of the total imports?

9. Using the data in Ex. 8, find the per cent of ad valorem duty charged, if the customs on the dutiable imports amounted to $551,850,000.

10. If in Ex. 8 there had been no free list, what would have been the per cent of ad valorem duty charged for the customs to have amounted to $551,850,000?

DIRECT AND INDIRECT TAXATION

The money that is paid for the support of the government is obtained in two different ways, by **direct taxation** and by **indirect taxation. Direct taxes** are taxes, like the income tax, paid directly to the government. **Indirect taxes** are taxes like the **tariff** and **internal revenue** taxes, which are paid by the company or corporation doing business and then shifted to the consumer. It makes little difference whether the tax is collected from the manufacturer, the dealer, or directly from the consumers. In each case the consumer is the one who pays the tax.

Exercises

1. If a house is taxed $360 yearly, how much of the monthly rent is represented by taxes?

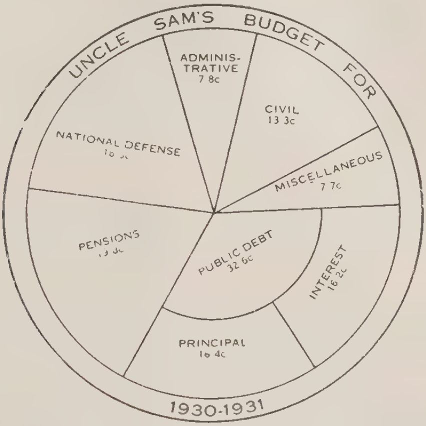
2. If an automobile has a factory price of $485 and there is a 3% tax on it, how much has the selling price been increased?

3. A silk kimono can be purchased in China for $18. If

80% duty must be paid on it and transportation charges are \$2.37, what is the price at which it must be sold to net the importer 20% profit on its cost? Who eventually pays the duty?

4. It is maintained that every individual who is a consumer pays an indirect tax of some kind. Discuss this.

WHERE UNCLE SAM GETS HIS MONEY



If we divide a Government dollar into its fractional parts, we get the following as the source of the dollar—given in cents and fractional parts of a cent.

WHERE IT COMES FROM

SOURCE	AMOUNT (CENTS)
Income and profits tax	49.16
Miscellaneous internal revenue	22.85
Customs revenue	14.43
Interest, premium, and discount	4.95
Fees, fines, penalties, and forfeitures	0.84
Repayments on investments	1.62
Trust fund receipts	2.16
Other miscellaneous receipts	3.99
	100.00

Exercises

1. What is the greatest source of income for the government?

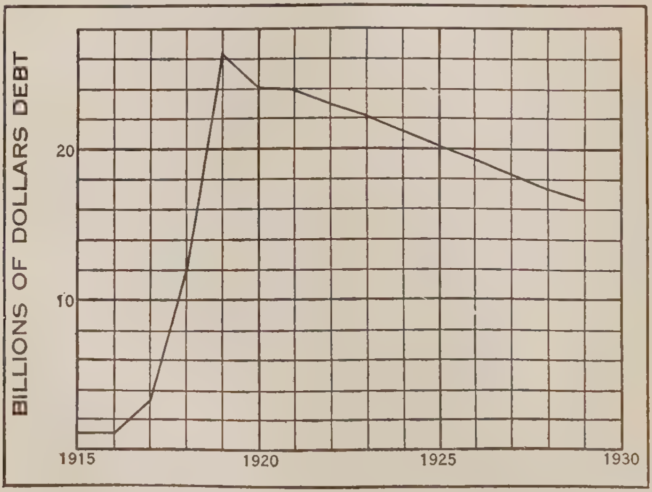
2. What is the greatest form of expenditure for the government?

3. How much of the dollar goes for national defense?

4. What has been the chief cause of the public debt?

5. President Coolidge pointed out that national defense expenditures for the year 1928 would tax the country $1,233 for every minute of the year. Find the amount of the expenditure for the year.

THE RISE AND DECLINE OF OUR NATIONAL DEBT



The graph given above appeared in the *New York Times*, July 7, 1929. It shows how the national debt has risen and declined.

6. In what year did the national debt reach its greatest amount?

7. Since 1921, the debt has been paid off at approximately what rate per year?

8. The debt in 1919 was approximately how many times what it was in 1917?

9. If the national debt is reduced at the same rate as it has been since 1922, about how long will it take to pay it off?

10. If the national debt in 1929 was $16,500,000,000, what will be the yearly interest charge for that amount at an average rate of $4\frac{1}{4}\%$?

11. Assuming the population of the United States to be 121,000,000, what was the amount of the national indebtedness per person in 1929?

12. If the government is able to reduce the debt $1,000,000,000 per year, what is the amount saved in interest charges at $4\frac{1}{2}\%$ each year?

THE COST OF GOVERNMENT IN A CITY

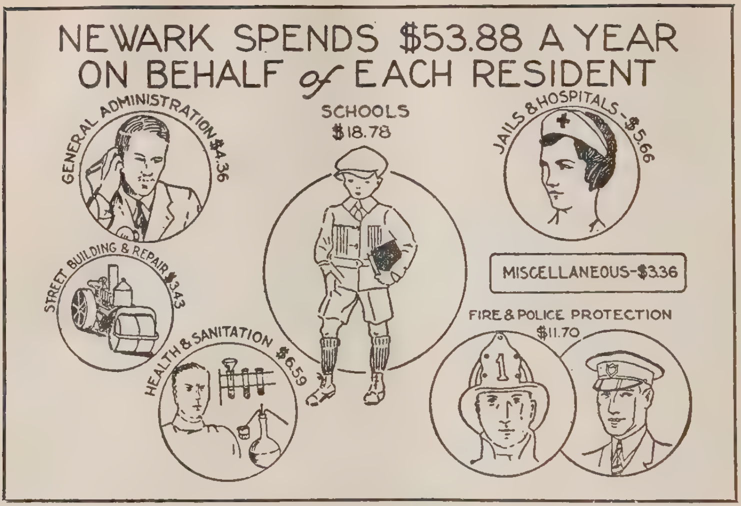
The cost of government in American cities represents the greatest part of the tax which a citizen in these cities pays. The part of the tax which the average citizen pays towards local government is much more than the part which goes towards the support of the federal government.

The graph on page 338 appeared in the *Newark Evening News*, July 5, 1929. It shows the cost of government per person in Newark, New Jersey.

Exercises

1. What item in the cost of government is the most costly?

2. Education represents what per cent of the total expenditure in Newark?



3. Assuming the population of Newark to be 500,000, what is the total amount raised by taxation each year?

4. What items are included in the group entitled "Health and Sanitation"?

5. If the tax rate in Newark is $3.94 per $100 of assessed valuation, what is the taxable wealth necessary to provide the amount given in Ex. 3?

6. What per cent is each item of the total cost of government.

7. In a recent survey of the cost of government in all cities of over 30,000 population, the percentage of costs for the different departments was as follows:

Education	38.9%
Fire and police protection	20.1%
Health and sanitation	9.8%
General administration	8.9%
Street building and repairs	8.7%
Jails and hospitals	6.3%
Miscellaneous	7.3%

How does Newark compare in its expenditures with the cities throughout the United States?

8. If you live in a city, find how the expenditures in your city compare with the table given in Ex. 7.

SPECIAL TAXES

All taxes are based on two things: first, the **ability to pay,** and second, **the benefit derived.** You have found that not all people pay income taxes because their incomes are not sufficient to justify being taxed. You have found that rich people pay more taxes than poor people do. In each case, this is true because of the ability to pay.

By **benefit derived** is meant the use or the benefit derived from the tax the individual will pay. If a new street is laid in front of a property owner's house, the benefit is going to be greater to this property owner than if the street had been constructed in another part of the town. For that reason, the property owner is made to pay a special tax to help defray the expenses of constructing the street.

One very common form of special tax is a gasoline tax. This is justified on the basis of benefit derived. The person who drives his car a greater number of miles pays a larger gasoline tax.

Exercises

1. If a man drove his car 8000 miles in one year and averaged 15 miles per gallon, what would be the amount of the gasoline tax he would pay at 4 cents per gal.?

2. If a man drove a car 12,000 miles in one year and averaged 15 miles per gallon, what would be the amount of the tax he would pay on the gasoline consumed at 6 cents a gal.?

3. In a recent year the consumption of gasoline in Ohio was 829,523,294 gallons. Find the amount of the tax if the rate is 4 cents a gal.

4. In California the tax in a recent year amounted to $29,566,800. How many gallons of gasoline were consumed in the state that year if the tax was 3 cents a gal.?

5. If a man, living in Virginia, should drive a truck 12,000 miles one year and average 6 miles per gallon, what would be the amount of the gasoline tax at 5 cents a gal.?

6. A man owns property with a frontage of 268 feet on a street. If property is assessed $3 a front foot for paving the street, what is the amount of his special tax?

THE MODEL TAX PLAN

In 1919 a commission composed of some of the leading tax authorities in this country recommended a tax plan for use in the United States. This is known as the **Model Tax Plan.** The recommendations of this committee may be summarized as follows:

1. *There should be a personal income tax to tax the net incomes of every one, above a small minimum exempt sum, regardless of the source from which the income is derived.*

2. *There should be a series of business taxes, levied on the net income of all business which is conducted within the state.*

3. *There should be a property tax on all tangible property, levied where the property is located, with a smaller rate on personal than on real property.*

4. *No attempt should be made to tax intangible property.*

Problems for Discussion

1. Why is it necessary to have taxes?

2. How will waste and extravagance in the city government affect the tax rate?

3. How does the tax rate in your city compare with the tax rate in another, nearby city? What may be some of the factors that could help to make a different rate?

4. Hundreds of new homes and apartment houses greatly increase the assessed valuation of a town. Will this tend to lower the tax rate? Why not?

5. If an individual pays rent, he may not need to pay taxes. The owner of the house would pay the taxes on the property. How would that affect the price of rent?

6. All large business corporations have to pay taxes. How does that affect the price of the article produced by the corporation?

7. Does the tax on imported goods affect the importer or the customer?

8. Is it just to charge the same rate of gasoline tax for a truck as for a touring car, if the truck will average but 6 miles per gallon and the touring car 18 miles? Why?

CHAPTER XI

REVIEW OF THE THINGS YOU HAVE LEARNED

This chapter may be used as a review of the work you have done during the year. No new material is presented, but many of the problems are different from the ones you solved in the chapters dealing with the subject at that place.

YOU HAVE LEARNED HOW TO SHOW RELATIONS BY GRAPHS

1. *Show by a bar-graph the following data:*

Comparison of the increase in the prices of commodities in 1930 over 1914.

	Increase		Increase
Rent	65.4%	Furniture	110.4%
Clothing	68.2%	Miscellaneous	103.3%
Fuel	80.7%	Combined costs of all.	74.8%

2. *Show by a bar-graph:*

According to data obtained by the United States Office of Education, out of every 1000 pupils who enter school, 634 reach the 8th grade, 342 enter high school, and 139 graduate from high school.

3. *Make a broken-line graph to show the variation in temperature in New York City:*

The mean temperature is:

January	31°	July	74°
February	31°	August	73°
March	38°	September	67°
April	49°	October	56°
May	61°	November	44°
June	69°	December	35°

4. *Make a broken-line graph to show the value of a car at the end of a given number of years:*

	% of Initial Cost		% of Initial Cost
First year	42%	Fifth year	18%
Second year	33%	Sixth year	16%
Third year	25%	Seventh year	14%
Fourth year	22%		

5. *Make two circular graphs to show the cost of living expenses for a family of* 6 *people with an income of between* \$2,100 *and* \$2,500 *in New York and in Chicago:*

	New York	Chicago
Food	\$845	\$816
Clothing	424	396
Rent	272	269
Other Expenses	552	701

6. *The following table shows how the annual income of a railroad is spent. The figures give the number of days' income necessary to pay for various items. Represent the information by a circular graph:*

	Days		Days
Labor	152	Taxes	23
Fuel	21	Rentals and interest	43
Supplies	65	Dividends	26
Other operating expenses	25		

7. *Make a distribution graph to show the uses of milk:*

DATA FOR THE YEAR 1927

Milk	45%	Cheese	4%
Butter	35%	Ice Cream	3%
Calves	5%	Waste	2%
Condensed	4%	All others	2%

8. *Make a distribution graph to show how a teacher's marks were distributed:*

A's	8%
B's	15%
C's	55%
D's	17%
F's	5%

9. *Make a circular graph to show how a ton of coal is used:*

Loss in the mines	600 lbs.
Needless transportation before it is consumed	95 lbs.
Not burned properly	446 lbs.
Loss of heat from furnace	102 lbs.
Loss from low efficiency of boilers	640 lbs.
Amount actually utilized	117 lbs.

YOU HAVE LEARNED TO EVALUATE A FORMULA

You have found that often it is convenient to use letters to express relations in numbers. Such expressions are called formulas and they are easier to remember than a rule expressed in words.

To evaluate a formula, substitute the numerical value of the letter and perform the computation.

In every case tell what rule is expressed by the formula.

1. Evaluate $A = lw$ when $l = 15$ and $w = 8$.
2. Evaluate $A = \frac{bh}{2}$ when $b = 24$ and $h = 8$.

3. Evaluate $A = \frac{h}{2}(b+b')$ when $h=8$, $b=12$, and $b'=10$.

4. Evaluate $C = \pi r^2$ when $r = 6$.

5. Evaluate $C = 2\pi r$ when $r = 8$.

6. Evaluate $V = Bh$ when $B = 60$ and $h = 15$.

7. Evaluate $V = \frac{Bh}{3}$ when $B = 24$ and $h = 6$.

8. Evaluate $V = \pi r^2 h$ when $r = 4$ and $h = 9$.

9. Evaluate $V = \frac{4}{3}\pi r^3$ when $r = 5$.

10. Evaluate $S = 4\pi r^2$ when $r = 6$.

11. Interpret the formula $A = \sqrt{s(s-a)(s-b)(s-c)}$. When $a = 12$ in., $b = 15$ in., and $c = 17$ in., find A.

12. The area of a triangle is 68 sq. in. What is the height, if the base is 14 in.?

13. A rectangular garden is twice as long as it is wide and requires 240 ft. of fencing to inclose it. What is the area of the garden?

14. How many more revolutions does a 30-inch wheel make than a 34-inch one in going a mile?

15. What is the area of a circular flower bed 12 ft. in diameter?

16. Compare the area of a circle 6 in. in diameter with one 9 in. in diameter.

17. What is the weight of a block of stone which is 8 ft. long, 2 ft. 6 in. wide, and 2 ft. thick, allowing 165 lb. to the cubic foot?

18. Allowing .8 bu. to the cubic foot, how many bushels will a bin hold that is 8 ft. long, 6 ft. wide, and 5 ft. deep?

19. What must be the depth of a bin that will hold 240 bu., if the base is 8 ft. wide and 10 ft. long?

20. How many cubic yards of excavation in a cut for a road bed that is 60 ft. wide at the top and 40 ft. wide at the bottom and which has an average depth of 12 ft., if the cut is 75 ft. long?

21. Allowing $7\frac{1}{2}$ gallons to the cubic foot, how many gallons

of water will a cistern hold that is 8 ft. in diameter and 10 ft. deep?

22. What will be the cost to paint a cylindrical smoke stack 75 ft. high and 4 ft. in diameter at 45¢ per square yard?

23. Allowing .8 bu. per cubic foot, find the number of bushels of wheat in a conical pile that is 12 ft. in diameter and 4 ft. high.

24. Find the surface of a basketball, if it is 12 in. in diameter. (A basketball is in the shape of a sphere.)

25. Find the weight of a steel ball 4 in. in diameter, if a cubic foot of steel weighs 460 pounds.

SQUARE ROOT AND THE RIGHT TRIANGLE

1. What is the side of a square that has the same area as a rectangle 16 ft. wide and 36 ft. long?

2. One leg of a right triangle is 76 ft. and the hypotenuse is 95 ft. What is the length of the other side?

3. What is the area of an equilateral triangle whose sides are each 12 inches?

4. If an isosceles triangle has its base 8 in. and a side 10 in., what is its area?

5. Find the length of the diagonal of a room from one corner at the floor to the opposite corner at the ceiling, if the room is 16 ft. long, 10 ft. wide, and 8 ft. high.

OTHER MEASURES YOU HAVE LEARNED

You have learned how to measure gas, electricity, temperature, and lumber; to draw to a scale; to find the median and the average; and how to use the metric system. Solve the following problems which involve one or more of these:

1. On March 1st the reading of a gas meter was 17,800, and at the close of the month it was 19,600. At $1.25 per M, what was the cost of the gas consumed during the month?

2. Mr. Brown's electric meter registered 14,380 (K.W.H.) on May 1st and at the end of the month it registered 14,960 (K.W.H.). At 9¢ per K.W.H., find the cost of the electricity used during the month.

3. When the temperature registers 65° F, what would be the reading on the Centigrade scale?

4. Change 32° C to the Fahrenheit scale.

5. The distance between two cities is 275 miles. On a map they are represented by a distance of $2\frac{1}{2}$ inches. To what scale is the map drawn?

6. Find the area inclosed by a rectangle drawn to a scale of 1 in. = 25 mi., if the rectangle is 8 in. long and 6 in. wide.

7. What is the amount of error of $\frac{1}{64}$ in. made in finding the distance from a map between two cities, if the map is made to a scale of 1 in. = 250 mi.?

8. Draw to a scale a map which will represent a lot 60 ft. wide and 125 ft. long.

9. Find the cost of laying a new barn floor at $56 per M, if the floor is 40 ft. long, 16 ft. wide, and the lumber is 2 in. thick.

10. Find the cost at $52 per M of a piece of lumber 14 ft. long and 4 in. square.

11. The weights of a high school football team were as follows: 132 lb., 160 lb., 148 lb., 178 lb., 210 lb., 126 lb., 162 lb., 140 lb., 158 lb., 191 lb., and 137 lb. Find the average weight and also the median weight.

12. The heights of the members of a basketball team were as follows: 5 ft. 6 in., 6 ft. 1 in., 5 ft. 8 in., 5 ft. 10 in., and 6 ft. What was the average height and also the median height?

13. In a recent airplane contest, an aviator covered 300 kilometers in 46 min. 45 sec. What was his rate in miles per hour?

14. When an airplane made a record of 246.49 miles per hour, how many kilometers was that per hour?

15. A runner covered the 100-meter course in $11\frac{2}{5}$ sec. How many feet per second was that?

YOU HAVE LEARNED TO USE PER CENT

1. In the year 1921, 185,158 corporations in the United States reported no profit whatever, but 171,239 reported a profit. What per cent of the whole number reporting showed a profit and what per cent did not report a profit?

2. A merchant sold a table costing \$48 at a 35% margin on the cost. What was the selling price?

3. A merchant sold a pair of shoes costing \$4.50 for \$7. What per cent of the cost was the margin?

4. A clothier sold a suit costing \$22.50 at a margin of 40% of the cost, but the overheads were 12% of the selling price. What was the amount of profit?

5. A suit marked \$45 was discounted 20% and then gave a margin of \$5.50. What was the cost?

6. A radio manufacturer gave two successive discounts of 15% and 15% to retailers. If the radio was marked \$150, what was the net price?

7. A salesman was working on a salary plus a commission of 2% of his sales. If his salary was \$150 a month, what was the amount of the sales if he earned \$3000 a year?

8. What is the per cent of saving by buying potatoes at \$2 per bushel instead of at 60¢ a peck?

9. A margin of 40% of the cost is what per cent of the selling price?

10. Mr. Smith's semi-annual interest on his savings bank deposit, paying 4% yearly, amounted to \$45.30. How much money did he have in his savings account?

YOU HAVE LEARNED ABOUT INVESTMENTS

1. What is a bond? A stock? What are the chief differences between them?

2. Do you consider a mortgage a safe investment? What is the difference between a first and a second mortgage? Which pays the higher rate? Why?

3. What is meant by the yield on a bond?

4. What are United States Government Bonds?

5. If a person owns a $1000 Liberty Bond paying $4\frac{1}{2}\%$, what is the amount of the semi-annual interest?

6. If one buys 10 shares of stock having a par value of $100 but selling for 112, how much does the stock cost, not counting the brokerage?

7. If the stock in Ex. 6 pays a 7% dividend, what will be the amount of income from the 10 shares?

8. If a man buys a 2-family house for $12,000 and rents each half for $50 a month, is he making more or less than if he should invest the money at 6%, allowing $225 for taxes, $110 for repairs, and $150 for depreciation?

9. Not including brokerage, how many shares of stock listed at 125 can be bought for $1000? If an $8 dividend is declared, what will that yield on the investment?

10. Which will yield the greater amount, the stock as given in Ex. 9, or a bond bought for $1000, paying 6%?

11. A man has $5000 to invest. He can buy 7% preferred stock at par or loan his money on a first mortgage at 6%. Which would you advise him to do, and why?

12. A man is able to save $300 a year for 30 years with interest compounded at 4%. What will be the amount of his savings? (Use the table on page 234.)

13. How much would the savings fund in Ex. 12 earn if loaned at 6%?

14. Find the yield on a 6% bond listed at $104\frac{7}{8}$, and maturing in 10 years.

15. A company sells a radio for $130 cash, or for $30 cash and 8 monthly payments of $15 each on the installment plan.

What rate of interest is one paying when buying it on the installment plan?

16. A saving of $1 per week ($52 per year) for 25 years will amount to how much when placed in a savings bank paying 4%? (Use the table on page 234.)

17. If you had $5000 to invest, discuss the ways you could invest it, the probable returns from each way, and the safety of each investment.

YOU HAVE LEARNED TO SOLVE PROBLEMS ORALLY

1. When cloth is selling for $1.50 per yard, what will be the cost of $2\frac{1}{2}$ yards?

2. When 5 yards of cloth can be bought for $4, what is the price per yard?

3. What is the perimeter of a rectangle 20 ft. wide and 30 ft. long?

4. Find the dimensions of a square whose area is 1600 square feet.

5. If a man's car averages 12 miles per gallon of gasoline, how many gallons will be needed to drive 75 miles?

6. A farmer dug 300 bu. of potatoes from a 2-acre field. At that rate how many bushels should he get from 5 acres?

7. When steak is selling for 60¢ a pound, what will be the cost of 12 ounces?

8. If a gallon of milk weighs 8.6 lb., what will be the weight of a 10-gallon can full of milk, allowing 12 lb. for the weight of the empty can?

9. When ice cream sells for 35¢ a pint, what is the cost of a gallon at that rate?

10. When potatoes are selling at the rate of 5 lb. for 16¢, what is the cost per bushel (60 lb.)?

11. If posts are set 10 ft. apart, how many posts will it take for 100 ft. of fencing?

12. Find the circumference of a circle having a diameter of 10 ft.

13. The area of a rectangle is 48 sq. ft. If the base is 8 ft., what is the width?

14. The volume of a prism is 480 cu. in. What is the area of the base if the height is 12 in?

15. What per cent of 36 is 24?

16. What per cent of 40 is 50?

17. An article marked for $50 was sold at a 25% discount. Find the selling price.

18. When an article marked $40 is sold for $32, the discount is what per cent of the marked price?

19. A merchant sold a suit costing $24 at a margin of $66\frac{2}{3}\%$ of the cost. What was the selling price?

20. A fruit dealer sold a carload of oranges for $1600 at a loss of $37\frac{1}{2}\%$ of the cost. What was the cost of the oranges?

21. A real estate agent's commission for selling a farm was $400 and his rate was 2% of the selling price. What was the selling price of the farm?

22. When lemons are bought for 20¢ a dozen and sold at the rate of 2 for 5¢, the margin is what per cent of the cost?

23. What is the interest on $500 for 6 mo. at 6%?

24. A 3-month note is made on March 1st. How many days before it is due?

25. Mr. Jones owns 15 shares of stock which he bought at 104. If the stock pays a $7 dividend, what will be the amount of his yearly dividends?

YOU HAVE LEARNED HOW TO ANALYZE AND SOLVE A NEW PROBLEM

You have learned that when you meet a problem whose solution is not evident at once, it is better to follow a definite outline in your method of attack. You have found that the following things helped:

1. **Read the problem carefully to determine what is asked to be found and what is given.**

2. **Discover what process or processes are to be used.**

3. **When there are several steps involved, decide upon the order in which they must be done.**

4. **Next do the computation very carefully and, when completed, be sure you check your work.**

5. **When possible, estimate your answer and see if it is reasonable for the data given in the problem.**

In the following exercises, note carefully the suggestions just given:

Exercises

1. When $1.75 will buy 5 yards of cloth, what will 8 yards cost?

2. Mr. White in starting on a 200-mile trip, drove the first 80 miles in 4 hours. If he increased his speed 5 miles per hour for the remaining distance, how long did it require for the entire trip?

3. A man rode a motorcycle 180 miles in 4 hours. If he had gone just $\frac{2}{3}$ as fast, how long would it have taken him?

4. John can add 12 columns of figures while Mary adds 10. How many columns can John add while Mary adds 15?

5. Frank is 8 years of age and his father is 4 times as old. When Frank is 20, how old will his father be?

6. Mary is 20 years of age and her mother is twice as old. What was the relation between their ages 10 years ago?

7. A man planned to make a trip in 5 hours, but he increased his speed $\frac{1}{5}$; how long did it take him?

8. A recipe calls for $2\frac{1}{2}$ lb. of sugar to 4 lb. of fruit. If 10 lb. of fruit are used, how much sugar would be needed?

9. A recipe for making jam calls for $1\frac{1}{2}$ pints of sugar to $2\frac{1}{2}$ pints of berries. If there were 8 pints of fruit and sugar, how much of each ingredient was used?

10. A formula for making a mixed feed called for $3\frac{1}{2}$ lb. of cracked corn to 5 lb. of oats. To make 51 lb. of mixture, how much of each is needed?

11. When an automobile runs $\frac{3}{8}$ mi. in $\frac{3}{4}$ of a min., what is the rate per hour.

12. A photographer wished to take a picture of a train moving at the rate of 60 mi. per hour. How many inches did the train move in $\frac{1}{1000}$ sec. while the camera shutter was open?

13. A farmer dug 51 bu. of potatoes from .3 acre. At that rate, how many bushels should he get from a field containing 6 acres?

14. If it requires 14 yd. of material 36 in. wide to make 3 dresses, how many yards of the same material 42 in. wide will it take to make the dresses?

15. When selling coffee at 30¢ per pound, a merchant made a margin of $\frac{1}{5}$ of the cost. If he wishes to make a margin of $\frac{2}{5}$ of the cost, at what price must he sell it?

16. When tea is sold at 42¢ per pound, it gives a margin of $16\frac{2}{3}\%$ of the cost. If it is sold at 48¢ per pound, the margin will be what per cent of the cost?

17. A boy sold papers at a profit of 3¢ on 5 papers. How many papers must he sell each month to make $15?

18. A boy made $6.30 one week by selling papers, at a profit on week days of 90¢ for 100 papers; and on Sunday he made 3¢ on each paper. If he sold 75 Sunday papers, how many papers did he sell each week day?

19. The receipts from an afternoon show at the movies amounted to $174. The admission prices were 20¢ for children and 30¢ for adults. If there were 180 children's tickets sold, how many 30¢ tickets were sold?

20. At a "senior benefit" in a high school, the class sold 920 tickets at 30¢. If they had to pay $225 for the reel, what was the profit from the performance?

21. Statistics show that the average car in operation in this country travels 6750 miles per year. Allowing 15 miles to the gallon of gasoline, what will be the cost of the gasoline at an average price of 22¢ per gallon?

22. A man, when idle $\frac{1}{3}$ of the time, finished a job in 36 days from the time he began it. If he had been idle only $\frac{1}{4}$ of the time, when could he have finished it?

23. A man received an increase of $3 per week, which was $\frac{1}{8}$ of his salary. What was his increased salary?

24. A boy increased in weight from 84 lb. to 96 lb. in one year. At that rate of increase, what should be his weight at the end of the next year?

25. Ruth wished to reduce her weight by dieting. The first month she reduced from 150 lb. to 147 lb. What was the per cent of reduction?

26. A boy buys maple syrup at $2 a gallon and sells it at 85¢ a quart. When the expressage amounts to 40¢ on a gallon, his profit is what per cent of the cost including expressage?

27. If a gallon of milk weighs 8.6 lb. and a gallon of water 8.36 lb., what is the ratio of the weight of milk to that of water?

28. If a cubic foot of water weighs 62.5 lb. and a gallon of water weighs 8.36 lb., how many gallons are there in a cubic foot?

29. In a recent year one of the major league teams finished the season having won 98 games and lost 56. What was the final percentage for this team? (Express to three decimal places.)

30. A man bought 8 tons of coal in September and noticed that when the winter was half gone he had but 3 tons remaining. At that rate how many more tons does he need to buy?

31. When the winter was half gone, a man had used $\frac{2}{3}$ of his fall supply of coal. At that rate he needed to buy what part of his fall's supply to last throughout the entire winter?

32. A man said, "My mileage this year is $\frac{1}{4}$ more than last

year." If he is now getting 15 miles per gallon of gasoline, what was his average mileage last year?

33. If living expenses are $\frac{1}{5}$ less now than they were in 1920, what did it cost a family to live in 1920 that now is paying $36 a week?

34. The purchasing power of a dollar in 1924 was 55¢ compared to a dollar of 100¢ in 1913. In 1913, $10 would have purchased as much as what sum in 1924?

35. Using the data in Ex. 34, what would be the purchasing power in 1913 of $5 in 1924.

36. A grocer bought cabbage in carload shipments at $14 per ton and sold it at 3¢ per pound. If the freight cost $12 per ton, his margin was what per cent of the cost?

37. A fruit dealer bought a carload of 1200 boxes of Florida oranges, at $1.25 per box. The freight amounted to $275 for the carload and he lost 50 boxes because they were decayed. If he sold the remainder at $3 per box, his margin was what per cent of the cost?

38. A merchant found that he lost 1 out of every 5 melons that he bought. How many would he expect to lose out of a shipment of 20 crates of 24 melons each?

39. If there are only 28 qt. of berries that are salable out of every crate of 32 qt., what is the actual cost per quart sold from a crate that cost a grocer $4.20?

40. Using the data in Ex. 39, at what price per quart must the grocer sell the berries to make a margin of 40% of the cost?

41. The average amount of wheat sowed per acre in the United States is 1.38 bu. At that rate how many bushels will it take to sow a rectangular field 60 rd. long and 48 rd. wide?

42. It takes 160 ft. of fencing to inclose a square garden. How much will it take to inclose a square garden twice as large?

43. To allow 216 cu. ft. of air for each pupil in a room 12 ft.

high, what must be the area of the floor space for a room containing 40 pupils?

44. If the ratio of the width to the length of the flag of the United States is 10 to 19, what must be the length of a flag that is 12 ft. wide?

45. What will it cost to fill a bin with coal at $14.75 per ton, if the bin is 8 ft. wide, 12 ft. long, and 6 ft. deep, allowing 32 cu. ft. per ton?

46. It cost Mr. Scott $210 to fill his coal bin, which is 6 ft. by 8 ft. by 10 ft. Allowing 32 cu. ft. per ton, how much per ton did it cost?

47. Compare the area of a garden plot 60 ft. by 80 ft. with one 55 ft. by 75 ft.

48. A cake 6 in. in diameter sold for 60¢. What should be the price of one 9 in. in diameter, if they are of the same thickness and quality?

49. Compare the circumference of a wheel 30 in. in diameter with one 34 in. in diameter.

50. Compare the area of a circle 10 ft. in diameter with one 15 ft. in diameter. The area of the first circle is what per cent of the area of the second? The area of the second is what per cent of the first?

51. Compare the size of a table glass 2 in. in diameter and 4 in. deep with one 3 in. in diameter and 6 in. deep.

52. John was paid $3 to mow a lawn 80 ft. wide and 150 ft. long. At that rate, what should he be paid to mow one 60 ft. wide and 125 ft. long?

53. It took a man 9 da. to pick $\frac{3}{5}$ of his apples. How long will it take him to finish?

54. After taking 12 dishes of cream from a can that was $\frac{3}{4}$ full, it was still $\frac{1}{2}$ full. How many dishes would the can hold when full?

55. A man 6 ft. tall casts a shadow 8 ft. long. At the same

time a flag pole casts a shadow 50 ft. long. What is the height of the pole?

56. When an article that was selling for $12 is increased to $15, the increase is what per cent of the former price?

57. A sign read, "This city has increased in population 150% within the past 20 years." The population is now 50,000. What was it 20 years ago?

58. An express train was averaging 1 mi. in 55 sec. That is an average rate of how many miles per hour?

59. When a ship is sailing at 24 knots, how many miles per hour is that? (1 knot = 1.152 mi. per hr.)

60. Mr. Smith had 500 hens and they laid 210 eggs per day. He sold 150 of the hens and then the ones remaining laid 175 eggs per day. What per cent better yield is he getting from the second group than from the first group?

61. A suit at a "clearance sale" was marked $\frac{1}{3}$ off. If it sold for $24, what was the former price?

62. What must be the selling price of an article costing $12 on which a merchant makes a margin of $\frac{1}{3}$ of the selling price? A margin of $\frac{1}{3}$ of the cost?

63. If a pair of shoes cost a merchant $3.60, at what price must he sell them to make a margin of $\frac{1}{4}$ of the selling price?

64. A dress costing $18 was marked so as to allow a 25% discount and leave a margin of $33\frac{1}{3}\%$ of the cost. What was the marked price?

65. A merchant received a bill of goods costing $125, which he marked so as to make a margin of 40% of the cost. He then discounted them 25%. What was his actual margin?

66. A merchant's margin is 40% of the cost of his goods, but his expenses are 22% of his sales. His profit is what per cent of the cost of the goods?

67. If goods are sold at a discount of 25% and still leave a margin of 20% of the selling price, they were marked to give a margin of what per cent of the cost?

68. Show by a diagram that a margin of $\frac{1}{5}$ of the cost is equal to $\frac{1}{6}$ of the selling price.

69. By using a diagram, find what part of the cost a margin of $\frac{2}{7}$ of the selling price is.

70. A man sold a second-hand car for \$90, which was 40% less than he paid for it. What did it cost him?

71. A merchant marked his goods $62\frac{1}{2}$% above cost, but gave a discount of 20% of the marked price. What would be his margin on an article costing \$40?

72. A camera was listed for \$60. Two successive discounts of 10% and 15% were given. What was the net price?

73. A merchant wished to mark a hat costing \$3, so that he could give a 25% discount and still have a margin of 25% of the cost. Find the selling price. The marked price.

74. A suit costing \$19.50 was sold at a margin of 35% of the selling price. What was the margin?

75. A dealer sold a radio for \$160 at a margin of 75% of the cost, but his overheads amounted to 25% of his sales. What was his profit on the set?

76. A fish loses 40% in dressing. How many pounds of undressed fish are needed to make 15 lb. of dressed fish?

77. What would be the selling price of dressed fowl, if the loss in dressing is $\frac{1}{3}$ on a 6 lb. hen, and the live fowl sells for 30¢ per lb., allowing 5¢ for the labor in dressing?

78. A dealer paid \$7 per ton for ice and sold it for 60¢ per 100 lb. If he lost 20% in melting, his margin was what per cent of the cost?

79. What is the per cent of saving by buying soap at 3 cakes for 25¢ instead of 10¢ per cake?

80. At a "special sale" a 12-lb. sack of flour sold for 60¢. The usual price was 8¢ per pound. What per cent of saving was made by buying at the special sale?

81. If ham, in boiling and slicing, loses 45% in weight,

what is the cost of sliced boiled ham when raw ham costs 36¢ per pound?

82. In making wheat flour, 72% of the wheat goes into flour. How many bushels of wheat will it take to make 20 barrels of flour? (A bushel of wheat weighs 60 lb. and a barrel of flour weighs 196 lb.)

83. When an article is bought at 20% below the list price and sold 20% above the list price, the margin is what per cent of the cost?

84. An increase of $12\frac{1}{2}$% in a man's monthly salary amounted to $25. What was his new salary?

85. A dealer buys milk at 25¢ per gallon and bottles it. He sells the milk at 6¢ per pint bottle. His margin is what per cent of the cost?

86. It costs $76.90 to make a certain article.

(a) It must be sold for how much in order to give a margin of 35% of the cost?

(b) It must be sold for how much in order to give a margin of 30% of the selling price?

(c) At what must it be listed in order to give a trade discount of 40% and leave a margin of 20% of the cost?

(d) At what must it be listed in order to allow a discount of 30% and still have a margin of 20% of the list price?

87. The delivered cost of a bill of goods is $1600.

(a) If the buyer sells them at a margin of 35% of the cost, how much does he get for them?

(b) If he sells them at a margin of 25% of the selling price, how much does he get for them?

(c) If the margin is 40% of the cost and the cost of selling is 25% of the sales, what is his profit?

(d) In part (c), can you find what per cent the profit is of the cost without using the $1600?

88. A dealer sold a bill of goods for $2400.

(a) If he made a margin of 20% of the selling price, what was the margin?

(b) If he made a margin of 20% of the cost. what was the margin?

(c) If he sold them at a discount of 20% from the marked price, at what price were they marked?

(d) If his margin was 40% of the sales and the cost of selling was 20% of the sales, what was his profit?

(e) If his margin was 50% of the cost and the cost of selling was 25% of the sales, what was his profit?

(f) Had he sold the goods at a margin of 25% of the cost and the cost of selling was 20% of the sales, did he make or lose and how much? Can you tell what per cent of the cost he made, without using the $2400?

89. If wages are increased 40% and are later reduced 30%, they are what per cent of the former wage before the increase?

90. If a man's salary is increased 50% and later decreased 40%, he is getting how much less than his original salary?

91. If goods sell at a margin of 60% of the cost and the cost of selling is 40% of the sales, a merchant is losing what per cent of the cost?

92. If goods are marked to sell at a margin of 40% of the cost and discounted 30%, they are selling at what per cent below cost?

93. A merchant whose overheads amounted to 20% of his sales, marked his goods to sell at 50% above cost, but had to discount them 20%. His loss was what per cent of the cost?

94. After discounting his goods 40%, a merchant still made a margin of 20% of the marked price. What per cent above cost were they marked?

95. After discounting his goods 40%, a merchant still made a margin of 20% of the selling price. How much above cost were they marked?

96. In one year a large automobile manufacturing company

reduced the number of minor parts from 13,000 to 2100. What was the per cent of reduction?

97. A certain soap manufacturing company advertises that its product is $99\frac{44}{100}\%$ pure. How many parts of purity are there to one part of impurity?

98. The dollar-bills prior to July 10, 1929, were $7\frac{1}{2}$ in. long and $3\frac{1}{8}$ in. wide, but after that date the bills were $6\frac{5}{16}$ in. long and $2\frac{11}{16}$ in. wide. What was the per cent of reduction in the size of the bills?

99. The regular postal rate for sealed matter is 2¢ per ounce, but "via air mail" the rate is 5¢ for the first ounce and 10¢ for each additional ounce. How much more will it cost to send a sealed parcel weighing 8 ounces by air mail than by the regular postal rate?

100. Between 1914 and 1930, living costs increased 63% and wages 118%. In 1914 a man earned $1200 and his living costs represented 80% of his earnings. Assuming that he had the same earning power and that he maintained the same standard of living in 1930 as in 1914, how much would he earn in 1930 and how much would he spend for living during that same year?

A Test in Fundamental Principles

1. If the sum of two numbers and one of them are known, how is the other found?

2. If the sum of several numbers and all but one of them are known, how is the unknown number found?

3. If the larger of two numbers and their difference are known, how is the smaller number found?

4. If the smaller of two numbers and their difference are known, how is the larger number found?

5. If the product of two numbers and one of them are known, how is the other found?

6. If the product of three numbers and two of them are known, how is the third number found?

7. Multiplying or dividing the dividend without changing the divisor has what effect upon the quotient?

8. Multiplying or dividing the divisor without changing the dividend has what effect upon the quotient?

9. If the divisor, quotient, and remainder are known, how may the dividend be found?

10. If the dividend, quotient, and remainder are known, how may the divisor be found?

11. Multiplying the numerator of a fraction has what effect upon the fraction?

12. Dividing the denominator has what effect upon a fraction?

13. Multiplying the denominator has what effect upon a fraction?

14. Moving the decimal point one place to the right has what effect upon a decimal?

15. Annexing a zero to a whole number has what effect upon the number?

16. Annexing a zero to a decimal has what effect upon the number?

An Inventory Test

1. Give and illustrate the three uses of subtraction.
2. Give and illustrate the two uses of division.
3. Give and illustrate three ways of expressing the remainder.
4. Give the formulas for five areas you have studied.
5. Give the formulas for five volumes you have studied.
6. Give six different kinds of graphs you studied.
7. Name four methods of investing money.
8. Name three types of taxes.
9. Name two types of insurance.
10. Name five important things to observe in solving a new problem.

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ANSWERS

ANSWERS TO PROBLEMS IN HIGHER ARITHMETIC

Page 2

1. 2871.
2. 3178.
3. 3215.
4. 3005.
5. 2953.
6. 2513.

Page 3

1. 3673.
2. 5087.
3. 5199.
4. 4596.
5. 4641.
6. 4542.
7. 4803.
8. 4299.
9. 4412.
10. 5022.
11. 5428.
12. 4130.
13. 4709.
14. 4714.

Page 4

1. 46,212.
2. 39,977.
3. 52,304.
4. 51,303.
5. 57,388.

Pages 6–7

Exercise A

1. $\frac{3}{4}$.
2. $1\frac{1}{6}$.
3. $\frac{7}{12}$.
4. $\frac{1}{2}$.
5. $1\frac{5}{12}$.
6. $\frac{5}{8}$.
7. $\frac{5}{8}$.
8. $1\frac{3}{8}$.
9. $1\frac{1}{2}$.
10. $1\frac{1}{8}$.
11. $1\frac{1}{8}$.
12. $1\frac{1}{9}$.
13. $\frac{8}{9}$.
14. $\frac{7}{9}$.
15. $1\frac{1}{6}$.
16. $1\frac{1}{4}$.
17. $\frac{13}{16}$.
18. $1\frac{5}{16}$.
19. $1\frac{7}{12}$.
20. $\frac{13}{16}$.
21. $1\frac{1}{18}$.
22. $\frac{5}{6}$.
23. $1\frac{1}{18}$.
24. $1\frac{7}{12}$.
25. 1.
26. 2.
27. $1\frac{3}{8}$.
28. $1\frac{3}{8}$.
29. $1\frac{7}{8}$.
30. $1\frac{1}{12}$.
31. $1\frac{1}{3}$.
32. $1\frac{5}{12}$.
33. $1\frac{1}{3}$.
34. $1\frac{2}{3}$.
35. $1\frac{5}{12}$.
36. $\frac{15}{16}$.
37. $1\frac{11}{16}$.
38. $1\frac{7}{12}$.
39. $1\frac{1}{12}$.
40. $1\frac{3}{16}$.
41. $1\frac{7}{8}$.
42. $1\frac{1}{2}$.
43. $2\frac{1}{8}$.
44. $2\frac{1}{4}$.
45. $1\frac{11}{16}$.
46. $1\frac{2}{3}$.
47. $2\frac{3}{8}$.
48. $2\frac{11}{24}$.

Exercise B

1. 10.
2. 8.
3. $11\frac{3}{8}$.
4. $9\frac{7}{8}$.
5. $8\frac{1}{12}$.
6. $7\frac{7}{8}$.
7. $11\frac{5}{12}$.
8. $10\frac{3}{8}$.
9. $10\frac{1}{4}$.
10. $10\frac{1}{8}$.
11. $10\frac{7}{18}$.
12. $7\frac{7}{12}$.
13. $14\frac{1}{8}$.
14. $9\frac{3}{4}$.
15. $10\frac{7}{12}$.
16. $11\frac{5}{12}$.
17. $10\frac{1}{2}$.
18. 17.

Page 8

1. 19 ft. 7 in.
2. 24 bu.
3. 16 yd. 2 ft.
4. 27 lb. 8 oz.

1. $230\frac{3}{4}$.
2. $183\frac{1}{6}$.
3. $142\frac{3}{8}$.
4. 96.
5. $103\frac{5}{8}$.
6. 61.75.
7. 102.375.
8. 31.782.
9. 24 ft. 6 in.
10. 28 lb.

Pages 11–12

1. 352.
2. 751.
3. 568.
4. 474.
5. 563.
6. 686.
7. 1384.

8. 2370.
9. 3376.
10. 3314.
11. 6506.
12. 38,307.
13. 8879.
14. 43,307.
15. 15,778.
16. 38,639.
17. 40; 18; 81; 68; 28; 25; 56; 84; 20; 39; 58; 47; 65; 61; 9; 76; 73; 55; 62; 12; 19; 44; 34; 41; 8; 72; 85; 53; 16; 23; 75; 64; 43; 37; 86; 79; 63; 6; 24; 11.

1. 58, 148, 328.
2. 48, 035, 202.
3. 21, 453, 606.
4. 1, 757, 363.
5. 55, 454, 985.
6. 36, 976, 783.
7. 24, 899, 167.
8. 34, 772, 052.
9. 22, 557, 696.
10. 16, 589, 718.
11. 39, 774, 358.
12. 19, 096, 491.

Page 13

1. $\frac{1}{2}$.
2. $\frac{1}{4}$.
3. $\frac{1}{4}$.
4. $\frac{1}{12}$.
5. $\frac{1}{6}$.
6. $\frac{1}{12}$.
7. $\frac{1}{6}$.
8. $\frac{5}{12}$.
9. $\frac{1}{6}$.
10. $\frac{1}{12}$.
11. $\frac{1}{8}$.
12. $\frac{1}{8}$.
13. $\frac{7}{12}$.
14. $\frac{2}{9}$.
15. $\frac{1}{9}$.
16. $\frac{5}{9}$.
17. $\frac{1}{2}$.
18. $\frac{1}{8}$.
19. $\frac{4}{9}$.
20. $\frac{1}{3}$.
21. $\frac{5}{8}$.
22. $\frac{1}{6}$.
23. $\frac{1}{9}$.
24. $\frac{7}{16}$.

Page 14

1. $\frac{3}{4}$.
2. $\frac{1}{3}$.
3. $\frac{5}{8}$.
4. $\frac{5}{6}$.
5. $\frac{2}{5}$.
6. $\frac{5}{7}$.
7. $\frac{4}{5}$.
8. $\frac{5}{12}$.
9. $\frac{11}{16}$.
10. $2\frac{5}{8}$.
11. $3\frac{1}{8}$.
12. $3\frac{1}{3}$.
13. $2\frac{7}{12}$
14. $4\frac{3}{8}$.
15. $4\frac{1}{12}$.
16. $2\frac{1}{2}$.
17. $2\frac{2}{3}$.
18. $3\frac{1}{4}$.
19. $2\frac{3}{8}$.
20. $5\frac{1}{12}$.
21. $4\frac{1}{6}$.
22. $4\frac{1}{2}$.
23. $1\frac{1}{4}$.
24. $2\frac{3}{8}$.
25. $3\frac{1}{3}$.
26. $3\frac{1}{8}$.
27. $5\frac{3}{4}$.
28. $1\frac{1}{2}$.
29. $3\frac{3}{4}$.
30. $2\frac{2}{3}$.
31. $3\frac{3}{8}$.
32. $1\frac{1}{2}$.
33. $3\frac{5}{8}$.
34. $2\frac{3}{8}$.
35. $2\frac{1}{2}$.
36. $2\frac{5}{8}$.
37. $6\frac{5}{6}$.
38. $1\frac{3}{4}$.
39. $5\frac{2}{3}$.

Pages 15–17
Exercise A

1. $2\frac{1}{6}$.
2. $3\frac{1}{8}$.
3. $2\frac{1}{3}$.
4. $5\frac{1}{2}$.
5. $3\frac{5}{12}$.
6. $3\frac{1}{12}$.
7. $2\frac{1}{4}$.
8. $3\frac{1}{8}$.
9. $2\frac{1}{6}$.
10. $2\frac{5}{12}$.
11. $5\frac{1}{4}$.
12. $3\frac{5}{8}$.

Exercise B

1. $1\frac{1}{2}$.
2. $2\frac{3}{4}$.
3. $1\frac{1}{4}$.
4. $3\frac{2}{3}$.
5. $1\frac{1}{3}$.
6. $4\frac{5}{8}$.
7. $2\frac{5}{6}$.
8. $1\frac{1}{6}$.
9. $1\frac{1}{8}$.
10. $3\frac{3}{8}$.
11. $2\frac{7}{8}$.
12. $4\frac{8}{9}$.
13. $1\frac{4}{9}$.
14. $2\frac{7}{12}$.
15. $3\frac{5}{12}$.
16. $5\frac{7}{16}$.
17. $1\frac{13}{16}$.
18. $3\frac{2}{9}$.

Exercise C

1. $2\frac{3}{4}$.
2. $2\frac{3}{4}$.
3. $1\frac{7}{8}$.
4. $2\frac{5}{8}$.
5. $2\frac{7}{8}$.
6. $1\frac{7}{8}$.
7. $2\frac{3}{8}$.
8. $1\frac{5}{8}$.
9. $1\frac{2}{3}$.
10. $1\frac{5}{6}$.
11. $2\frac{1}{2}$.
12. $1\frac{7}{12}$.
13. $1\frac{7}{9}$.
14. $2\frac{5}{12}$.
15. $5\frac{11}{12}$.
16. $2\frac{5}{6}$.
17. $1\frac{5}{12}$.
18. $4\frac{7}{12}$.

Exercise D

1. $2\frac{3}{4}$.
2. $2\frac{7}{8}$.
3. $4\frac{1}{3}$.
4. $4\frac{5}{6}$.
5. $2\frac{1}{4}$.
6. $3\frac{1}{2}$.
7. $2\frac{1}{3}$.
8. $4\frac{1}{9}$.
9. $4\frac{2}{5}$.
10. $4\frac{3}{4}$.
11. $3\frac{5}{6}$.
12. $2\frac{5}{16}$.

Exercise F

1. $\frac{1}{4}$.
2. $\frac{5}{8}$.
3. $\frac{5}{18}$.
4. $\frac{1}{12}$.
5. $1\frac{1}{4}$.
6. $\frac{9}{16}$.
7. $4\frac{7}{12}$.
8. $4\frac{1}{20}$.
9. $5\frac{2}{3}$.
10. $2\frac{1}{3}$.
11. $3\frac{1}{12}$.
12. $3\frac{5}{12}$.
13. $2\frac{5}{8}$.
14. $2\frac{1}{6}$.
15. $4\frac{1}{9}$.
16. $1\frac{5}{8}$.
17. $3\frac{1}{12}$.
18. $2\frac{5}{6}$.
19. $5\frac{2}{5}$.
20. $2\frac{2}{3}$.
21. $3\frac{1}{8}$.
22. $1\frac{1}{2}$.
23. $4\frac{1}{8}$.
24. $\frac{3}{4}$.
25. $2\frac{7}{16}$.
26. $2\frac{1}{3}$.
27. $5\frac{7}{9}$.
28. $\frac{2}{3}$.
29. $2\frac{3}{8}$.
30. $3\frac{5}{12}$.
31. $\frac{1}{8}$.
32. $\frac{1}{18}$.
33. $4\frac{5}{6}$.
34. $1\frac{1}{8}$.
35. $2\frac{3}{4}$.
36. $\frac{1}{10}$.
37. $2\frac{19}{24}$.
38. $5\frac{1}{8}$.
39. $6\frac{11}{15}$.
40. $2\frac{15}{16}$.

Page 17

1. 23.75.
2. 20.20.
3. 30.39.
4. 21.32.
5. 34.84.
6. 7.95.
7. 13.48.
8. 15.422.
9. 107.72.
10. 0.009.
11. 5.00.
12. 88.63.

Page 18

1. 4 lb. 7 oz.
2. 3 ft. 3 in.
3. 3 gal. 1 qt.
4. 1 yd. 1 ft.
5. 3 lb. 10 oz.
6. 1 bu. 3 pk.
7. 2 lb. 2 oz.
8. 1 yd. 1 ft.

Page 20

1. 112.
2. 232.
3. 222.
4. 414.
5. 406.
6. 657.
7. 420.
8. 1740.
9. 3654.
10. 4060.
11. 54,855.
12. 62,448.
13. 1032.
14. 2916.
15. 1974.
16. 20,140.
17. 7063.
18. 74,192.

Page 21
Exercise A

1. 26,037.
2. 32,916.
3. 63,966.
4. 30,360.
5. 32,190.
6. 52,758.
7. 47,915.
8. 23,504.
9. 58,113.
10. 38,552.

Exercise B

1. 637,812.
2. 413,952.
3. 701,775.
4. 152,988.
5. 154,323.
6. 581,064.
7. 139,854.
8. 511,252.
9. 886,289.
10. 224,804.
11. 538,859.
12. 313,306.

Page 22

2. 507,327.
3. 816,314.
4. 334,208.
5. 711,708.
6. 256,088.
7. 600,416.
8. 239,512.
9. 355,152.
10. 645,414.
11. 235,776.
12. 667,644.
13. 195,684.
14. 599,814.
15. 199,082.

Page 24

1. $1\frac{1}{2}$.
2. $4\frac{1}{6}$.
3. $5\frac{1}{4}$.
4. 30.
5. $16\frac{1}{4}$.
6. $98\frac{2}{3}$.
7. 56.
8. $201\frac{1}{9}$.
9. 6.
10. $3\frac{1}{3}$.
11. 12.
12. $17\frac{1}{2}$.
13. $8\frac{1}{3}$.
14. $6\frac{3}{4}$.
15. $4\frac{1}{2}$.
16. $19\frac{1}{2}$.
17. $71\frac{1}{4}$.
18. 201.
19. $122\frac{2}{3}$.
20. $\frac{10}{27}$.
21. $\frac{3}{5}$.
22. $\frac{3}{16}$.
23. $\frac{7}{18}$.
24. $\frac{2}{3}$.
25. $\frac{7}{12}$.

Exercise A

1. $\frac{27}{32}$
2. $5\frac{5}{8}$.
3. $3\frac{1}{18}$.
4. $3\frac{5}{6}$.
5. $2\frac{3}{16}$.
6. $1\frac{7}{18}$.
7. $4\frac{11}{16}$.
8. $2\frac{1}{12}$.
9. $2\frac{1}{2}$.
10. $2\frac{3}{8}$.
11. $3\frac{5}{24}$.
12. 1.
13. $4\frac{4}{9}$.
14. $2\frac{11}{12}$.
15. $2\frac{4}{7}$.

Exercise B

1. $10\frac{1}{2}$.
2. $8\frac{1}{3}$.
3. $14\frac{2}{3}$.
4. $11\frac{4}{15}$.
5. $12\frac{5}{6}$.
6. $7\frac{1}{2}$.
7. 3.
8. $2\frac{1}{2}$.
9. 9.
10. $17\frac{1}{2}$.

Page 25

1. 252.28.
2. 0.1956.
3. 365.4.
4. 5.80.
5. 23.14.
6. 2815.2.
7. 154.14.
8. 1.484.
9. 258.23.
10. 1.197.
11. 85.5.
12. 1.38.
13. 163.76.
14. 350.32.
15. 8.28.
16. \$22.26.
17. \$412.39.
18. \$20.23.

Page 26

1. 18 bu. 3 pk.
2. 38 ft. 6 in.
3. 75 lb.
4. 30 bu. $2\frac{1}{2}$ pk.
5. 26 yd. 2 ft.
6. 35 lb. 10 oz.
7. 32 ft.
8. 23 yd. $2\frac{1}{2}$ ft.
9. 49 yd. $2\frac{1}{2}$ ft.

1. 350.
2. 35.
3. 400.
4. 17.5.
5. 5.6.
6. 1700.
7. 2000.
8. 365.
9. 420.
10. 35.
11. 0.6.
12. 6.
13. 24.5.
14. 31.2.
15. 360.

Page 27

1. 420.
2. 160.
3. $112\frac{1}{2}$.
4. 160.
5. $83\frac{1}{3}$.
6. 800.
7. 3200.
8. 3750.
9. 2150.
10. 2400.
11. 300.
12. 225.
13. 3200.
14. 1500.
15. 1200.
16. 1600.
17. 400.
18. 300.

Pages 28–29

1. 2025.
2. .1225.
3. 42.25.
4. 625.
5. .0225.
6. 30.25.
7. 9025.
8. .7225.
9. .5625.
10. 12.25.

1. 122.5.
2. 56.25.
3. .225.
4. 4.225.
5. 30.25.
6. 7.225.
7. 6.25.
8. 20.25.
9. 9.025.
10. 422.5.

1. 1216.
2. 2021.
3. 9009.
4. 3016.
5. 4221.
6. 616.
7. 224.
8. 2016.
9. 5624.
10. 9021.

1. 2.16.
2. 122.4.
3. 3.021.
4. 9.024.
5. 42.09.
6. 2.016.
7. 122.1.
8. 30.24.
9. 2.21.
10. 6.24.

Page 29

1. $3\frac{1}{3}$¢.
2. 3.
3. $4\frac{1}{5}$ ft.
4. $6\frac{2}{3}$.
5. 28 in.
6. $7\frac{1}{2}$.
7. 7.
8. 8 ft.
9. 6.
10. 7 ft.
11. 9.
12. 5¢.
13. 8.
14. 9¢.

Page 31

1. 993.
2. 545, 27 r.
3. 1596, 15 r.
4. 544, 27 r.
5. 1349, 37 r
6. 1340, 2 r.
7. 490, 66 r.
8. 1372, 45 r.
9. 2370, 32 r.
10. 1273, 18 r.

1. 114, 6 r.
2. 73, 6 r.
3. 148, 24 r.
4. 94, 43 r.

5. 30, 23 r.
6. 24, 32 r.
7. 72, 46 r.
8. 32, 56 r.
9. 91, 14 r.
10. 113, 37 r.
1. 593.
2. 726.
3. 834.
4. 519.
5. 396.
6. 277, 60 r

Page 32

1. $\frac{3}{8}$.
2. $\frac{5}{16}$.
3. $\frac{5}{24}$.
4. $\frac{1}{5}$.
5. $\frac{1}{16}$.
6. $\frac{5}{36}$.
7. $\frac{1}{16}$.
8. $\frac{7}{32}$.
9. $\frac{3}{16}$.
10. $\frac{7}{32}$.
11. $\frac{1}{10}$.
12. $\frac{2}{21}$.
13. $\frac{1}{10}$.
14. $\frac{1}{18}$.
15. $\frac{5}{16}$.
16. $\frac{4}{27}$.
17. $\frac{1}{12}$.
18. $\frac{1}{12}$.
19. $\frac{1}{12}$.
20. $\frac{1}{6}$.

Page 33

1. $\frac{7}{8}$.
2. $\frac{5}{6}$.
3. $\frac{25}{32}$.
4. $6\frac{1}{6}$.
5. $2\frac{1}{16}$.
6. $2\frac{1}{6}$.
7. $5\frac{2}{9}$.
8. $2\frac{1}{12}$.
9. $\frac{1}{2}$.
10. $2\frac{5}{8}$.
11. $2\frac{3}{4}$.
12. $2\frac{1}{2}$.
13. $5\frac{7}{12}$.
14. $2\frac{17}{30}$.
15. $2\frac{3}{8}$.
16. $\frac{17}{32}$.
17. $\frac{11}{12}$.
18. $2\frac{1}{16}$.
19. $8\frac{1}{9}$.
20. $6\frac{3}{4}$.
21. $1\frac{5}{12}$.
22. $\frac{11}{12}$.
23. $11\frac{1}{9}$.
24. $3\frac{25}{32}$.

Pages 34–35

1. 8.
2. 12.
3. 32.
4. $4\frac{4}{5}$.
5. $2\frac{2}{7}$.
6. 18.
7. 25.
8. $6\frac{2}{3}$.
9. 20.
10. 36.
11. $6\frac{6}{7}$.
12. $5\frac{2}{5}$.

Page 35

1. $\frac{3}{5}$.
2. $1\frac{1}{3}$.
3. $\frac{3}{4}$.
4. $\frac{5}{8}$.
5. $1\frac{1}{2}$.
6. $1\frac{7}{8}$.
7. $1\frac{1}{20}$.
8. $1\frac{1}{6}$.
9. $\frac{4}{5}$.
10. $\frac{1}{2}$.
11. $\frac{7}{9}$.
12. $\frac{1}{4}$.
13. $1\frac{1}{5}$.
14. $1\frac{1}{2}$.
15. $\frac{1}{3}$.
16. $\frac{3}{4}$.
17. $1\frac{1}{5}$.
18. $\frac{5}{12}$.
19. $\frac{1}{4}$.
20. $1\frac{1}{5}$.
21. $\frac{8}{15}$.
22. $\frac{1}{2}$.
23. $\frac{1}{3}$.
24. $1\frac{1}{2}$.

Pages 36–37
Exercise A

1. $\frac{3}{8}$.
2. $\frac{1}{8}$.
3. $1\frac{1}{9}$.
4. $2\frac{1}{12}$.
5. $2\frac{3}{10}$.
6. $\frac{2}{3}$.
7. $\frac{2}{9}$.
8. 12.
9. $\frac{3}{4}$.
10. $2\frac{1}{15}$.
11. $4\frac{4}{9}$.
12. $4\frac{4}{9}$.
13. $9\frac{3}{5}$.
14. $\frac{25}{32}$.
15. $\frac{3}{4}$.
16. $\frac{3}{4}$.
17. $\frac{7}{24}$.
18. $\frac{7}{12}$.
19. 12.
20. $30\frac{5}{16}$.
21. $5\frac{5}{9}$.
22. $\frac{17}{18}$.
23. $\frac{8}{15}$.
24. $\frac{7}{9}$.
25. $4\frac{1}{2}$.
26. 15.
27. $\frac{2}{5}$.
28. $\frac{1}{2}$.
29. $\frac{4}{15}$.
30. $72\frac{1}{12}$.

Exercise B

1. $6\frac{9}{10}$.
2. $9\frac{3}{16}$.
3. $4\frac{8}{9}$.
4. $4\frac{5}{8}$.
5. $4\frac{11}{32}$.
6. $7\frac{3}{8}$.
7. $6\frac{17}{24}$.
8. $3\frac{1}{3}$.
9. $3\frac{2}{5}$.
10. $2\frac{1}{5}$.
11. $1\frac{5}{6}$.
12. $1\frac{1}{5}$.

Exercise C

1. $1\frac{1}{2}$.
2. $\frac{4}{5}$.
3. $1\frac{1}{8}$.
4. $1\frac{1}{5}$.
5. $\frac{8}{15}$.
6. $2\frac{1}{4}$.
7. $5\frac{1}{4}$.
8. $\frac{3}{8}$.
9. $3\frac{1}{8}$.
10. $\frac{9}{14}$.
11. 2.
12. $1\frac{5}{6}$.
13. $\frac{5}{8}$.
14. $1\frac{1}{8}$.
15. $\frac{2}{3}$.

Pages 37–38

1. $\frac{3}{20}$.
2. $\frac{1}{6}$.

3. $\frac{3}{16}$.
4. $\frac{1}{6}$.
5. $\frac{5}{12}$.
6. $\frac{3}{35}$.
7. $\frac{1}{10}$.
8. $\frac{7}{16}$.
9. $\frac{1}{12}$.
10. $\frac{1}{14}$.
11. $\frac{1}{8}$.
12. $\frac{7}{64}$.
13. $\frac{5}{6}$.
14. $1\frac{2}{3}$.
15. $1\frac{2}{5}$.
16. $\frac{8}{15}$.
17. $\frac{13}{18}$.
18. $\frac{11}{16}$.
19. $1\frac{11}{16}$.
20. $1\frac{1}{4}$.
21. $1\frac{11}{15}$.
22. $\frac{11}{16}$.
23. $\frac{4}{5}$.
24. $2\frac{1}{2}$.
25. $1\frac{1}{8}$.
26. $1\frac{1}{9}$.
27. $\frac{3}{4}$.
28. $\frac{5}{6}$.
29. $\frac{4}{15}$.
30. $\frac{15}{32}$.
31. $\frac{2}{3}$.
32. $1\frac{1}{2}$.
33. $1\frac{5}{16}$.
34. $1\frac{1}{14}$.
35. $1\frac{1}{4}$.
36. $\frac{2}{3}$.
37. $1\frac{7}{9}$.
38. 2.
39. $2\frac{4}{15}$.
40. $1\frac{11}{64}$.
41. $2\frac{6}{23}$.
42. 2.
43. $1\frac{13}{32}$.
44. $2\frac{\text{[illegible]}5}{\text{[illegible]}8}$.
45. $\frac{13}{22}$.
46. $1\frac{1}{8}$.
47. $4\frac{5}{16}$.
48. $8\frac{14}{27}$.
49. $11\frac{11}{28}$.
50. $17\frac{1}{3}$.
51. $12\frac{41}{48}$.
52. $14\frac{67}{72}$.
53. $34\frac{3}{5}$.
54. $24\frac{1}{6}$.
55. $34\frac{5}{6}$.
56. $93\frac{11}{12}$.

Page 39

1. 10,100.
2. 101.
3. 1010.
4. .0101.
5. 101.
6. 1.01.
7. .0101.
8. 1010.
9. 101.
10. 101,000.
11. 1.01.
12. .00101.
13. 10.1.
14. 10.1.
15. 10,100.

Page 40

1. 10.5.
2. 30.4.
3. 4700.
4. 33.
5. 93.
6. 1.23.
7. 1202.
8. 100.
9. 27.6.
10. 43.6.
11. .0092.
12. 970.
13. 75.3.
14. .106.
15. 5600.
16. 4030.
17. 231,900.
18. 3.82.
19. 40.
20. 1.35.
1. 1.69.
2. 2.11.
3. 16.
4. 2.12.
5. 22.41.
6. 3.33.
7. 203.54.
8. 71.38.
9. 0.031.
10. 20.21.
11. 0.023.
12. 1.59.

Pages 40–41

1. 70.
2. 503.
3. 32.7.
4. 9.
5. 2000.
6. 3.
7. $1\frac{1}{8}$.
8. $\frac{2}{7}$.
9. $59\frac{7}{12}$.
10. $3\frac{1}{9}$.
11. $\frac{5}{12}$.
12. $3\frac{9}{14}$.
13. $1\frac{1}{4}$.
14. 8.67.
15. 5.6.
16. 3.2.

Pages 41–42

2. 1 yd. 26 in.
3. 19.
4. 49 da.
5. 2 yd. 6 in.
6. $6\frac{1}{2}$.
7. 1 gal. 3 qt.
8. 7.
9. 2 hr. 5 min.
10. 125.
11. 1 bu. 3 pk.
12. 7.

Pages 42–44

4. 10 curtains; $\frac{1}{4}$ yd. rem.
5. 25 pay 41¢ each; 5 pay 42¢ each.
7. $43\frac{3}{4}$¢.
9. \$1.08$\frac{3}{4}$.
10. 109.
11. 108.

Page 45

3. 17.44.
4. 14.08.
5. 13.92.
6. 23.52.
7. 40.5.
8. 68.4
9. 12.78.
10. 10.5.
11. 19.2.
12. 23.2.
13. 1.456.
14. 2.202.
15. 6.584.
16. 3.276.
17. 9.024

Page 47

1. 48.275.

2. $20\frac{3}{4}$.
3. 18,747.
4. $2\frac{3}{8}$.
5. 223.04.
6. 8.1345.
7. 11.
8. $14\frac{2}{5}$.
9. 1119, 17 r.
10. 131.73.
11. $1\frac{11}{16}$.
12. $2\frac{1}{28}$.

Pages 47–48

1. $\frac{5}{6}, \frac{5}{7}, \frac{5}{8}, \frac{5}{9}, \frac{5}{13}$.
2. $\frac{9}{13}, \frac{8}{13}, \frac{7}{13}, \frac{5}{13}, \frac{4}{13}, \frac{2}{13}$.
3. $\frac{5}{6}, \frac{3}{4}, \frac{2}{3}, \frac{5}{8}, \frac{7}{12}, \frac{1}{2}$.
4. .482, .48, .45, .413, .4058, .4.
5. $\frac{3}{5}, \frac{11}{20}$, .534, $\frac{1}{2}$, .48.
6. .8, $\frac{3}{4}$, .735, .704, $\frac{2}{3}$.
7. $\frac{3}{4}, \frac{13}{18}, \frac{2}{3}$, .65, .635.
8. $1\frac{3}{5}, 1\frac{1}{2}, \frac{149}{100}$, 1.35, $1\frac{1}{3}$.
9. $\frac{2}{5}, \frac{3}{8}, \frac{37}{100}$, .365, $\frac{1}{3}$.
10. $\frac{3}{4}$, .7, .675, $\frac{2}{3}, \frac{65}{100}$.

Pages 61–62

1. 1920.
2. $2.37.
3. $15.61.
4. $4685; $3554.
5. $2738.
6. Loss of $177.
7. $1052.

Page 66

9. 1000 mi.
10. About 440 mi.
11. About 7200 ft.
12. About 4400 ft.
13. 11,800 ft.

Page 68

1. 44.3%.
2. $6\frac{1}{4}$%.
3. 2.24%.
4. 2.14.
5. About 1 to every 43 people.

Page 74

1. 21.4%.
2. 56.4%.
3. 10.2%.
4. 20.8%.

Pages 76–78

1. About 8 times as large.
2. About $1\frac{2}{3}$.
2. 1000.
3. 120; about 900.

Pages 88–90

Exercise A

1. 18 mi.
2. $11.44.
3. $4\frac{1}{2}$ hr.
4. 24 mi.
5. 5 dishes.
6. 8 in. and 16 in.
7. 4 yd. $32\frac{2}{3}$ in.
8. $\frac{2}{3}$.
9. 27 yr.
10. 11 hr.

Exercise B

1. .36 in.
2. $113\frac{1}{3}$ ft.
3. $1\frac{1}{2}$ hr.; 3 hr.
4. 4 hr.
5. Cost is $\frac{1}{2}$ the margin.
6. $5\frac{1}{3}$ nr.
7. 15 lb.
8. 172.
9. 2.55 ft.
10. $\frac{1}{5}$.

Pages 90–92

5. 190 bu.
6. 3.9¢.
7. 186.9 mi.
8. $1.94.
9. 4 dresses; $\frac{1}{2}$ yd. rem.
10. $2\frac{10}{11}$ hr.
11. 1580 bu.
12. 299.2 gal.
13. 2535 steps.
14. 62.44 sq. ft.
15. 46.34 ft.
16. $2\frac{1}{4}$ pt.
17. 140.
18. $3\frac{3}{8}$ tbs.
19. $87\frac{1}{2}$ bu.
20. 9 pt. fr. ju.; 6 pt. sugar.
21. $34\frac{3}{7}$ mi.
22. $168\frac{3}{4}$ mi.
23. $4\frac{6}{7}$ mo.
24. 570 bu.
25. 128.8 mi.

Page 94

1. 16,000 A.
2. $31\frac{1}{4}$ ft.
3. $153\frac{3}{5}$ bu.
4. $465.
5. 220.4 lb.
6. 60¢.
7. $28.80.
8. About 5 qt.
9. $1.20.
10. 92 lb.

Pages 95–96

1. 17.58 in.
2. 8.12 lb.
3. 27.6 mi.
4. 10.03 cu. ft.
5. $58.82.
6. $1.13.
7. 4.48 hr.
8. 7.20 in.
9. 11.63 in.
10. 202 ft.

Pages 96–97

1. $36.
2. 50 mi.
3. $5\frac{9}{16}$ lb.
4. $4.40.
5. $\frac{3}{7}$.
6. $45,000.
7. 3 dresses; $2\frac{1}{4}$ yd. rem.
8. $23.94.
9. $\frac{1}{2}$.
10. 18.45 gal.

Pages 97–101

1. 405 bu.
2. $3\frac{1}{8}$ yd.
3. 1836.25 lb.
4. 1.467¢.
5. $820.31.
6. 83.5 lb.
7. Rent, 30%; food, 24%; expenses, 24%; saving, 22%.
8. 3 yd.
9. 3 sacks.
10. $35\frac{5}{8}$¢, or 36¢.
11. 1120 gal.
12. 45 min.
13. $2\frac{1}{2}$ hr.
14. 8 wk.
15. $4.08.
16. 10.8 lb. less.
17. 21.3 bu.
18. $\frac{1}{2}$ of cost.
19. $311.60.
20. 126.9 ft.
21. 25.5 ft.
22. 22.64 A.
23. 75 da.
24. $1\frac{3}{4}$¢.
25. 90.
26. $10.89.
27. $4.51.
28. 104.2 lb.
29. $4212.50.
30. Mrs. Brown, $10; Mrs. Cole, $6.50.
31. 7.25 hr.
32. $4.80.
33. $15.
34. $7\frac{1}{2}$ mi.
35. $21.26.
36. $84.
37. $27; $21.
38. $40.
39. $17\frac{1}{2}$ bu.
40. 36¢.
41. $42.50.
43. .391.
44. $4.80.
45. $52.32.
46. 16.
47. 2555.5 lb.
48. $35.
49. 40 mi.
50. $2.73.

Pages 102–106

1. $160.
2. $\frac{2}{7}$ of S.P.
3. $45.
4. $28.125.
5. $\frac{1}{3}$ of C.; $\frac{1}{4}$ of S.P.
6. $\frac{7}{9}$ of C.
7. $19.20.
8. $375.
9. $7200.
10. $40.
11. $\frac{1}{4}$ of C.
12. 512.
13. A, $\frac{5}{8}$; B, $\frac{3}{8}$.
14. 96.
15. $22.50.
16. 32.
17. $\frac{4}{5}$ of M.P.
18. $42\frac{6}{7}$ mi.
19. 18.
20. $3000.
21. 7 times as old.
22. 4.36 bu.
23. $40.
24. $1.05.
25. Milk 56¢ cheaper per gal.
26. $1.50.
27. 18 da.
28. $12.
29. $\frac{1}{15}$ of C.
30. 18 lb.
31. $16.
32. 15 mi.
33. $\frac{1}{3}$.
34. $9.
35. $\frac{4}{5}$ of C.
36. 15 boys; 25 girls.
37. $\frac{1}{8}$ of C.
38. $180.
39. $31.25.
40. $30.
41. 45 mi.
42. 3 gal. $\frac{1}{2}$ qt.
43. $2\frac{1}{2}$ ft.
44. $103.13.
45. $\frac{1}{3}$.
46. $27.
47. $7\frac{1}{2}$ lb. oats; $12\frac{1}{2}$ lb. corn.
48. 40 mi. per inch.
49. 128 copies.
50. $3.50.

Pages 106–109

Test A

1. $6\frac{1}{4}$ yd.
2. 5 hr.
3. 25 ft.
4. 40 yr.
5. 6 ft.; 8 ft.
6. 250.
7. 50 mi.
8. 65 in.
9. .391.
10. $\frac{1}{3}$¢ cheaper at rate of 12 for 25¢.

Test B

1. $12\frac{3}{16}$ in.
2. 83.
3. 400 bu.
4. 25 lb.
5. Bulbs, $2.50; markers, $1.00; fertilizer, $2.00.
6. $2.25.
7. $4\frac{1}{2}$ hr.
8. 25¢.
9. $60.
10. $5\frac{1}{2}$ hr.

Test C

1. $1.68.
2. 22.22 mi.
3. $\frac{1}{3}$ of cost.
4. 10 lb. of c.c.; 15 lb. of oats.
5. 140.17 lb.
6. sugar, 8 lb.; fruit, 13 lb.
7. 24 lb.
8. 4 ft. 2 in.
9. 4 in.
10. $26\frac{2}{3}$ da.

Pages 109–111

Test A

1. $2.
2. $2\frac{1}{2}$ hr.

3. 20 yr.
4. $7\frac{1}{2}$ hr.
5. $1500.
6. 80.
7. 30.
8. 144 sq. ft.
9. 30 ft.
10. $225.

Test B

1. 45.
2. 40¢.
3. 194.
4. $1.40.
5. 15 gal.
6. $12.
7. 60 ft.
8. 2 ft.; 6 ft.
9. $2\frac{1}{2}$ doz.
10. $9.30.

Test C

1. 30¢.
2. 27 mi.
3. $5.
4. $3.25.
5. 52¢.
6. 16 pages.
7. 25.
8. 280.
9. $35.
10. $16.

Page 114

12. 96.
13. 105.
14. 441.
15. 630.
16. 600.
17. 343.
18. 420.
19. 576.
20. 525.

Page 116

1. 8.66.
2. 7.07.
3. 9.54.
4. 9.90.
5. 10.25.
6. 10.95.
7. 10.77.
8. 11.75.
9. 12.25.
10. 12.85.
11. 10.39.
12. 13.60.
13. 14.14.
14. 14.70.
15. 15.56.

Page 118

1. 31.4.
2. 33.5.
3. 41.5.
4. 51.4.
5. 71.8.
6. 89.7.
7. 80.8.
8. 97.1.
9. 93.5.
10. 82.4.
11. 5.1.
12. 6.2.
13. 7.2.
14. 8.1.
15. 9.1.
16. 11.7.
17. 12.3.
18. 14.8.
19. 26.2.
20. 31.3.

Pages 121–124

1. 53.8 ft.
2. 68 ft.
3. 10 ft.
5. 127.3 ft.
6. 67.3 ft.
7. 127.3 ft.
8. 217.3 ft.
9. 44.7 ft.
10. 197.8 ft.
11. 19.2 mi.
12. 4 mi.
13. 41 ft.
14. 32.65 ft.
15. 180 ft.
16. 357.2 ft.
17. 40 sq. in.
18. 11.6 in.
19. 43.3 sq. in.
20. 161.55 ft.
21. 40.25 ft.
22. 38.96 rd.
23. 3888 sq. rd.
24. 149.66 yd.

Pages 124–125

1. 52 in.
2. 67 in.; 134 in.
3. 10 in.
4. 5.66 in.
5. 12.81 in.
6. About 6 in.
7. About 22.1 in.
8. 75 sq. ft.; 10 ft.
9. 60.39 ft.
10. 81.1 ft.
12. 60 sq. in.
13. 294.57 sq. rd.
14. 47.9 sq. ft.

Page 127

5. $83.34.
6. $63.09.
7. 385 sq. ft.
8. $90.
9. $27.
10. $26.88.

Pages 128–129

1. 5.
2. 8.
3. 8.
4. $7\frac{1}{2}$.
5. 3.
6. $4\frac{3}{8}$.
7. 5.
8. 6.6.
9. 0.
11. 64 ft.
12. 30 ft.
13. 180 sq. ft.
14. 6 ft.
15. 54 ft.
16. Doubled.
17. Doubled.
18. 4 times.
19. 3 times.
20. 9 times.

Pages 130–131

3. 15 sq. in.
4. $5\frac{1}{2}$ in.
5. 10 ft.

Pages 131–132

1. 36 sq. in.
2. 18 sq. ft.
3. 340 sq. rd.
4. 62.755.
5. $72.45.
6. $59.50.
7. 56 sq. ft.
8. Doubled.
9. Doubled.
10. 4 times.

Page 132

2. $7'' \times 5''$; 35 sq. in.
3. 17.5 sq. in.

Page 133

1. 60 sq. in.
2. 40 sq. ft.
3. 156 sq. rd.
4. 61.69.
5. 81.56.
6. 42.8.
7. 320 sq. rd.

Pages 134–135

1. 0.016 ft.
2. 25,132.8 mi.; 25,120 mi.
4. 37.68 ft.
5. 210.2 ft.
6. 672.6 rev.
7. 15.2 mi. per hr.
8. 888.9 ft.
9. 62.8 ft.

Pages 136–137

1. 113.04 sq. in.
2. 78.5 sq. ft.
3. 176.63 sq. rd.
4. 72.35 sq. rd.
5. 1 to 4.
6. 9420 lb.
7. $111.30.
8. 10 in.
9. 314.
10. $1.125.
12. The circular barn; 1404.7 sq. ft.
13. Doubles.
14. 4 times.
15. 3 times; 9 times.
16. 4 times.

Pages 139–140

1. 205 cu. ft.
2. 56.1 gal.
3. 230.4 bu.
4. 4950 lb.
5. $74.67.
6. 201,960 gal.
7. 56¢.
8. 16.45 T.
9. 1862.19 T.
10. Doubled.
11. 4 times.
12. 8 times.
13. 456 lb.

Page 140

1. 3 ft.
2. 4.23 ft.
3. 5.5 ft.
4. 18 sq. ft.
5. $6\frac{1}{4}$ ft.
6. 11.5 ft.

Pages 141–142

9. 132.
10. 32.
11. 91.
12. 262.
13. 90.
14. 154.
15. $14\frac{2}{3}$.
16. 114.

Pages 142–143

1. 15 bd. ft.
2. 10 bd. ft.
3. 8 bd. ft.
4. 20 bd. ft.
5. $2\frac{1}{4}$ in.
7. 400 bd. ft.; 600 bd. ft.; 2000 bd. ft.

Pages 144–145

1. 29.44 gal.
2. 191.3 gal.
3. 117.4 gal.
4. 72.06 T.
5. About 6 A.
6. About 192 da.
7. 4239 cu. in.
8. 565,488 bu.
9. 4 times.
10. Doubles.

Page 145

3. $2.83.
4. $42.39.
5. 75.36 sq. ft.

Page 147

1. 96 cu. in.
2. 12.56 cu. in.
3. 104.72 bu.
4. $13.08.
5. 64 sq. ft.
6. 4.85 cu. yd.
7. 565.2 sq. ft.

Page 149

1. 1256.64 sq. in.; 3217 sq. ft.
2. 201,062,400 sq. mi.
3. 523.6 cu. ft.
4. 2143.68 cu. in.
5. 8.9 lb.
6. 1.95 gal.
7. $2441.66.
8. 20.7 lb.
9. $\frac{1}{8}$.
10. $\frac{27}{64}$.
11. 8 times; 27 times.
12. 4 times.
13. 9 times.

Page 150

1. 8.
2. 4.
3. 8.
4. 8.
5. 37.68.
6. 50.24.
7. 4.3.
8. 1.99.
9. 3.
10. 40.
11. 157.
12. 452.16.
13. 33.5.

Pages 150–151

1. −.
2. −.
3. +.
4. −.
5. +.
6. +.
7. +.
8. +.
9. +.
10. −.
11. +.
12. +.
13. +.

14. −.
15. +.
16. −.
17. +.
18. −.
19. +.
20. +.

Page 154

1. 1 mm. = .039 in.; 1 cm. = .394 in.; 1 m. = 39.37 in., 3.281 ft., 1.094 yd.; 1 km. = 3281 ft., 1094 yd., .621 mi.; 1 in. = 2.538 cm.; 1 ft. = 30.456 cm., .305 m.; 1 yd. = .914 m.; 1 rd. = 5.027 m.; 1 mi. = 1610.4 m., 1.610 km.
2. 39.75 mi.
3. 2.3 mi.
4. 11.1 hr.
5. The first; 0.7 ft. per sec.
6. 5.17 mi.
7. 111.3 cu. cm.
8. 1.625 sq. m.

Pages 155–156

3. $\frac{5}{9}$°.
4. $\frac{9}{5}$°.
5. 5°.
6. 64.8°.
8. 7.8°.
9. 14.4°.
10. 41.1°.
11. 12.2° below zero.
12. 59°.
13. 113°.
14. 177.8°.
15. 15.8°.

Pages 157–160

8. 10 times.
15. About 2 mi.
16. 325 mi.
17. $3\frac{3}{4}$ in.
18. 6.25 mi.
19. $\frac{5}{8}$ ft.

Pages 160–161

1. $1.65.
2. $3.24.
3. $8.55.
4. Nearly 15¢.
5. 21¢.
6. $18.

Pages 161–163

1. $46.50.
2. $20.25.
4. $2.90.
5. 0.15 k.w.h.; $1\frac{1}{2}$¢.
6. 20¢.
7. 18¢.
8. 31¢.
9. $1\frac{1}{2}$¢.
10. $1.80.
11. $31\frac{1}{2}$¢.
12. 40¢.
13. 48¢.
14. $1.00.
15. $7262.40.
16. 40.5¢.
17. 20,667.4% increase.

Pages 164–165

1. 95, med.; 95.2, avg.
2. 166 lb., med.; 169.9 lb., avg.
3. 14 yr. 3 mo., med.; 13 yr. 10.1 mo., avg.
5. 87, med.; 84.5, avg.
6. 82.5.
7. 142 lb.

Pages 165–167

1. 800 sq. ft.
2. 50 ft. by 100 ft.
3. 12 in.
4. 72 sq. ft.
5. 45 ft.
6. 78.5 ft.
7. About 168 rev.
8. $108.85.
9. 33.8%.
10. Areas, $\frac{1}{4}$; circumferences, $\frac{1}{2}$.
11. 204.4 lb.
12. 204 sq. ft.
13. 7.81 ft.
14. 37.70 sq. ft.
15. 8.15 gal.
16. 32 cu. ft.
17. 30.1 bu.
18. 153.9 sq. in.
19. 9.5 lb.
20. 64.

Page 167

1. 6562 ft.
2. 72.45 km.
3. 20°.
4. 77°.
6. 9.2 in.
7. 4.69 mi.
8. $3.76.
9. $42.30.
10. median, 5 ft. 10 in.; average, 5 ft. 10.6 in.

Pages 170–172

4. 61.1%.
5. 93.3%.
6. 400%.
7. $316\frac{2}{3}$%.
8. 25% of S. P.; $33\frac{1}{3}$% of C.
9. 15%.
10. 16.9%.
11. 48.7% boys; 51.3% girls.
12. $15,000 house; 0.4%.

13. 1500%.
14. 37.5%.

Pages 172–173

1. \$98.
2. 474.21 lb.
3. \$402.50.
4. \$720.
5. \$1840.
6. \$242.10.
7. \$1827.50.
8. 43.8 bu.
9. Rm. and bd., \$728; cl. amd la., \$327.60; re. and inc., \$218.40; adv., \$273; sav., \$273.
10. 338 girls; 182 boys.
11. Tuition, \$200; rm. and bd., \$360; bk., \$50; inc., \$390.

Pages 174–175

12. \$160.
13. 270 mi.
14. \$125.
15. 800.
16. \$75.
17. \$120.
18. 120 mi.
19. 40 ft.
20. \$55.
21. 18 lb.
22. 33 in.
23. 52 mi.
24. 150%.
25. 200%.
26. 100%.
27. $133\frac{1}{3}\%$
28. 300%.
29. 100%.

Pages 175–176

1. \$6.
2. \$22.
3. 9 mi.
4. \$0.50.
5. \$3.
6. \$60.
7. 3.
8. \$1.
9. 9 ft.
10. \$300.
11. \$30.
12. \$3.
13. 50%.
14. 30%.
15. 60%.
16. 6%.
17. .6%.
18. 20%.
19. 1%.
20. 50%.
21. 5%.
22. .5%.
23. 200%.
24. 200%.
25. 150%.
26. 400%.
27. 150%.
28. \$0.15.
29. \$35.
30. 80%.
31. 37.5%.
32. 150%.
33. 25%.
34. 25%.
35. 80%.
36. 25%.
37. 20%.
38. 72.
39. 40.
40. 16.
41. \$13.50.
42. \$103.50.
43. \$76.50.
44. 1.5%.
45. $1\frac{1}{4}\%$.
46. $\frac{1}{2}\%$.
47. $\frac{3}{4}\%$.
48. $\frac{1}{2}\%$.
49. 1%.
50. $\frac{1}{2}\%$.

Page 176

The corrected series is given in the following:

1. $10\% = \frac{1}{10}$; .1.
2. $25\% = .25$; $\frac{1}{4}$; $\frac{25}{100}$.
3. $33\frac{1}{3}\% = \frac{1}{3}$; $\frac{100}{300}$.
4. $37\frac{1}{2}\% = \frac{3}{8}$; .375.
5. $50\% = .5$; $\frac{1}{2}$; 0.50; $\frac{50}{100}$.
6. $66\frac{2}{3}\% = \frac{2}{3}$; .666.
7. $\frac{1}{2}\% = .5\%$.
8. $\frac{3}{4} = 75\%$; .75; $\frac{75}{100}$.
9. $1 = 100\%$.
10. $2\frac{1}{2} = 250\%$; 2.5; $\frac{500}{200}$.

Pages 177–178

1. 500.
2. 40 qt.
3. \$30.
4. \$60.
5. \$120.
6. 600 lb.
7. 80 mi.
8. \$300.
9. 80 lb.
10. 69.4 lb.

Pages 179–183

5. 44.2 lb.
6. 200 lb.
7. 2631.6 lb.
8. 45 lb.
9. 40%.
10. \$19,000.
11. \$224.
12. \$60.
13. 96 lb.; 600 lb.
14. 10%; 7%.
15. \$50 per mm.
16. 30%.
17. 16 mi.
18. 22.85%.
19. 21.16%.
20. \$2000.
21. \$3000.
22. (*a*) 39.1%; (*b*) 43.85%; (*c*) 46.8%; (*d*) 49%; (*e*) 56.87%; (*f*) 67.05%.
23. 30.77%.
24. \$160.60.
25. \$10 more at 3%.
26. \$11.04.

Pages 184–185

2. 60.2%.
3. 19.3%.
4. 233.8%.
5. Ia., 14.6%; Ill., 9.77%.
6. (*a*) 191.5%; (*b*) 52.2%.
7. (*a*) 125.6% Tex. with Ia.; (*b*) 79.5% Ia. with Tex.
8. 46.5%.
9. 69.8%.
10. Gold = 119.2% of silver; silver = 83.87% of gold.
11. 155.85%.
12. 76.2%.
13. 131.6%.
14. 63,881.
15. 1890–1900.

Pages 186–187

55. 112.5.
56. 1.6.
57. 562.5.
58. 5.25.
59. 937.5.
60. .875.
61. 787.5.
62. $556\frac{2}{3}$.
63. $59\frac{2}{3}$.
74. 300.
75. 480.
76. 400.
77. 800.
78. 500.
79. 1300.
80. 900.
81. 20%.
82. 2%.
83. $\frac{1}{5}$%.
84. 200%.
85. 25%.
86. 50%.
87. 360.
88. 4.
89. 75%.
90. 3.75%.
91. 9.
92. 80.
93. 200.
94. 200%.
95. 48.
96. 16.
97. 200%.
98. 400%.
99. .64.
100. 1200.

Pages 188–189

1. True.
2. True.
3. False.
4. False.
5. False.
6. False.
7. False.
8. True.
9. False.
10. True.
11. True.
12. False.
13. True.
14. True.
15. False.
16. True.
17. True.
18. False.
19. True.
20. False.
1. $\frac{1}{200}$.
2. 20%.
3. .0005.
4. 3.5.
5. $\frac{1}{2}$.
6. 200%.
7. $\frac{1}{2}$%.
8. 40%.
9. 30 mi. per hr.
10. $\frac{1}{2}$%.
11. 1%.
12. 1¢.
13. $\frac{1}{300}$.
14. 140%.
15. $1.
16. $\frac{1}{2}$%.
17. 8.
18. 200.
19. 800.
20. $\frac{1}{500}$.

Pages 189–191

1. $17.31; $2.47.
2. $40.
3. $420; $105.
4. Shelter, 13.3%; food, 30%; clothing, 15%; operating, 10%; advancement, 5%; incidentals, 6.7%; saving, 20%.
5. $50.
6. $300.
7. $14.42.
8. 20%.
9. $36.42; $41.40.
10. Housing, 20.3%, 22.3%; fuel, light, 4.6%, 5%; food, 40.2%, 35.4%; clothing, 13.1%, 13.5%; sundries, 21.7%, 23.8%.
11. Manual workers', 18.72%; office workers', 7.6%.
12. $44.81.
13. 13.7%.
14. 38.78%.

Pages 191–192

1. $20.
2. $4.32.
3. $16\frac{2}{3}$%.
4. $160.63.
5. $192.50.
6. $73.50; $100.63; $192.50; $243.25.
7. $140; $100; $172.

9. On wool velvets, $20\frac{2}{3}$%.
10. 25%.
11. 50%.
12. $33\frac{1}{3}$%.
13. $66\frac{2}{3}$%.
14. 100%.

Pages 192–195

1. $413.10.
2. $843.75.
7. $198.90.
8. $174.96.
9. $25.
11. 58.7%.
13. 25%.
14. 25%.
16. 25%.
17. $33\frac{1}{3}$%.
18. 29.8%.
19. 30.7%.
20. $500.

Pages 196–197

1. $7.20.
2. $510.
3. $19.25.
4. $35.25.
5. $150.
6. $2500.
7. $1.129.
8. $32.49.
9. $66,666\frac{2}{3}$ bu.
10. $163.00.
11. $1000.
12. $312.50.
13. $1640.
14. 2%.
15. $1000.
16. $650,000.
17. $70.
18. 30%.
19. $240 to $360.
20. $750.

Pages 198–202

1. $50.
2. $6.25.
3. 48%; 32.43%.
4. 25%.
5. $33\frac{1}{3}$%.
6. 25%.
7. 20%.
8. $1.35.
9. 10.1%.
13. $32.67.
14. $11.25.
15. $40.
16. $1248.
17. $1225.
18. $4500.
19. $3000.
20. $495.
21. $36\frac{2}{3}$%.
22. $25 loss.
23. 22.5%.
24. 11.84%.
25. 11.84%.
27. $33\frac{1}{3}$%.
28. $28\frac{4}{7}$%.
29. $33\frac{1}{3}$%.
30. $42.67.
31. 35.13%.
32. $27.
33. $30.
34. 23.07%.
35. 17.64%.
36. 17%.
37. 9.37%.

Page 203

1. $13.50.
2. $42.79.
3. $37.50.
4. $28.93.
5. $76.45.
6. $90.75.
7. 85.18%.
8. 71.69%.
9. 51.23%.
10. 7.45%.
11. 39.65%.
12. 38.2%.
13. $6.23.
14. $5.36.
15. $9.45.
16. $42.78.
17. $30.
18. $68.85.

Page 204

1. $30.
2. $24.
3. $33.
5. $1214.
6. $15; $515.
7. $99; $1899.
8. $22.50; $922.50.
9. $72; $1672.
10. $16.50; $1216.50.
11. $15.33; $815.33.
12. $25; $1525.
13. $29.33; $2029.33.
14. $32; $2432.
15. $51; $3651.

Pages 205–207

1. $9.
4. $22.50; $772.50.
5. $16.29; $866.29.
6. $32; $992.
7. $22; $1222.
8. $20; $1520.
9. $10.33; $610.33.
10. $16.67; $2016.67.
11. $4.38; $1504.38.
12. $40.50; $940.50.
13. $19.50; $669.50.
14. $8.50; $1708.50.
15. $77.78; $4077.78.
16. $25; $2525.
17. $3.20; $203.20.
18. $0.31; $150.31.
19. $0.90; $90.90.

Page 208

1. $2.
2. $4.
3. $20.
4. $1.80.
5. $2.40.
6. $1.40.
7. $3.50.
8. $10.
9. $10.50.
10. $16.
11. $21.
12. $12.50.
13. $18.

Page 209

2. \$27.60; \$1772.40.
3. \$49.20; \$2350.80.
4. \$6.10; \$593.90.
5. \$13.80; \$886.20.
6. \$30.50; \$1469.50.
7. \$9.32; \$990.68.
8. \$33.73; \$2366.27.
9. \$5.90; \$294.10.
10. \$1.50; \$898.50.

Page 210

4. \$50.
5. \$3.
6. \$1.17.
7. \$333.33.
8. \$25.
9. \$1.
10. \$150.

Page 211

3. \$32.65.
4. \$28.33.
5. \$140.
6. \$191.67.
7. \$219.
8. \$54.
9. \$114.17.
10. \$250.
12. \$152.
13. \$216.
14. \$29.50.
15. \$3.20.

Page 212

13. .12.
14. .05.
15. 1.2.
16. 4.
17. .003.
18. .0035.
19. .005.
20. .0525.
21. .165.
22. .00002.
23. .002.
24. \$13.20.
25. 12.
26. 75%.
27. 2¢.
28. $\frac{1}{2}$%.
29. 20,000%.
30. 72.
31. 25%.
32. $33\frac{1}{3}$%.
33. 10¢.
34. 1%.
35. 50%.
36. 400%.
37. 100%.
38. $\frac{1}{4}$.

Pages 212–215

Test A

1. 50%.
2. \$10.50.
3. 16.
4. .005.
5. \$25.50.
6. $12\frac{1}{2}$%.
7. 28%.
8. \$8.75.
9. 150%.
10. $\frac{1}{2}$ of S.P.

Test B

1. \$30.
2. \$8.
3. \$4.50.
4. \$96.
5. \$150.
6. 60%.
7. \$25.
8. \$7500.
9. $44\frac{4}{9}$%.
10. 225%.

Test C

1. $\frac{3}{4}$%.
2. $66\frac{2}{3}$%.
3. $33\frac{1}{3}$%.
4. \$10.50.
5. \$162.
6. \$250.
7. 3%.
8. \$10.
9. 90%.
10. $8\frac{1}{3}$%.

Test D

1. 1%.
2. 100 in.
3. 400%.
4. \$500.
5. 20.
6. \$12.75.
7. $33\frac{1}{3}$%.
8. \$62.50.
9. \$3000.
10. \$1,000,000.

Pages 215–220

1. 2.04 yr.
2. 89.9%.
3. 50%.
4. 48.15%.
5. \$60,000.
6. \$1375.
7. $16\frac{2}{3}$%.
8. \$5.25.
9. \$7.50.
10. \$400.
11. \$26.32.
12. \$5.16.
13. \$2.30.
14. 27.35%.
15. John, $37\frac{1}{2}$%; Tom, 60%; Henry, 45%.
16. \$492.
17. 15.2%.
18. $16\frac{2}{3}$%.
19. $22\frac{1}{2}$ qt.
20. 20¢.
21. $66\frac{2}{3}$%.
22. $33\frac{1}{3}$%.
23. 16%.
24. \$15.25.
25. \$1230.83.
26. 30¢.
27. 36¢.
28. 3.57%.
29. 4.1%.
30. 200 mi.
31. 44.44%.
32. 192.8%.
33. None.
34. 96%.
35. 108%.
36. 56.25%.
37. $66\frac{2}{3}$%.
38. 17%.
39. 75%.
40. 38.75%.
41. 59.36%.
42. col. grad., 1,284,000;

hi. sch., 15,048,000; el. ed., 36,678,000.
43. col. grad., 57.89%; hi. sch., 11.35%; el. ed., 7.74%.
44. col. grad., 1 to about 125; hi. sch., 1 to about 6000; el. sch., 1 to about 18,000.
45. Low, $2520; average, $3080; liberal, $4160.
46. $15,400.
47. $1080.

Pages 222–223

1. +.
2. −.
3. +.
4. −.
5. −.
6. +.
7. +.
8. +.
9. +.
10. −.
11. +.
12. −.
13. +.
14. +.
15. −.
16. −.
17. +.
18. +.
19. −.
20. −.

Pages 227–228

1. Minimum, $1050.10; middling, $1750.05; master, $3750.
2. Minimum, $35; middling, $58; master, $126.
3. $1920.
4. Minimum, $591.20; middling, $983.60; master, $1979.10.
5. About 47 yr.

Page 229

5. Quarterly.
6. $1\frac{1}{8}$%.

Pages 231–232

3. $1314.66, annually; $1324.92, semi-annually.
4. $994.90.
5. $1440.72.
6. $453.33.
7. $416.28.
8. $533.74.
9. $1391.16.
10. $2786.25.
11. $508.20.
12. $1154.63.
13. $1578.15.
14. 5%.
15. $6172.84; $3773.58.
16. $6755.84; $4563.92.
17. About 7 yr. at 3%; 5 yr. at 4%; 4 yr. at 5%; $3\frac{1}{2}$ yr. at 6%.
19. 24 yr. at 3%; 18 yr. at 4%; 15 yr. at 5%; 12 yr. at 6%.

Page 235

2. $3121.50 in 10 yr.; $7742.25 in 20 yr.
3. $20,702.40.
4. $1499.25 more by first plan.
5. $1246.25.
6. $2265.70.

Pages 237–238

1. $4009.
2. $1597.
3. $8689.
4. $2301.50.
5. Interest, 12%; principal, 88%; interest, 23%; principal, 77%.
6. Interest, 33%; principal, 67%; interest, 42%; principal, 58%.
7. $43.95.
8. Mr. Jones, $43.97 per mo.; Mr. Black, $136.35 per mo.; difference, $92.38.
10. $3074.60.
11. $43.94.
14. No.

Page 241

3. Book value, $1753.20; withdrawal value, $1659.20.
4. $38.
5. Withdrawal value $33.70 less.

Pages 242–243

1. 20 shares; $20.
2. $20; total payment, $40.

3. About 12 yr.
4. About 13 yr.
5. $2000; $3000.
6. $300; 25 shares.
7. About $6\frac{1}{2}$ yr.
8. $47.
9. $15 per mo.
10. About $2800.

Page 246

2. $390.
3. $495.
4. (*a*) $6000; (*c*) Mr. Taylor, $180 semi-annually.
5. $660,000,000.
6. $5000.
7. 10%.

Pages 249–251

1. Cincinnati Street Railway Co.
2. No.
3. Part entitled "Security."
4. Part entitled "Earnings."
6. For 1929, 76%.
7. For 1929, 23.95%.
8. About 33.9%.

Pages 251–252

2. Armour & Co., $892.50, cost; interest, $45; Beth. Steel, $1070, cost; interest, $55; Am. Tel. & Tel., $1056.25, cost; interest, $50; Penn. R.R., $978.75, cost; interest, $45; Chile Copper, $955, cost; interest, $50; South R.R., $1227.50, cost; interest, $65.
5. Ital. Gov., $70; Dom. of Can., $50.
6. $6180, cost; $155, interest.

Page 255

3. 5.85%.
4. 6.38%.
5. 5.90%.
6. 7.15%.
7. 5.52%.
8. 5.26%.
9. 5.19%.
10. 5.49%.
11. 5.15%.
12. 5.20%.

Pages 256–257

1. $12.50.
3. $2934.33.

Pages 259–260

1. 5000; $\frac{1}{50}$.
2. $\frac{1}{10}$; $800.
3. 2000; $\frac{1}{2000}$.
4. 15%.
5. $160.
6. $100.
7. $60,000; $90.
8. 12%.

Page 267

1. $280.
2. $250.
3. 5.87%.
5. $275.

Pages 271–273

3. 121.6%.
4. 42.67%.
5. 42.86%.
6. 69%.
7. 11.4%.
8. 21.7%.
9. 23.4%.
10. 25%.
11. 24.4%.
12. 35.75%.
13. 16.93%.
14. $750,000,000.
15. $525,000,000.

Page 278

1. No.
2. 1920.
3. About $100,000,000.
4. About $400,000,000.

Pages 279–282

9. $36.
10. $100.
11. $90.
12. $75.
14. .24%.
15. 1.62%.
16. $57.60.
17. $60.
18. $85 on S.C.; $18 on H.
19. $1600; $2400.
20. $23.04; $9600; $6000; $500.
21. 60%.
22. $9600.
23. $8000; $\frac{3}{4}$.
25. co-insurance is $19.20 cheaper than insurance

at full value.
26. $1317.32.
27. $25,000.
28. $3\frac{1}{3}$ mills.
29. $1.00.

Pages 285–286

2. New York City: W, $104.40; X, $128.40; Y, $163.20; outside of New York City: W, $31.20; X, $38.40; Y, $48.
3. $288.
4. $2179.
5. $2222.62.
6. $184 less.
7. Insurance $28 cheaper.
10. The Oranges, $5.07; New York City, $17.16.
11. The Oranges, $10.41; New York City, $39.42.
12. $7733.33.
14. 1st yr., $35.96; 2nd yr., $25.38; 3rd yr., $16.92; 4th yr., $10.58.
15. 1st yr., $4.59; 2nd yr., $4.32; 3rd yr., $3.60; 4th yr., $2.70.

Pages 289–290

1. 112.
2. 70.
3. 62.5%.
4. 55.25%.
5. 16.26%.
7. $692.65.
8. $1000.
9. $5618.84; $13,936.19.

Pages 290–292

2. $4936.
3. $23,103.
4. $2,005,661.
5. $1082.60.
7. $7.50.
8. $7.75.
9. $8.11.
10. $8.60.
11. $9.42.
12. $10.73.
13. $13.25.
14. $25.67.
15. $226.48.
16. $961.50.
18. $14.71.
19. $15.91.
20. $22.38.
21. $40.42.

Pages 294–295

4. $100.70; $154.70.
5. O. L., $131.75; 20-Pay., $181.10; 20-yr. End., $249.25.
6. O. L., $5141.25; 20-Pay., $7395.75; 20-yr. End., $10,986.75.
7. $5122.16.
8. $2923.58 less in bank.
9. $7146.95 less in bank.
10. $12,746.62 more in bank.
11. $2768.84 less in bank.
12. $737.80.
13. 483.9%.
15. 6.94%.

Pages 297–298

1. $170.75.
2. $567.24.
3. $2353.30.
4. $66.73 less than paid out.
5. $97.82 less than paid out.
6. $576 less than paid out.
7. $835.64 more in bank.
8. $1590.86 more by insurance.

Pages 301–302

1. $31.35.
2. $69.45.
3. $158.35.
4. $13,942.50.
5. $510.94.

Pages 307–308

1. $394.21.
2. $398.35.
3. $509.77.
4. $275.57.
5. $185.66.

Pages 309–311

3. 1,050,000 shares.
6. $133,434.92.
7. $21,941.37.
8. 74.5%.
9. 15%.
11. $72.
12. $55.
13. $7.71.
14. $4.50.
15. $35.
16. $25.50.
17. $8.63.
18. $4.58.

19. $8.33.
20. $33.25.
21. $30.80.

Page 311

1. $2.50; $497.50.
2. $7.50; $742.50.
3. $4.90; $975.10.
4. $11.48; $753.52.
5. $6.25; $1243.75.
6. $15.75; $1559.25.
7. $22.50; $1477.50.
8. $12.38; $1637.62.
9. $9.30; $1850.70.
10. $26.70; $1753.30.
11. $18.20; $1541.80.
12. $4.50; $1345.50.

Pages 315–316

4. $20.03.
5. $37.10.
6. $68.45.
7. $44.55.

Page 317

1. $5.57.
2. $4.50.
3. $3.80.
4. $1.31.

Pages 317–318

2. May 2, $3.43; June 1, $9.27.
5. $161.
6. M. C. Lewis is credited $2.01.
7. $16.35.

Page 321

1. 29.6 mills.
2. 34.7 mills.
3. $12.
4. $24.40.
5. $32\frac{3}{4}$ mills.

Pages 321–323

3. $1.50; 1.5%.
4. $144.
5. $80.75.
6. $1.80.
7. $8100.
8. $96.20.
9. 1.1%.
10. $.75.
11. $1.75.
12. $1.47.
13. $.857.
14. $.51.
15. $.519.
16. $.496.
17. $1.039.
18. $56.25.
19. $92.63.
20. $173.25.
21. $148.50.
22. $83.06.
23. $198.
24. $236.67.
25. $330.27.
26. $.025 per $100.
27. $1.00.
28. $224.64.
29. $8000.

Page 325

1. $2\frac{1}{4}$%.
2. $3120.
3. $42.90.
4. $218.40.
5. 5.2%.

Page 326

2. Federal, 157.1%; state, 76.9%; local, 108.7%.
7. General gov.: 1904–15, 124.1%; 1915–22, 75.4%; Highways: 1904–15, 78.4%; 1915–22, 68.2%; education: 1904–15, 100%; 1915–22, 174.7%.

Pages 330–331

1. $27,814,-208.
2. $297,350,-221.
3. $67,711,-838.58.
4. $54,893,-233.

Pages 331–332

1. $12.50.
2. $12.50.
3. $10.
4. $440.
5. $800.
7. $200,-000,000.

Pages 333–334

1. $13,750.
2. $2.46.
3. $55,500.
4. $534.
5. $21; $29.40.
6. Imports, $4,000,-000,000; exports, $5,000,-000,000.
8. 35.1%.
9. 37.62%.
10. 13.21%.

Pages 334–335

1. $30.
2. $14.55.
3. $41.72.

Pages 336–337

5. $648,064,-800.
6. 1919.

7. $1,000,-000,000.
8. 12 times.
9. About 19 yr.
10. $701,250,-000.
11. About $136.
12. $45,000,-000.

Pages 337–339

1. Schools.
2. 34.85 %.
3. $26,940,-000.
5. $683,756,-000.
6. Ed., 34.85 %; admst., 8.09 %; streets, 6.37 %; health, 12.24 %; hospitals, 10.50 %; police, 21.71 %; misc., 6.24 %.

Pages 339–340

1. $21.32.
2. $48.
3. $33,180,-931.76.
4. 985,560,-000 gal.
5. $100.
6. $804.

Pages 344–346

1. 120.
2. 96.
3. 88.
4. 113.04.
5. 50.24.
6. 900.
7. 48.
8. 452.16.
9. 523.33.
10. 452.16.
11. 87.8.
12. 9.71.
13. 3200 sq. ft.
14. 79.1.
15. 113.04 sq. ft.
16. $\frac{4}{5}$.
17. 6600 lb.
18. 192 bu.
19. $3\frac{3}{4}$ ft.
20. 1666.67 cu. yd.
21. 3768 gal.
22. $47.10.
23. 120.6 bu.
24. 452.16 sq. in.
25. 8.92 lb.

Page 346

1. 24 ft.
2. 57 ft.
3. 62.28 sq. in.
4. 36.66 sq. in.
5. 20.5 ft.

Pages 346–348

1. $2.25.
2. $52.20.
3. 18.33°.
4. 89.6°.
5. 1″ = 110 mi.
7. 3.9 mi.
9. $71.68.
10. $.97.
11. median, 158 lb.; average, 158.4 lb.
12. median, 5 ft. 10 in.; average, 5 ft. 9.8 in.
13. 239.1 mi.
14. 396.85 km.
15. 28.78 ft.

Page 348

1. No profit, 48.05 %; profit, 51.95 %.
2. $64.80.
3. 55.55 %.
4. $5.22.
5. $30.50.
6. $108.38.
7. $60,000.
8. $16\frac{2}{3}$ %.
9. 28.57 %.
10. $2265.

Pages 348–350

5. $22.50.
6. $1120.
7. $70.
8. Interest $5 more.
9. 8 shares 6.4 %.
10. stock, 0.4 % more.
12. $17,498.40.
13. $1049.88.
14. 5.26 %.
15. 44.44 %.
16. $2252.22.

Pages 350–351

1. $3.75.
2. 80¢.
3. 100 ft.
4. 40 ft.
5. $6\frac{1}{4}$ gal.
6. 750 bu.
7. 45¢.
8. 98 lb.
9. $2.80.
10. $1.92.
11. 11 posts.
12. 31.4 ft.
13. 6 ft.
14. 40 sq. in.
15. $66\frac{2}{3}$ %.
16. 125 %.
17. $37.50.
18. 20 %.
19. $40.
20. $2560.
21. $20,000.
22. 50 %.
23. $15.
24. 92 da.
25. $105.

Pages 352–361

1. $2.80.
2. 8.8 hr.
3. 6 hr.
4. 18.

5. 44 yr.
6. mother 3 times as old.
7. $4\frac{1}{6}$ hr.
8. $6\frac{1}{4}$ lb.
9. 3 pt. sugar; 5 pt. of berries.
10. 21 lb. corn; 30 lb. of oats.
11. 30 mi.
12. 1.056 in.
13. 1020 bu.
14. 12 yd.
15. 35¢.
16. $33\frac{1}{3}$%.
17. 2500 papers.
18. 75.
19. 460.
20. $51.
21. $99.
22. 32 da.
23. $27.
24. 109.7 lb.
25. 2%.
26. $41\frac{2}{3}$%.
27. Milk is 1.028 times as heavy.
28. 7.48 gal.
29. .636.
30. 2 T.
31. $\frac{1}{3}$.
32. 12 mi.
33. $45.
34. $18.18.
35. $2.75.
36. 130.8%.
37. 94.4%.
38. 96.
39. 15¢.
40. 21¢.
41. 24.84 bu.
42. 226.2 ft.
43. 720 sq. ft.
44. 22.8 ft.
45. $265.50.
46. $14.
47. First is 116.36% of second.
48. $1.35.
49. First is 88.2% of second.
50. 44.44%; 225%.
51. First is 29.7% of second.
52. $1.88.
53. 6 da.
54. 48 dishes.
55. $37\frac{1}{2}$ ft.
56. 25%.
57. 20,000.
58. 65.45 mi.
59. 27.65 mi.
60. 8%.
61. $36.
62. $18; $16.
63. $4.80.
64. $32.
65. $6.25.
66. 9.2%.
67. $66\frac{2}{3}$%.
69. $\frac{2}{5}$.
70. $150.
71. $12.
72. $45.90.
73. $3.75: $5.
74. $10.50.
75. $28.57.
76. 25 lb.
77. $46\frac{1}{4}$¢.
78. 37.1%.
79. $16\frac{2}{3}$%.
80. $37\frac{1}{2}$%.
81. $65\frac{5}{11}$¢.
82. 90.7 bu.
83. 50%.
84. $225.
85. 92%.
86. (*a*) $103.82; (*b*) $109.86; (*c*) $153.80; (*d*) $153.80.
87. (*a*) $2160; (*b*) $2133.33; (*c*) $80 or 5% of cost; (*d*) 5%.
88. (*a*) $480; (*b*) $400; (*c*) $3000; (*d*) $480; (*e*) $200; (*f*) Yes.
89. 98%.
90. 10%.
91. 4%.
92. 2%.
93. 4%.
94. 150%.
95. $108\frac{1}{3}$%.
96. 85.39%.
97. 177.57.
98. 27.68%.
99. 59¢.
100. Earnings, $2616; living costs, $2092.80.

2 3
3.1416
31
188496
94248
1130976
60
6786.00 (29.30
462
2166
2079
870

2
239
2079

231/2
262

2930

29